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NONLINEAR DYNAMICS OF NORMAL AND OSTEOARTHRITIC HUMAN KNEE

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Abstract: Nonlinear dynamics analyses of the knee joint were performed to estimate the dynamic stability during the flexion and extension motion for humans with osteoarthritis. The experimental data were acquired with a complex goniometer system. Phase plane, time lag, correlation dimension, and the largest Lyapunov exponent were calculated from experimental data. The largest Lyapunov exponents obtained for each test of human knee were positives. The largest Lyapunov exponents for the osteoarthritis patients have, in general, higher values compared with healthy subjects.

Key words: nonlinear dynamics, knee joint, osteoarthritic knee, largest Lyapunov exponents.

1. INTRODUCTION

Osteoarthritis (OA) is the fourth most frequent cause of health problems in women and the eighth most frequent cause in men. About 40 percent of men over the age of 70 years are affected by osteoarthritis of the knee; about 80 percent of patients with osteoarthritis suffer from limited mobility, and 25 percent of these patients can no longer perform the most basic activities of daily life. Knee osteoarthritis involves a degenerative process of the knee joint cartilage, leading to its eventual loss. This degenerative process can be generally caused by obesity, excessive physical activity, joint trauma, immobilization or hyper-mobility. The range of knee flexion angle in the sagittal plane as well as the peak flexion angles are generally lower in patients with knee osteoarthritis [1, 2]. In addition, patients with knee osteoarthritis often report that they feel unstable. Joint laxity refers to static stability, but, taking into account the possibility of falling which can occur during movements, joint laxity also implies problems of dynamic stability [3, 4]. Increased gait variability has been associated with an increased risk of falling in elderly subjects. Stride-to-stride variability in the control of gait is known to be a predictor of falling [5, 6]. Consequently, locomotion analysis is a valuable complementary tool for diagnosis and treatment of orthopedic, muscular and neurological diseases.

Nonlinear analyses have been used in the field of biomechanics to study various aspects of human locomotion, including differences between normal and pathological walking gait, the effects of age and illness and the stability of walking subject to continuous perturbations [5, 7–9]. Furthermore, there is a growing literature on the possible relevance of the Lyapunov exponents to assess movement stability [8–11]. Lyapunov exponents have been used to quantify the local dynamic stability of human walking kinematics [8] and the nonlinear motion of the healthy human knee joint [12]; to measure the balance control of standing in humans [13]; to study the stability of amputees with prosthetic legs [14]; and to quantify the stability of the knee in the sagittal plane [15, 16]. The Lyapunov exponents have been used both in humans and in robots for the study of nonlinear dynamics. The effects of the task to be performed and the change in environment on robot behavior have been studied using experiments and chaos theory [17–19].

Based on analyses using Lyapunov exponents, control techniques have been developed for the biped humanoid robot that moves through human imitation [20]. Experimental results indicate that a deterministic nonlinear gait is possible in a passive dynamic walking robot [21]. This methods could be also used to study the stability of new hybrid robots used in medical field, for minimally invasive surgery [22].

The aim of this paper is to investigate the nonlinear motion of the human knee joint using nonlinear dynamics stability analysis. We compare the dynamics of the flexion-extension knee in a healthy subject group to that of a patient group with OA knees. A quantitative measure of knee stability using Lyapunov exponents can provide the clinical team an essential tool for the diagnosis of movement pathologies, for finding proper treatments and for monitoring patient progress.

2. EXPERIMENTAL STUDY

2.1. Apparatus

The experimental method which allows to obtain the kinematic parameter diagrams for the human knee joint is based on Biometrics data acqusition system with flexible electrogoniometers [8, 23–25]. The flexion-extension process was performed in sagittal plane. A data series for both human knee joints of each OA patient and for one human knee of each healthy subject during flexion-extension repetitive cycles are obtained.

