THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Special Issue/2018, pp. 273–278

ON THE DESIGN AND OPTIMIZATION OF CONSTRUCTAL NETWORKS OF HEAT EXCHANGERS BY CONSIDERING ENTROPY GENERATION MINIMIZATION AND THERMOECONOMICS

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Abstract. This study aims to integrate in constructal networks of heat exchangers (HEXs) two ways of optimization: entropy generation minimization and thermoeconomics. We obtained a complete (non-simplified) analytic solution of the entropy generation along of the HEX that allows exact evaluation of the irreversible processes in smooth or augmented tubes. By using this analytic solution, we proposed an adapted augmentation entropy generation number and compared it with Bejan's original formulation. We developed a thermoeconomic model of a HEX for periods of operation of 1 to 10 years, for which we obtained specific optimum points of cost. Representing the cost *versus* entropy generation opens the path for future advanced comparisons of complementary objectives that may conduct to balanced designs: thermal process optimum *vs* economical optimum. We outlined a new methodology of analysis of the constructal tree networks based on the network laws and thermo-hydro-electric analogy. The proposed methodology is characterized by compactness, a higher degree of abstraction of the problem and allows further generalizations, making it suitable for advanced objectives like optimization, irreversibility analysis, sensitivity analysis or inverse problems.

***Key words*:** Entropy generation, Thermoeconomics, Constructal theory, Heat exchangers with corrugated tubes optimization, Network analogy.

1. INTRODUCTION

Problems of heat transfer are present in most of the designs of energy systems. While for all thermal systems the first level of analysis consists in finding the solutions for the equations of process that condition the operation of the system, a second level of analysis is often concerned with optimization (Fig. 1). The common design of a system may suffer a qualitative evolution when objectives of thermal or/and economic efficiency are concerned (Fig. 1).

Constructal theory integrates such instruments considering that the design evolution is one of its bases. Bejan's approach [1, 2] introduced the concepts of analysis based on the criterion of entropy generation of the design variation towards an augmented one. In this regard, in this article we propose an adapted augmentation entropy generation number which, as a consequence of a new analytical solution for entropy generation, will complete Bejan's original formulation. Additionally, a thermoeconomic model of a HEX developed through the constraint equations of process will reveal how the cost function varies versus entropy generation.

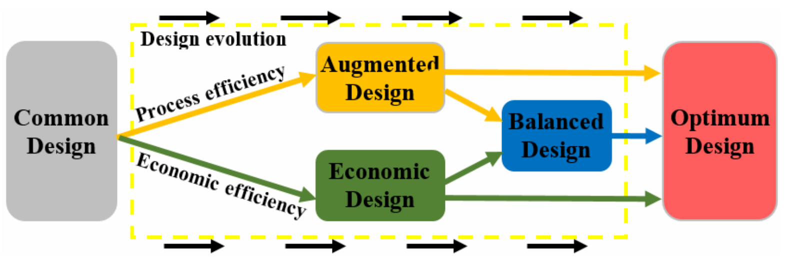

Fig. 1 – Criteria of optimization for the design of the thermal systems.

2. ANALYTIC INVESTIGATIONS AND RESULTS REGARDING THE ENTROPY GENERATION IN HEXS

2.1. Adapted augmentation entropy generation number: analytical investigation

In what follows we describe an adaptation of Bejan's formulation of the augmentation entropy generation number that avoids several simplifications or approximations. The new approach will be tested on a HEX with corrugated tubes of 25 kW thermal power. Several researches [3] considered, for similar thermal systems, different criteria (constant pumping power, fixed or variable geometry, etc). Bejan's original definition of the augmentation entropy generation number [2, 4] considers for the augmentation design transition the spatial partial derivative of the entropy generation, $N_S = S'_{gen,a} \big/ S'_{gen}$, that is a mathematical compromise in favor of simplicity. In this way, we evaluate that the consideration of an adapted augmentation entropy generation number $\left(N_S^*\right)$ will eliminate the existing insufficiency, reductionism as it contains a complete (non-altered) integration along the length of the HEX:

$$N_S^* = \frac{S_{gen,a}}{S_{gen}} \neq \frac{S'_{gen,a}}{S'_{gen}} = \frac{\partial S_{gen,a} \big/ \partial x}{\partial S_{gen} \big/ \partial x}. \tag{1}$$

