ACADEMIA ROMÂNĂ

SCOSAAR

HABILITATION THESIS

OPTIMAL CONTROL, VARIATIONAL AND ASYMPTOTIC METHODS FOR THE STUDY OF REAL WORLD PROBLEMS

ABSTRACT

Domeniul fundamental: MATEMATICĂ ŞI ŞTIINŢE ALE NATURII

Domeniul de abilitare: MATEMATICĂ

Autor: Cercetător ştiinţific I dr. RUXANDRA-MARINA STAVRE

Teză elaborată în vederea obţinerii atestatului de abilitare în scopul conducerii lucrărilor de doctorat în domeniul MATEMATICĂ.

BUCUREŞTI, 2018

ABSTRACT. This work presents the main scientific results obtained after defending the Ph. D. Thesis in the research field: mathematical modeling and application of variational, asymptotic and optimal control methods in the theoretical study of various practical problems arising from medicine, biology, engineering sciences. The reference list contains 50 works published either as sole author or in collaboration. Among these, 5 were published before the Ph. D. thesis, 42 after the Ph. D. thesis and 3 are in preparation.

For highlighting the continuity of my research activity with respect to the topics of interest, I begin the presentation with a short discussion concerning the results published before the Ph. D. thesis, [62]-[66]. The first results of my scientific activity are related to the study of some free boundary problems. Free boundary problems present further difficulties because of the presence of an additional unknown: the free boundary. An important problem of this type is known in the literature as "the dam problem". In the early 70's, the article [8] introduces a modern method for the study of the dam problem, method based on the variational inequalities, a new mathematical tool at that time. The idea proposed in this article was successful and it was followed by an important number of similar articles, such as: [4], [9], [14], [17], [28]. In 1985 I obtained the first significant results for the dam problem, in [64]. This work was cited, among other authors, by Prof. Avner Friedman, author of hundreds of articles in the field of free boundary problems, in [38]. After almost 30 years after the publication, the article [64] is still cited (see e.g. [30], [45]).

The present habilitation thesis contains 4 chapters and references. Chapters 1, 2, 3 present the most important results obtained and published after the defendence of the Ph. D. thesis, classified by topics. The last chapter deals with a short presentation of my research intentions in the near future. The complete reference list contains 140 works, 50 of them representing the results that I obtained either as sole author, or in collaboration.

Chapter 1 is devoted to the presentation of some free boundary problems. It contains results that continue the Ph. D. thesis topics, published in [67], [68], [69]. If previously we talked about the free boundary problem named "the dam problem", in [67]-[69] we introduce another model of free boundary problem: the fluid flow toward a porous wall. The problem introduced in [67] is a new one, different from those studied in the literature until then, that have analyzed jet flows, either with classical methods such as hodographic

method (see, e.g., [15], [37], [39]), or with variational methods ([5], [6]). I studied the jet flow toward a porous wall, by transforming the problem into a minimum problem. I obtained existence, uniqueness and monotonicity results. The most important idea of the article is that we succeed to prove properties of the unknown flow domain by means of a suitable choice of the test functions in the minimum problem. More precisely, physical intuitive properties are rigorously proved with pure mathematical arguments. In [68], [69] we generalize the results of [67] for the cases of the impact of a jet with two immiscible fluids on a porous wall and, respectively, a jet incident on a porous wall in a gravity field.

Chapter 2 presents optimal control problems, most of them associated with models from real world. From the rich literature dealing with optimal control, it is very difficult to select the most suggestive examples. We mention first some general works on optimal control theory such as [42], [29], [43], [47], [11]. Since we are interested in optimal control models constructed as direct applications to practical problems we give some articles with similar topics: [1], [12], [13], [10], [7], [46].

The results presented in this chapter were published in 15 articles and 1 book chapter: [22]-[26], [70]-[74], [18], [20], [78]-[80] and [75]. In [23], [24] we study from the theoretical and, respectively, numerical point of view a boundary control problem associated with stationary Navier-Stokes equations coupled with heat equation, considering for the temperature a general boundary condition of Robin type. The control problem consists in determining a temperature of the surrounding medium that leads to a desired configuration of the fluid temperature. In [23] we prove the existence of an optimal control and we obtain the necessary conditions of optimality by introducing a family of penalized optimal control problems. Using a finite element method, we introduce the discrete optimality system in order to determine numerically an optimal control and we prove the convergence of the proposed algorithms in [24].