2.2. Participants

In order to obtain the flexion-extension diagram developed by the human knee joints we first analyze seven healthy volunteers subjects, all men, for which experimental data were acquired for walking on the treadmill for three minutes at a speed of 3.6 km/h. They were pain-free, without any evidence or history of arthritic disease, or record of surgery to the lower limbs. The experimental tests were carried out in the Laboratory of Biomechanics from the INCESA – Center of Advanced Research – of the University of Craiova. The osteoarthritis patient group consisted of 5 elderly patients with OA knee, who were prior evaluated in the first or second stage of OA. For the suffering subjects, the experimental test consisted of walking on treadmill for three minutes at a speed of 3.6 km/h. The anthropometric data were collected from the healthy subjects, and from the patients, respectively. Table 1 shows the mean values and standard deviations for anthropometric data of healthy subjects. We noted Hip-Knee distance with H-K dist and Knee-Ankle distance with K-A dist.

Table 1

Mean values and standard deviations
of anthropometric data of healthy subjects

Indicat.	Age [years]	Weight [kg]	Height [cm]	H–K dist [cm]	K–A dist [cm]
Average	31.33	74.55	173.77	44.56	41.11
Standard Deviation	3.94	8.76	7.25	4.61	4.075

Table 2 shows the mean values and standard deviations for anthropometric data of subjects affected by OA.

Table 2

Mean values and standard deviations of anthropometric data of patients with OA

Indicat.	Age [years]	Weight [kg]	Height [cm]	H-K dist [cm]	K-A dist [cm]
Average	53.66	78.5	173	45.17	40.33
Standard Deviation	4.76	16.42	10.47	6.24	6.09

2.3. Procedure

The zero angular position for the knee was defined when the femoral axis and tibial axis are disposed vertically, Fig.1. A time series was constructed for each test joint from the flexion-extension angle. The angular displacement diagram during knee flexion-extension process obtained for a healthy human is shown in Fig. 2.

For characterizing the underlying complexity during movement, the experimental data are analyzed using the Largest Lyapunov Exponents (LLE) and correlation dimensions of angular amplitude in human knee joint; they were used as the dynamic characteristic from the time series of the flexion-extension angle of human knee joint in order to reveal the chaotic characteristic. The LLE is a nonlinear measure that can quantify the exponential divergence of the neighboring movement trajectories. It is a measure of its dependence on initial conditions and of a dynamical system stability. The LLE values for an unstable system that diverges with data that are described by mathematical chaos define complexity, or highly variable fluctuations in physiological processes resembling mathematical chaos, is positive.

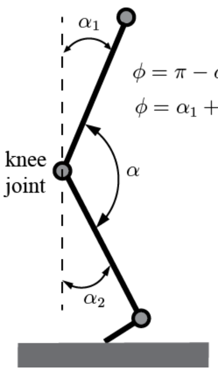

Fig.1 – Knee angle measurements.

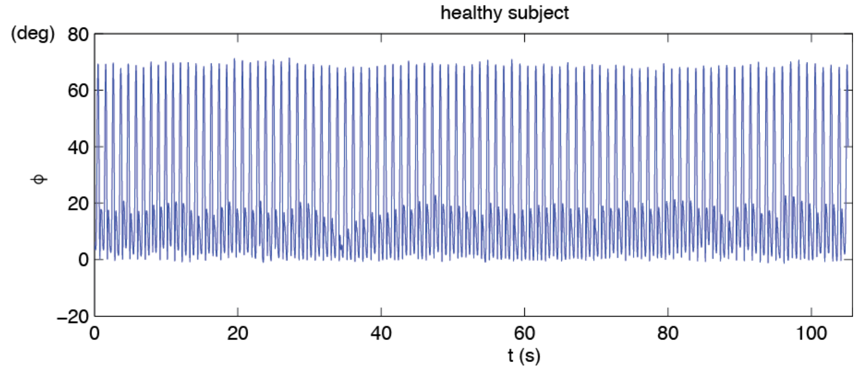

Fig. 2 – Angular diagram of the knee joint for a healthy subject.

In next section, we present the results obtained applying different tools of nonlinear dynamics.