The differential equation of entropy generation along a HEX with constant wall temperature of the tubes is:

$$\mathrm{d}\dot{S}^{gen}(x) = \frac{\partial \dot{S}^{gen}}{\partial x}\mathrm{d}x = \frac{Q_w}{NLT_w}\cdot\frac{\Delta T_w(x)\mathrm{d}x}{T(x)} + \frac{32 f\dot{m}^3}{\pi^2\rho^2 D^5}\cdot\frac{\mathrm{d}x}{T(x)} = \frac{Q_w}{NLT_w}\cdot\frac{\Delta T_w(x)\mathrm{d}x}{T(x)} + \frac{\pi\rho v^3 f\,\mathrm{Re}^3}{2D^2}\cdot\frac{\mathrm{d}x}{T}, \tag{2}$$

where the component variables of the equation are explained in Nomenclature. Integrating eq. (2), the entropy generation will be:

$$\dot{S}^{gen} = \int_0^L \mathrm{d}\dot{S}^{gen}(x) = \frac{Q_w}{NT_w}\overline{\left(\Delta T_w / T\right)} + \frac{\pi\rho v^3 L f\,\mathrm{Re}^3}{2D^2}\overline{\left(1/T\right)}. \tag{3}$$

The explicit relationship of the entropy generation along of a HEX with smooth tubes will be:

$$\dot{S}^{gen} = \underbrace{\frac{\mathrm{Pr}\cdot D\cdot\dot{Q}\cdot\mathrm{Re}}{4\cdot N\cdot L^2\cdot\mathrm{Nu}}\cdot\frac{1}{T_w}\ln\left[\frac{T_w}{T_1} - \frac{T_w}{T_1}\mathrm{e}^{-\frac{4\cdot L^2\cdot Nu}{D\cdot\mathrm{Re}\cdot\mathrm{Pr}}} + \mathrm{e}^{-\frac{4\cdot L^2\cdot\mathrm{Nu}}{D\cdot\mathrm{Re}\cdot\mathrm{Pr}}}\right]}_{\dot{S}^{gen}_{\Delta T}} + \underbrace{\frac{\pi\rho v^3\,\mathrm{Pr}\cdot f\cdot\mathrm{Re}^4}{8\cdot D\cdot L\cdot\mathrm{Nu}}\cdot\frac{1}{T_w}\ln\left[1 - \frac{T_w}{T_1} + \frac{T_w}{T_1}\mathrm{e}^{\frac{4\cdot L^2\cdot\mathrm{Nu}}{D\cdot\mathrm{Re}\cdot\mathrm{Pr}}}\right]}_{\dot{S}^{gen}_{\Delta p}}. \tag{4}$$

In a similar manner in [5] an equivalent form of (4) is obtained, but for different objectives. The adapted irreversibility distribution ratio will be:

$$\phi^* = \frac{\dot{S}^{gen}_{\Delta p}}{\dot{S}^{gen}_{\Delta T}} = \frac{\pi\rho v^3\cdot N\cdot L\cdot f\cdot\mathrm{Re}^3}{2\cdot D^2\cdot\dot{Q}_w}\cdot\ln\left[1 - \frac{T_w}{T_1} + \frac{T_w}{T_1}\mathrm{e}^{\frac{4\times L^2\cdot\mathrm{Nu}}{D\cdot\mathrm{Re}\cdot\mathrm{Pr}}}\right] / \ln\left[\frac{T_w}{T_1} - \frac{T_w}{T_1}\mathrm{e}^{-\frac{4\cdot L^2\cdot\mathrm{Nu}}{D\cdot\mathrm{Re}\cdot\mathrm{Pr}}} + \mathrm{e}^{-\frac{4\cdot L^2\cdot\mathrm{Nu}}{D\cdot\mathrm{Re}\cdot\mathrm{Pr}}}\right]. \tag{5}$$