The problem studied in [25] has direct applications in biology. Starting from the model of biconvective flow presented in [41], we introduce and study in [25] an optimal control problem related to biconvective flow. We establish an existence and uniqueness result for the stationary biconvective flow. The method we use allows us to obtain the existence of a solution with less restrictive assumptions than in [40]. Moreover, we study a control problem with the aim of optimize the microorganisms concentration in the culture fluid.

This article was cited 43 times (according to Google Scholar); we mention: [2], [3], [61].

Another work inspired directly from an effective physical problem is [26]; in this article we study from the theoretical and numerical points of view an optimal control problem in heat propagation. This problem corresponds to a situation when some physical quantity of interest, not directly accessible to measurement, has to be estimated indirectly using measured data for other, accessible, quantities. This article was cited in works as: [16], [60].

In [72] we study another optimal control problem associated with the nonlinear coupled system: Navier-Stokes and heat equations. The choice of the cost functional leads to an adjoint system which is not divergence free. For overcoming this difficulty, we propose a method based on the construction of several suitable functions.

The articles [73], [74] present optimal control problems associated with the micropolar fluid flows. The theory of micropolar fluids introduced in [35] allows the study of some physical phenomena which cannot be described by the classical Navier-Stokes equations.

In [78], [80] we propose a mathematical model for the blood flow optimization in venous insufficiency. When a leg vein loses its elasticity, phenomena as stagnation and recirculation of the blood may appear; these phenomena produce medical complications. We propose a mathematical model in order to diminish the negative consequences of the lack of vein elasticity. We begin with a simplified model in [78] and then we generalize the obtained results in [80].

In many practical problems, different physical characteristics or different solution behaviors are coupled across interfaces. Such situations arise in fluid-fluid or fluid-structure interaction problems, where two domains corresponding to different fluids or to a fluid and a deformable solid are separated by an interface. In [79] we study a mathematical model and an optimal control problem associated with the interaction between two non miscible fluids in a porous medium when on their interface there exists a jump of temperature. A physical motivation of this model is represented by the groundwater contamination by organic liquids or the phenomena that appear in a PEM fuel cell.

In the framework of research grants CEEX 189/2006 and CEEX 320/2006 we proposed and studied mathematical models for describing the complex phenomena that appear in a polymer electrolyte membrane (PEM) fuel cell. A PEM fuel cell is one of the most interesting new energy sources; it consists of several layers of basic cells. In these cells, a proton conductivity membrane separates the anode and the cathode. During the oxidation process, the hydrogen dissociates into protons and electrons and the results of this process are water, heat and electric energy. The results obtained for the mathematical models associated with these phenomena were published in [18]-[21].

From the 16 works described above, we selected for presentation in Chapter 2 the articles [20], [79] and [80].

Chapter 3 presents asymptotic methods in the study of micropolar fluid flows and fluid-structure interaction problems. The results were obtained during my collaboration of almost 20 years with Prof. Grigory Panasenko from the University Jean Monnet, Saint-Etienne, France, one of the most important specialists of the world in asymptotic methods. This collaboration started in the framework of the international grant Eurrommat No. ICA1-CT-2000-70022, to which I was project leader and materialized in 18 published works and 3 in preparation.