3. LYAPUNOV EXPONENT

The state space S is reconstructed using the delay coordinates vectors [26]:

$$x_n = \left\{ s\left(t_0 + nT_s\right), s\left(t_0 + nT_s + T\right), ..., s\left(t_0 + nT_s + \left(d_g - 1\right)T\right) \right\}, \tag{1}$$

where $s(.)$ is a measured scalar function, T_S is the sampling time, $n = 1,2,...,d_E$, $T = kT_S$ is an appropriately chosen time delay, and d_E is the embedding dimension.
The embedding dimension d_E must be large enough so that the reconstructed orbit does not overlap with itself. The dynamics in the reconstructed state space is equivalent to the original dynamics. An attractor in the reconstructed state space has the same invariants, such as Lyapunov exponents [27].

The false nearest-neighbor (FNN) method, introduced by [28], is one of the most used method for measuring the minimal embedding dimension. The minimum dimension needed to reconstruct the chaotic flow is marked by a vanishing fraction of FNN [29]. The embedding dimension is chosen where this percentage of false nearest neighbors approaches zero [17]. Lyapunov exponents, λ_i, provide a measure of the sensitivity of the system to its initial conditions. They exhibit the rate of divergence or convergence of the nearby trajectories from each other in state space and are fundamentally used to distinguish the chaotic and non-chaotic behavior [17]. If a 3-dimensional state space is considered, there will be an exponent for each dimension: all negative exponents will indicate the presence of a fixed point; one zero and the other negative indicate a limit cycle; one positive indicate a chaotic attractor [8].

In order to characterize the behavior of a dynamical system the sign of Lyapunov exponents must be determined. The value of the LLE is expressed in bits of information/second and is the main exponent that quantifies the exponential divergence of the neighboring trajectories in the reconstructed state space.
For this research the Lyapunov exponents are calculated using the approach defined by [30].

4. RESULTS

The angular amplitudes of human knee flexion-extension (fl-ex) have been obtained for each subject from the report generated by the Biometrics gathering system, as data files type *.txt*. We obtained a total of seven time series containing 17 000 data each, for the fl-ex movement of the each right knee of healthy subjects and 10 time series obtained from both knees of the five OA patients. For the data analysis the beginning and the final regions of time series were cut off in order to remove the transient data. For a more accurate representation of human joint variability, unfiltered data were analyzed in this study. The amplitudes range of knee fl-ex is between 67.22^{O} and 71.29^{O}. The medium amplitude for this healthy subject is 68.346 ^{O}. The medium amplitudes for all healthy subjects are given in Table 3. The medium knee amplitudes of the healthy subjects ranged from 61.444^{O} to 68.756^{O} with their mean value 63.765^{O}, as shown in Fig. 3a.

In Fig. 3b the medium cycle of healthy subject, and of the healthy and OA knees of patient are shown.The medium amplitudes for all patients are given in Table 4.