The adapted augmentation entropy generation number $\left(N_S^*\right)$ as a ratio between the total quantities of entropy produced from the fluid flow and heat exchange along the HEX is:

$$N_S^* = \frac{\dot{S}^{gen,a}}{\dot{S}^{gen}} = \frac{1}{1+\phi^*}N^*_{S,\Delta T} + \frac{\phi^*}{1+\phi^*}N^*_{S,\Delta p}, \tag{6}$$

where $N^*_{S,\Delta T} = \dot{S}^{gen,a}_{\Delta T} / \dot{S}^{gen}_{\Delta T}$ and $N^*_{S,\Delta p} = \dot{S}^{gen,a}_{\Delta p} / \dot{S}^{gen}_{\Delta p}$ when a smooth-to-augmented design transition is considered. We consider that N_S^* as an adapted formulation adds improvement to N_S because the approximations are avoided in the estimation of the irreversible processes. A numerical comparison between the two variables will be performed in what follows.

2.2. Adapted augmentation entropy generation number: numerical evaluation

In this section we surveyed the efficiency of a HEX with corrugated tubes and we identified the better augmentation solution. Thus, we used the aforementioned analytical advancements and we performed a comparison between Bejan's original formulation and an adapted formulation of the augmentation entropy generation number. Figure 2 illustrates the effect of the augmentation techniques from two perspectives: Bejan's original formulation and the adapted Bejan formulation. A sensible difference of the results of the two formulations can be observed. At high Reynolds numbers of the flow regime Bejan's original formulation shows a slight increase of the N_S number while the N_S^* number appears relatively constant. It is also noticeable that Bejan's original formulation of the entropy generation number tends to overestimate the irreversibility effect of the augmentation (corrugation), most probably due to its simplifications.

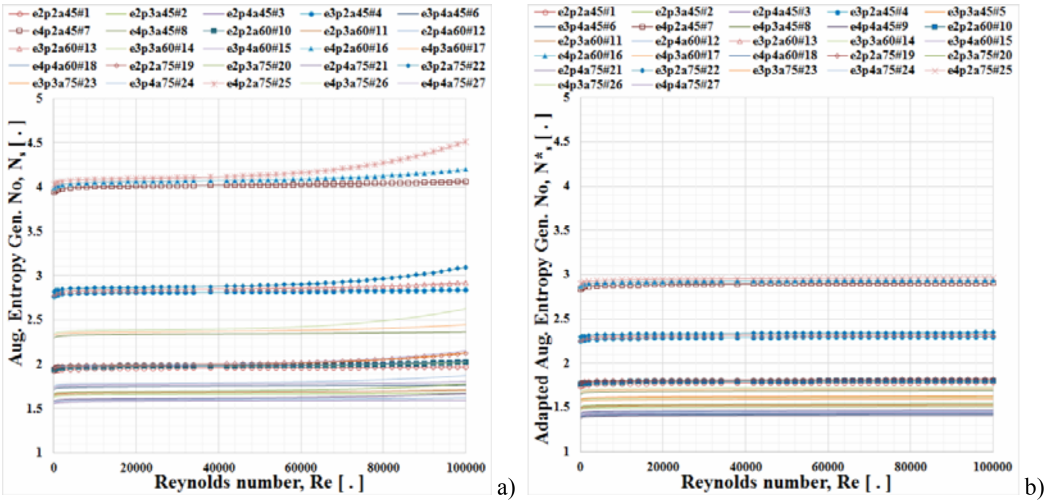

Fig. 2 – Augmentation entropy generation number characterizing the design transition:
a) Bejan's original formulation; b) adapted formulation.