In [31], [32] we introduce the asymptotic analysis of a micropolar fluid flow through a wavy periodic tube, with the period and the thickness of order ε, where ε is a small parameter. The main results announced in [31] are presented and proved in [32]. The solution of the physical problem is approximated by an asymptotic expansion and the error estimate justifies this asymptotic construction. Then the asymptotic partial decomposition of the domain(see [48], [49]) is applied for our problem. The article [33] represents a generalization of [31], [32]. In [33] an asymptotic analysis of a micropolar fluid flow in a structure with one bundle of tube is performed. The asymptotic analysis for a micropolar fluid flow in a more general, and consequently, more complicated case is presented in [34]. We consider a micropolar flow in a curvilinear channel, depending on a small parameter ε. Using a suitable change of coordinates, we replace the initial problem with a more complicated one, with variable coefficients, but set in a rectangle. We construct the asymptotic solution using the boundary layer method. The main difficulty in the study of this problem is determined by the fact that the problem satisfied by the exact solution has variable coefficients which depend on the small parameter ε while the problems satisfied by the boundary layer functions must have coefficients independent of ε, since the solution of these problems must be independent of ε. Another difficulty is that the asymptotic solution is not divergence free and we have to construct a divergence free function which verifies some suitable boundary conditions.

The article [51] represents the first of a sequence of works devoted to the asymptotic approach for fluid-structure interaction problems. A non steady-state viscous flow in a thin channel with an elastic wall is considered. The problem depends on two small parameters. For various ratios of these two small parameters, an asymptotic expansion of a periodic solution is constructed and justified by a theorem on the error estimates. In [52] we generalize the results from [51] for the non periodic flow. There is a major difference with respect to the previous work, since in the periodic case the asymptotic solution verifies the same boundary conditions as the exact solution, while in the non periodic case the trace of the asymptotic solution on the lateral boundaries is, in general, different from that of the exact solution. This leads to the necessity of introducing boundary layer functions and the problem they satisfy is not standard. The theoretical approach of this case is completed with some numerical results in [50]. The numerical simulations confirm the theoretical results highlighting the boundary layer formation near the entry sections. For some of the last articles I was awarded with the Romanian Academy Prize "Spiru Haret" for 2008.

The purpose of [53] is to extend the previous results to a more interesting case from the physical viewpoint. This article presents the asymptotic analysis of a Stokes fluid flow with variable viscosity in a thin channel with elastic membranes as upper and lower parts of the boundary. The approach is more difficult in this case since it is necessary to introduce additional correctors which correspond to the variable viscosity. The asymptotic construction is rigorously justified by the error estimates between the exact solution and the asymptotic one. The results obtained in [50]-[53] concern two-dimensional problems. Since as an application of a viscous fluid flow through a thin elastic tube we have in mind the blood flow in an artery or vein, it is more interesting to study this problem in 3D domains. In [54], [55] we study the viscous fluid-elastic structure interaction problem when the fluid domain is cylindric, by considering an axisymmetric flow.

For treating arterial stenoses, angioplasty is indicated; during angioplasty, a small wire mesh tube called a stent may be permanently placed in the newly opened artery or vein to help it remain open. In this way, the boundary of the fluid domain contains elastic parts and rigid parts, as well, the part where the stent was placed being considered a rigid boundary. In [36] we consider the viscous fluid flow through a domain with mixed rigid-elastic boundary. The domain contains two rectangular parts with elastic boundaries connected by a domain with rigid boundaries. We perform a variational analysis of the problem and we construct the asymptotic solution. The existence of a junction region between the two rectangles imposes to consider, as part of the asymptotic solution, some boundary layer correctors that correspond to this region.

In all the works previously presented in this chapter the elastic structure has a much smaller thickness than the fluid domain dimensions and, for this reason, it was neglected. In this way, the elastic medium became a part of the fluid domain boundary and so, some aspects of the physical phenomenon were omitted. In [56]-[58], [44] we analyze from the variational and asymptotic points of view the viscous fluid-elastic structure interaction when the thickness of the elastic structure is not neglected anymore. In other words, in these last four articles the fluid and the elastic medium occupy domains with the same dimension and the mathematical model is different with respect to that when the elastic medium was "a boundary". In [56], [57] we study the interaction viscous fluid-thin elastic plate in the case when the rigidity and the density of the elastic medium are "great". More precisely, considering that the thickness of the plate, ε, is the small parameter of the problem, we suppose that the density and the Young's modulus are of order ε^{-1} and ε^{-3}, respectively. We present a complete asymptotic construction and we prove that the main term satisfies the problem from [51].