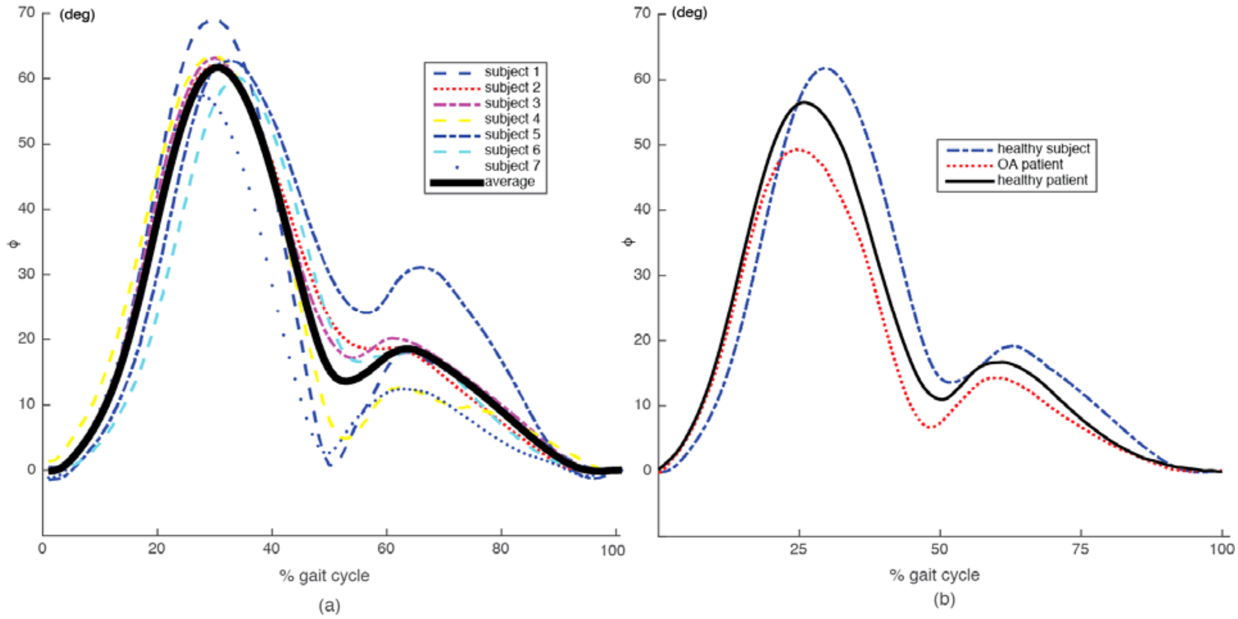

Fig. 3 – a) Averaged knee angle time series for the healthy subjects;
b) averaged knee angle time series for all subjects and patients.

Table 3

Mean amplitude healthy subjects

Subject	1	2	3	4	5	6	7	Mean value
Amplitude [^{O}]	68.756	62.672	63.127	63.484	62.743	64.128	61.444	63.674

Table 4

Mean amplitude for OA patients

Patient	1	2	3	4	5	Mean value
Amplitude OA knee [^{O}]	57.012	51.648	51.727	49.031	47.527	51.389
Amplitude healthy knee [^{O}]	64.232	61.431	60.021	59.557	56.788	60.405

From Tables 5 and Table 6 we can observe that the OA step length is shorter than normal step length and OA speed is slower. This could be a defensive system against the disease and pain. Reduced walking speed and step length denoted decreasing knee moments as an adjusting procedure.

Table 5

Gait measures for healthy subjects

Subject	1	2	3	4	5	6	7	Mean value
Distance [m]	180	176	184	188	197	201	180	186.571
No. of steps	280	284	294	314	302	310	279	294.714
Time [s]	180	180	180	180	180	180	180	180
Velocity [m/s]	1.000	0.9778	1.0222	1.0444	1.0944	1.1167	1.000	1.0365
Step length [m]	0.6429	0.6197	0.6259	0.5987	0.6523	0.6484	0.6452	0.6333

Table 5

Gait measures for OA patients

Patient	1	2	3	4	5	Mean value
Distance [m]	168	165	164	166	159	164.4
No. of steps	316	311	314	304	293	307.6
Time [s]	180	180	180	180	180	180
Velocity [m/s]	0.9333	0.9167	0.9111	0.9222	0.8833	0.9133
Step length [m]	0.5316	0.5305	0.5223	0.5461	0.5427	0.5346

For the patients, the amplitudes of the OA knee ranged from 47.527^{O} to 57.012^{O}, with the mean value of 51.389^{O}, as shown in Fig. 4. For the healthy knee of the patients, the amplitudes ranged from 56.788^{O} to 64.232^{O}, the mean value was 60.405^{O}, Fig. 4b. The maximum fl-ex angles were significantly different for the healthy knees of patients and for the OA knees of patients ($p = 0.0025 < 0.05$). The maximum flexion angles were not significantly different for the healthy knees of patients and for the knees of healthy subjects ($p = 0.0568 > 0.05$). The flexion angle during the gait cycle revealed differences with respect to flexion amplitude between the OA patients and the healthy subjects. The OA subjects had less range of motion during the gait cycle than the healthy subjects. A large difference between the amplitude of knee flexion during 25–50% of gait cycle phase and 65–80% of gait cycle phase is revealed. The statistical analysis revealed that the OA knees were less flexed throughout the gait cycle than the healthy subjects were ($p = 0.000051$) but, also, the healthy knees of the patients were less flexed than of the healthy subjects. This is explained by the influence of the OA knees pain and by body tendency of maintaining the stability when reaching the knee flexion amplitude.