2.3. Optimization using the thermoeconomic approach

While the thermoeconomic optimization is focused on cost function, some connections with the entropy generation that is strictly linked to the thermal process can be made. The economic optimum will recommend different solutions than the second law optimum, yet a direct comparison between those independent criteria can be done in a common diagram and a balanced design can be obtained. In several works [7–11] various ways of performing cost optimization are described, and often the cost is related to exergy destruction [7, 8]. We developed the criteria of cost optimization of a heat exchanger with smooth tubes (the thermal power is 25 KW) by considering different periods of operation (1…10 years). The total cost function of the investment cost and operation cost is:

$$C = C_{inv} + C_{op} = \underbrace{c_M \times \rho_S \times \delta \times S}_{investment} + \underbrace{c_e \times \mathrm{PP}_e \times \tau_{OT}}_{operation}, \tag{7}$$

where τ_{OT} is the operation time (of 1…10 years). The constraints of the process of flow and heat transfer are used. The thermoeconomic model of the HEX (Fig. 3) represents a field of design options that satisfy the phenomenological constraints of the process of flow and heat exchange.

Unlike the optimum of the thermal process, the optimum of cost acknowledges a relative status as the extremum point can widely vary with the unit costs afferent to the HEX. An economic scheme guarantees that the design is opportune in an economic environment and the second law optimization satisfies the preoccupation for conservation of useful energy. Both paths are complementary and recommend in most cases a balanced decision of design. We observed that the increase of period of operation coincides with the deviation of the point of optimum cost towards a higher entropy generation. At this stage the conditions for the implementation description of both optimization tools in a constructal network of heat exchangers can be prepared.

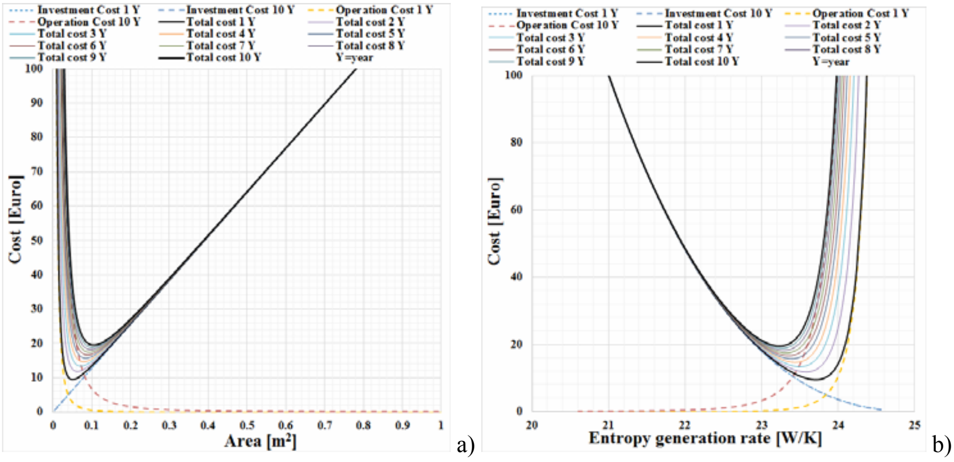

Fig. 3 – Cost diagram of the thermoeconomic model of the HEX for different operation periods (1…10 years): a) the cost-area diagram; b) the cost-entropy generation diagram.

3. A PATH OF ANALYSIS OF CONSTRUCTAL NETWORKS THROUGH THE NETWORKS THEORY

This section prepares a methodology of analysis of tree-shaped constructal networks where the laws of network are implemented through matrix equations. Constructal networks have many similarities with the networks of heat exchangers but with the addition of general concepts regarding the design evolution, objectives of equilibrium or optimization, scaling rules, flow architecture exploration concerning point-area or point-volume performance criteria of flux access.

Several papers [12–14] used the principles of the constructal theory for solving tree flow structures with the aim of process or economic optimization. Their results are representative for simple systems, but the classical approach meets a conceptual limit for high levels or factors of branching. Thus, the necessity of advanced methodologies that use the laws of networks in matrix formalisms appears. We intend to define the matrix relations for the constructal tree networks in Fig. 4 that have their specific incidence matrices.