The article [58] represents the generalization of [56], [57]. We present an asymptotic construction for a viscous fluid-elastic structure interaction problem when the flow domain is a thin, elastic, cylindrical tube. We study the axisymmetric problem and, like in the plane case, the asymptotic analysis is performed for a thickness of the tube wall, ε, that tends to zero, while the density and the Young's modulus are of the same order as in [57]. In the three works previously cited, [56]-[58], the fluid density and viscosity are supposed to be of order 1, which means that the elastic medium density is much greater that the fluid density. Since in many practical applications the two densities are of the same order, in [44] the asymptotic analysis was performed for the case when the densities of the two media are of order 1. In [44] we considered a 3D model of the interaction between a thin, elastic, stratified plate and a viscous fluid. Like in [56]-[58], the Young's modulus is considered of order ε^{-3}, but the plate density is of order 1. In these conditions, an asymptotic expansion of the solution is constructed and justified. The limit problem contains a non-standard boundary condition for the Stokes equations. The asymptotic analysis is applied to the partial asymptotic dimension reduction of the solid phase and the derivation of the asymptotically exact junction conditions between two-dimensional and three-dimensional models of the plate.

The article [59] represents the first step in the development of the asymptotic analysis for the Kelvin-Voigt model for a visco-elastic thin stratified strip. The dimension reduction combined with the homogenization procedure allows to construct a complete asymptotic expansion of the solution and to justify the one dimensional limit model.

From the works described above, we present with more details in Chapter 3 the articles [34], [51], [57].

The last part of this abstract is devoted to the description of some elements of my future research activity, represented by works in preparation or submitted for publication, which are presented in Chapter 4.

In [85] we study an optimal control problem, with the control in coefficients, with medical applications. We propose a mathematical model which describes the method for obtaining a normal blood pressure, controlled by the blood variable viscosity.

The article [81] deals with the asymptotic analysis of the interaction between a thin, stratified elastic plate and a viscous fluid layer. In [57] we presented this asymptotic analysis for a certain type of elastic material, characterized by "great" values of the density and of the Young's modulus of the elastic medium, as described above. In [44] the asymptotic approach is performed considering that only the Young's modulus has great values, the solid density being of order one. Since in real life the values of the densities of the solid and fluid phases may be very different from one medium to another, as well as the values of the Young's modulus of the solid phase and the dynamic viscosity of the fluid, the particular cases analyzed in [57] and [44] represent an important limitation in the study of physical phenomena. The novelty of [81] is that we propose an asymptotic expansion construction, rigorously justified by the error estimates, corresponding to various magnitudes (great and small) of the density and rigidity of the plate and we succeed to solve the limit problems that are very different from one case to another. In this very general context, the cases studied in [57] and [44] represent two particular cases of the 24 cases analyzed in this article. In [82] we perform an asymptotic analysis in the cylindrical case, without any restriction concerning the magnitude of the elastic medium characteristics. So, the problem studied in [58] becomes a particular case of the 6 analyzed in [82].

If in [59] we give some details of the asymptotic analysis for the Kelvin-Voigt model for a visco-elastic thin stratified strip in the quasi-static case, in [84] these details are developed and extended to the dynamic case.

Finally, in [83] we consider a fluid layer separated by a 3D thin elastic, stratified plate. The small parameter of the problem, ε, is defined as the ratio between the plate thickness and that of one fluid layer. We suppose that the Young's modulus is of order ε^{-3}, while the viscosity of the fluid and the densities of the two media are of order one. We present the variational and asymptotic analysis of the considered model. We analyze the limit problem that contains a non-standard boundary condition for the Stokes equations.

In the end of the description of my already published works, I mention other articles that proposed a mathematical model in connection with some mechanical problems of real world. Between January and June 1995 I got a postdoctoral scholarship at the University Jean Monnet, Saint-Etienne, France, in the laboratory of Numerical analysis led by Prof. C. Carasso. The article [27] presents the numerical model proposed for the simulation of a jet of ink flow. The articles [76], [77] were developed in the framework of the national grant CERES 4-96/2004, which I coordinated as member of "Simion Stoilow" Institute of Mathematics of the Romanian Academy, grant in collaboration with the Metallurgy Institute. They present the hydrodynamic behavior of the molten powder in meniscus zone, when the solidifying shell is either rigid, or with elastic properties.