Phase plane portraits can be used to characterize the kinematics of the system and provide a better understanding of the steady state dynamics. The phase plane plot is a two-dimensional plot in which the time derivative $d\phi$ is plotted versus ϕ at each data point. The phase plane plots are shown in Fig. 4. For the healthy human subjects, Fig. 4a, no structure is evident in the plot. For periodic data the phase plane plot is a closed curve. The phase plane plot for OA knee is shown Fig. 4b and the phase plane plot for an OA patient for the healthy knee is shown in Fig. 4c. The phase plane diagrams for OA patients look more irregular than the phase plane diagram for healthy subjects. Next for all the subjects we apply a Fast Fourier Transform algorithm for the angular positions. Random and chaotic data give rise to broad spectra (Fig. 5). The periodic data produce a dominant peak in the spectrum.

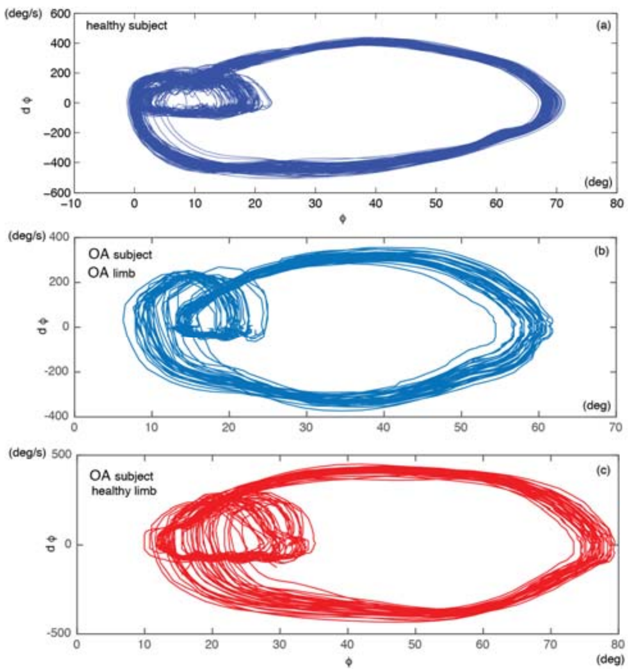

Fig. 4 – Phase-plane portraits.

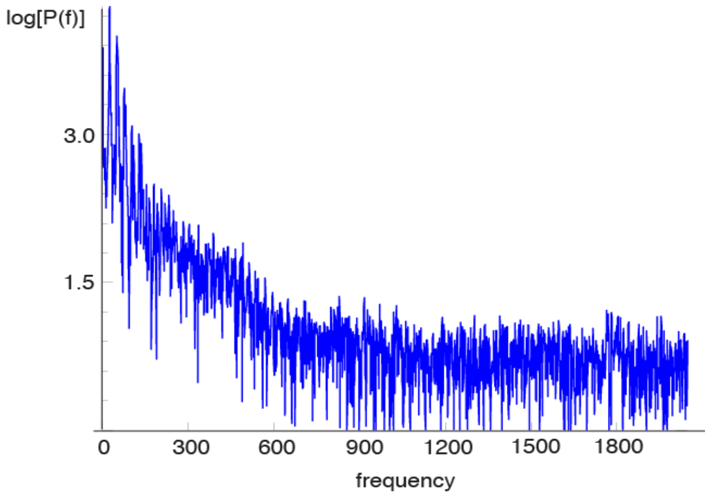

Fig. 5 – Fast Fourier Transform (FFT) for the angular position of an OA subject.