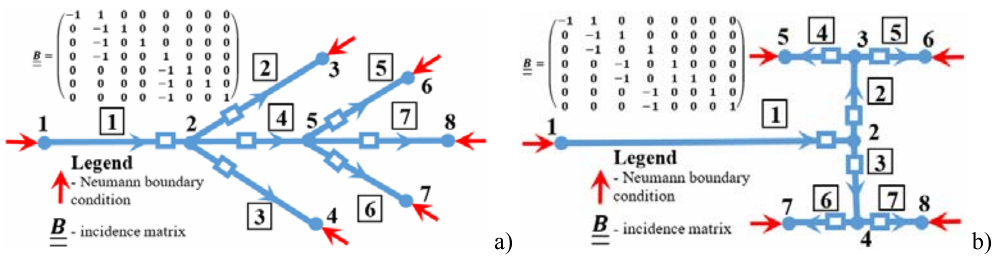

Fig. 4 –Constructal tree network of mass and heat exchange: a) simple network associated to a central trunk; b) network with a branching factor of two

Along the network, the temperatures and pressures are nodal potentials, while the flow rates and heat rates are fluxes. The metrics of the connected unidirectional graph (G) associated to the tree network (Fig. 4) are: $n = 8$ nodes; $e = n - 1 = 7$ edges; $L = 0$ loops (cycles or fundamental cycles) because $n - e + 1 = 0$ that is specific for tree graphs with no cycles; NST $= n - 1 = 7$ spanning tree edges. The number of independent equations (i.e. the rank of the graph) provided by the Kirckhhoff Current Law (KCL) from the total of n nodes is $N_{KCL} = n - 1 = 7$ [15]. Considering a matrix formalism, the pressure difference has the following form (the other variables are defined in a similar manner) [16]:

$$\underline{\Delta p} = \left[\Delta p_1, \Delta p_2, \Delta p_3, \ldots, \Delta p_n\right] = \underline{\underline{B}} \cdot \underline{p}\,. \tag{9}$$

Energy conservation according to the first law [17] involves the balance of heat rates of the Neumann boundary conditions:

$$\underbrace{\frac{\mathrm{d}E}{\mathrm{d}\tau}}_{=0}-\sum_i \dot{Q}_i+\underbrace{W}_{=0}-\sum_{in}\dot{m}_{N,in}h_{N,in}+\sum_{out}\dot{m}_{N,out}h_{N,out}=-\sum_i \dot{Q}_i-\underbrace{c_p\sum_j \dot{m}_{N,in}T_{N,in}}_{\dot{Q}_{N,in}}+\underbrace{c_p\sum_k \dot{m}_{N,out}T_{N,out}}_{\dot{Q}_{N,out}}. \tag{10}$$

According to the second law, the entropy generation [17] on the entire network will be:

$$\dot{S}_{gen,tot}=\underbrace{\frac{\mathrm{d}S}{\mathrm{d}\tau}}_{=0}-\sum_i \frac{\dot{Q}_i}{T_i}-\sum_{in}\dot{m}_{N,in}s_{N,in}+\sum_{out}\dot{m}_{N,out}s_{N,out}{}^{3}0. \tag{11}$$

The exergy balance of the system by considering the Neumann boundary conditions is:

$$\begin{aligned}&\underbrace{\frac{\mathrm{d}e}{\mathrm{d}\tau}}_{=0}-\dot{E}_Q+\underbrace{\dot{E}_W}_{=0}+\dot{E}_D-\sum_{in}\dot{m}_{N,in}e_{in}+\sum_{out}\dot{m}_{N,out}e_{out}=\\&=\underbrace{\frac{\mathrm{d}e}{\mathrm{d}\tau}}_{=0}-\sum_i \dot{Q}_i\left(1-\frac{T_0}{T_i}\right)+\dot{E}_D-\sum_j \dot{Q}_{N,in}\left(1-\frac{T_0}{T_{N,in}}\right)+\sum_k \dot{Q}_{N,out}\left(1-\frac{T_0}{T_{N,out}}\right)=0.\end{aligned} \tag{12}$$