The common feature of my works is represented by the proposal or the identification of mathematical models that describe as exactly as possible specific physical phenomena, followed by the analysis of these models. The more complex is the considered phenomenon, the more complicated is the associate mathematical model and the more difficult is its study. That is way, in general, we deal with nonlinear and/or coupled problems for which we provide qualitative information by using mathematical tools such as: variational formulations, minimization problems, boundary layer methods, fix point theorems, compact embeddings, *a priori* estimates, penalized problems, etc.

The results included in this thesis have been obtained either as sole author, or in collaboration with professors and researchers from Romania or from abroad, as it can be seen from the reference list. There are several names of my collaborators that should be mentioned here, but I'll limit myself to two of them, which have decisively marked my professional evolution. These are the late Prof. dr. Horia I. Ene, the scientific adviser of my Ph. D. thesis and Prof. dr. Grigory P. Panasenko.

The meeting with the first allowed me to begin my scientific career at the Department of Mathematics of INCREST, a desired position for any young graduate of the Faculty of Mathematics of those times. The chance of working with the second, one of the world's leading specialists in asymptotic methods, woke my interest in this research field and allowed me to establish a long collaboration that began 18 years ago and materialized in 21 works, with 18 already published.

References

[1] F. Abergel, E. Casas, Some optimal control problems of multistate equations appearing in fluid mechanics, *ESAIM-Math. Mod. Numer. Anal.*, **27**, 223-247, 1993.

[2] E. A. Adomavichus, G. V. Alekseev, Theoretical analysis of inverse extremal problems of admixture diffusion in viscous fluids, *Journal of inverse and ill-posed problems,* **9**, 435-468, 2001.

[3] G. V. Alekseev, O. V. Soboleva, D. A. Tereshko, Identification problems for a seady-state model of mass transfer, *J. Appl. Mech. Tech. Phy.*, **49**, 537-547, 2008.

[4] H. W. Alt, Strömungen durch inhomogene poröse Medien mit freiem Rand, *J. reine angew. Math.*, **305**, 89-115, 1979.

[5] H. W. Alt, L. A. Caffarelli, A. Friedman, Asymmetric jet flows, *Comm. Pure Appl. Math.*, **35**, 29-68, 1982.

[6] H. W. Alt, L. A. Caffarelli, A. Friedman, Axially symmetric jet flows, *Arch. Rat. Mech. Anal.*, **81**, 97-149, 1983.

[7] V. Arnautu, J. Sprekels, D. Tiba, Optimization problems for curved mechanical structures, *SIAM J. Control Optim.*, **44**, 743-775, 2005.

[8] C. Baiocchi, Su un problema di frontiera libera connesso a questioni di idraulica, *Annali. Mat. Pura Appl.*, **92**, 107-127, 1972.

[9] C. Baiocchi, A, Friedman, A filtration problem in a porous medium with variable permeability, *Annali. Mat. Pura Appl.*, **114**, 377-393, 1977.

[10] M. Barboteu, M. Sofonea, D. Tiba, The control variational method for beams in contact with deformable obstacles, *ZAMM*, **92**, 25-39, 2012.

[11] V. Barbu, T. Precupanu, *Convexity and optimization in Banach spaces*, Springer 2012.

[12] V. Barbu, G. Marinoschi, An optimal control approach to the optical flow problem, *Systems&Control Letters*, **87**, 1-9, 2016.

[13] M. Beldiman, I. Boaca, G. Marinoschi, Optimization of a singular flow in a porous medium, *ZAMM*, **93**, 633-647, 2013.

[14] V. Benci, On a filtration problem through a porous medium, *Annali. Mat. Pura Appl.*, **100**, 191-209, 1974.

[15] G. Birkhoff, E. H. Zarantonello, *Jets, wakes and cavities*, Academic Press, New York, 1957.

[16] T. Boulanger, A. Chrysochoos, C. Mabru, A. Galtier, Calorimetric analysis of dissipative and thermoelastic effects associated with the fatigue behavior of steels, *Int. J. Fatigue*, **26**, 221-229, 2004.