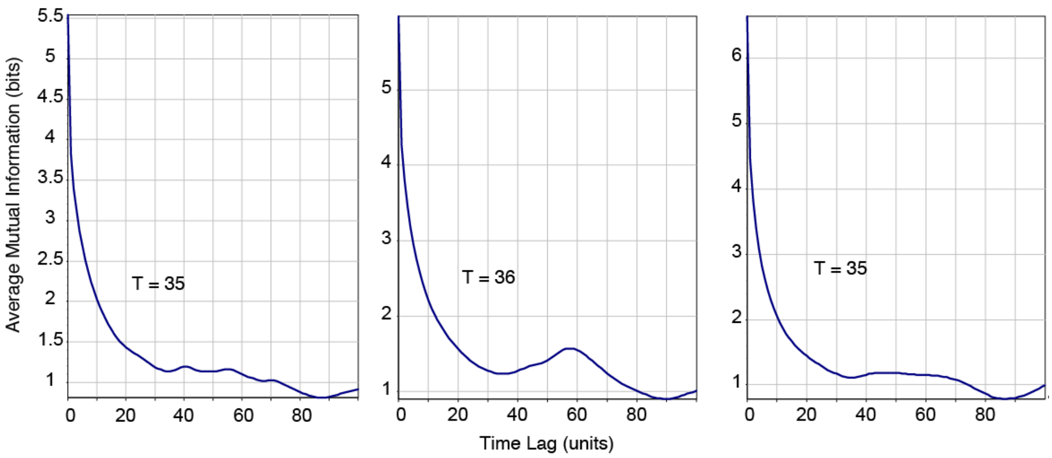

Fig. 6 – Time lag, T, for three healthy subjects.

An appropriate time lag, *T*, was determined by using the average mutual information (AMI) function which evaluates the amount of information shared between two data sets over a range of time delays. This criterion sets the time lag equal to the value of delay corresponding to the first minimum of the AMI function. Figure 6 shows the time lags calculated for three healthy subjects.

A suitable embedding dimension was chosen by using the false nearest neighbour (FNN) method. Embedding dimension is the minimum value that trajectories of the reconstructed state vector may not cross over each other in state space. Total percentage of false neighbours for the human knee was computed by FNN method, and the number of dimensions was chosen where this percentage approaches zero. The results were similar for all seven time series and indicated an appropriate embedding dimension of $d_E = 5$.

In this research we calculated the LLE using the method of Rosenstein *et al*. [30]. The results can provide the clinical and design, an essential mean to diagnose gait pathologies, administer proper treatments, and monitor patient progress. The LLE is a measure of the rate at which nearby trajectories in phase space diverge. The LLE calculated for all time series were positive. Figure 7 shows the average logarithmic divergence defined by Rosenstein *et al*. [30], solid curve, function of time for a healthy subject. For this curve there is a linear region that is used to calculate the Lyapunov exponent and the curve saturates for longer times. The linear best-fit line to the curve obtained by the average logarithmic divergence is the dashed line in Fig. 7. The LLE was approximated from the experimental knee joint angle data as the slope of the linear best-fit line to the curve (the slope of dashed line). For an OA patient the linear best-fit line to the average logarithmic divergence is the dotted line Fig. 7. Figure 8 shows the LLE for the knee joint of all seven healthy subjects.

The minimum value for LLE was 2.228, the maximum value was 2.607 and the mean value was 2.418.

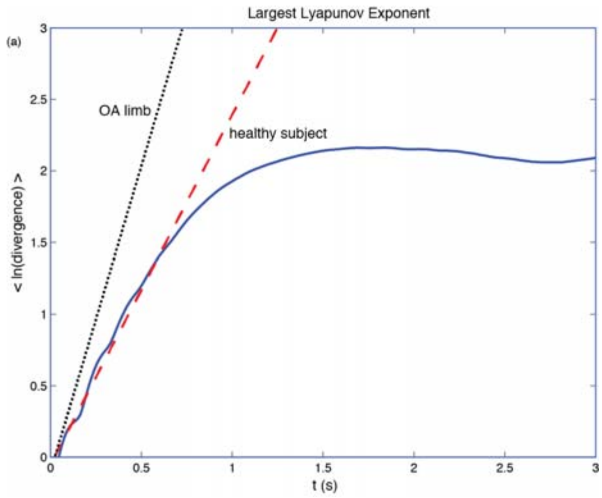

Fig. 7 – Average logarithmic divergence function of time.