When the system is determined after considering the previous equations, strategies of optimization can be applied, regarding the entropy generation, cost, flow path, flow resistance, surface-to-point or volume-to-point resistance [18], etc. Sensitivity analysis and inverse boundary value problems can be also explored in specific conditions. The process optimization will consist in minimizing the total entropy generation:

$$\dot{S}_{gen}=\sum \dot{S}_{gen,i}=\dot{S}_{gen}(\mathrm{Re}_1\ldots\mathrm{Re}_n;\ \ D_1\ldots D_n;A_1\ldots A_n)=\text{minimum}. \tag{13}$$

At the same time, the thermoeconomic optimization of the network will involve the minimization of the cost function of economic operation:

$$C_{tot}=C_{tot}(\mathrm{Re}_1\ldots\mathrm{Re}_n;D_1\ldots D_n;A_1\ldots A_n)=\text{minimum}. \tag{14}$$

We presented a methodology of analysis for tree-shaped constructal networks that can be further developed for complex cases. Its advantage consists in compactness of the algorithm and it is applicable for networks with multiple nodes and branches.

4. CONCLUSIONS

In this paper we defined new methodologies of performance evaluation and optimization of heat exchangers (HEX) by considering entropy generation minimization and thermoeconomics. We illustrated the method of implementation for such objectives in constructal networks using matrix equations.

We used the analytic solution of integration of the entropy generation on the domain of a HEX to define the adapted augmentation entropy generation number (N_S^*). This approach does not fundamentally change the original augmentation entropy generation number (N_S) defined by Bejan, yet it offers the possibility of exact evaluation of the impact of augmentation techniques. A comparison of the two suggested a difference of evaluation that ranges between 7% and up to 55% when the level of corrugation of the surface of the tubes is ample. Bejan's original formulation still remains more practical due to its simplicity, but we recommend the adapted formulation for important design objectives. We identified the most efficient design for different sizes of augmentation of the corrugated tubes and the different recommendations justify the necessity of using the adapted formulation as an alternative to the original one.

The thermoeconomic analysis of HEXs offers different perspectives regarding the interpretation of efficiency, as the quantified thermal processes are combined with cost objectives. A comparison of the cost function versus entropy generation suggests that the increase of operation period corresponds to the deviation of the position of the optimum point towards higher entropy generation of the process associated to a higher irreversibility. The second law and thermoeconomic optimizations offer the instrumental conditions to approach the objectives of a constructal network. We obtained meaningful matrix relations associated to the hydraulic and thermal phenomena in order to model and solve the contructal tree networks. Through this holistic and compact methodology we provided a full determination of the system. Furthermore, the methodology allows for advanced objectives of optimization, sensitivity analysis or inverse problems to be pursued.

The main advantage of this approach is the level of generalization that allows an overall description of the evolution of the system. It opens possibilities to multidisciplinary surveys and further developments.

Nomenclature

$\dot{Q}$ – heat rate	e – exergy (corrugation) height	N – no. of tubes; no. of nodes	δ – wall thickness
$\dot{S}$ – entropy generation rate	E – exergy, number of edges	Nu – Nusselt number	D – (exergy) destruction
$\dot{m}$ – mass flow rate	f – fanning factor	Re – Reynolds number	e – electrical
$\underline{\underline{B}}$ – incidence matrix	L – length	T – temperature	w – wall
c – unit cost	p – (corrugation) pitch; pressure	$\dot{W}$ – mechanical power	gen, generation
C – total cost	Pr – Prandtl number	ϕ – irrev. distribution ratio	tot, total
D – diameter	PP – pumping power	ρ – density	s – steel

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