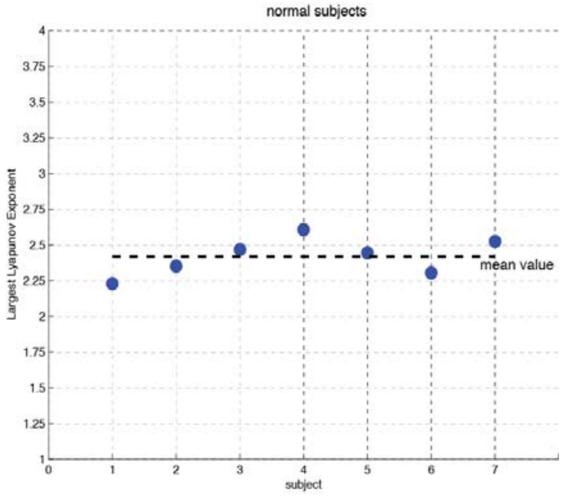

Fig. 8 – Lyapunov exponents for the seven subjects.

Figure 9 shows the LLE for all the OA patients and the mean value for the healthy subjects. For the OA patients we observe higher values than the mean value for the healthy subjects. The minimum value for OA patients is 2.922 and the maximum value is 5.080. For the patients the OA knee has larger value for the LLE than the healthy patient knee. Subject 1 makes exception from this rule.

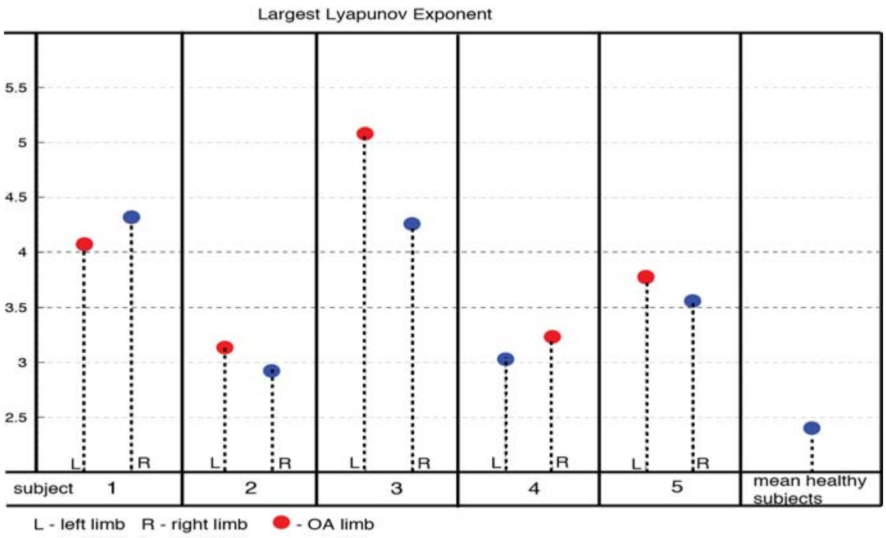

Fig. 9 – Lyapunov exponent for all the patients
and mean value for healthy subjects.

5. CONCLUSIONS

In this research we presented an analysis for a bipedal system with osteoarthritis in the knee joint. The paper is intended to be used in robotics, in rehabilitation and in the medical field for prosthetic devices. The kinematic data of the fl-ex angles for human knee motion were analyzed. We applied the nonlinear analysis to the fl-ex movements of the human knee. The LLE obtained for each test of human knee were positive values. We observed the increasing of the LLE for the OA patients with respect to healthy subjects. The ability to analyze the nonlinear dynamics on individuals and robots has the potential to provide insight into normal and abnormal motions in humans, and a better design for robotics used for rehabilitation.

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