[17] H. Brezis, D. Kinderlehrer, G. Stampacchia, Sur une nouvelle formulation du problème de l'écoulement à travers une digue, *C.R. Acad. Sci. Paris Sér. A*, **287**, 711-714, 1978.

[18] A. Capatina , H. Ene, G. Pasa, D. Polisevski, **R. Stavre**, Optimal relations between the parameters of a P.E.M. fuel cell, *Math. Rep.*, **10(60)**, 299-308, 2008.

[19] A. Capatina , H. Ene, G. Pasa, D. Polisevski, **R. Stavre**, Mathematical Model for the P.E.M. Fuel Cells using Sulfuretted Hydrogen, *Math. Rep.*, **11(61)**, 1-10, 2009.

[20] A. Capatina , H. Ene, G. Pasa, D. Polisevski, **R. Stavre**, Variational approach and optimal control of a PEM fuel cell, *Nonlin. Anal. TMA*, **74**, 3242-3260, 2011.

[21] A. Capatina , H. Ene, D. Polisevski, **R. Stavre**, Parametric study of fluid dynamics in PEM fuel cells, *Proc. Rom. Acad. Ser. A*, **10**, 101-109, 2009.

[22] A. Capatina, **R. Stavre**, Optimality conditions for control problems in heat conducting Navier-Stokes fluid, *Rev. Roum. Sci. Tech.-Mec. Appl.*, **40**, 189-199, 1995.

[23] A. Capatina, **R. Stavre**, Optimal control of a non-isothermal Navier-Stokes flow, *Int. J. Engng. Sci.*, **34**, 59-66, 1996.

[24] A. Capatina, **R. Stavre**, Numerical analysis of a control problem in heat conducting Navier-Stokes fluid, *Int. J. Engng. Sci.*, **34**, 1467-1476, 1996.

[25] A. Capatina, **R. Stavre**, A control problem in biconvective flow, *J. Math. Kyoto Univ.*, **37**, 585-595, 1997.

[26] A. Capatina, **R, Stavre**, Algorithms and convergence results for an inverse problem in heat propagation, *Int. J. Engng. Sci.*, **38**, 575-587, 2000.

[27] C. Carasso, **R. Stavre**, Numerical simulation of a jet of ink, *Ann. Univ. Buc.*, **1-2**, 31-36, 2001.

[28] J. Carrillo-Menendez, M. Chipot, On the dam problem, *J. Diff. Eqns*, **45**, 234-271, 1982.

[29] J. Cea, *Optimization. Théorie et algorithmes*, Dunod, Paris, 1971.

[30] S. Challal, A. Lyaghfouri, The heterogeneous dam problem with leaky boundary condition, *Comm. Pure Appl. Anal.*, **10**, 93-125, 2011.

[31] D. Dupuy, G. P. Panasenko, **R. Stavre**, Asymptotic analysis for micropolar fluids, *C. R. Mécanique*, **332**, 31-36, 2004.

[32] D. Dupuy, G. P. Panasenko, **R. Stavre**, Asymptotic methods for micropolar flows in a tube structure, *Math. Mod. Meth. Appl. Sci. (M^3AS)*, **14**, 735-758, 2004.

[33] D. Dupuy, G. P. Panasenko, **R. Stavre**, Multiscale modeling for micropolar flows in a tube structure with one bundle of tubes, *Int. J. Multiscale Comput. Engng.*, **2**, 461-475, 2004.

[34] D. Dupuy, G. P. Panasenko, **R. Stavre**, Asymptotic solution for a micropolar flow in a curvilinear channel, *Z. Angew. Math. Mech. (ZAMM)*, **88**, 793-807, 2008.

[35] A. C. Eringen, Theory of micropolar fluids, *J. Math. Mech.*, **16**, 1-18, 1966.

[36] R. Fares, G. P. Panasenko, **R. Stavre**, A viscous fluid flow through a thin channel with mixed rigid-elastic boundary. Variational and asymptotic analysis, *Abstract Appl. Anal.*, **Volume 2012, ID 152743**, 47 pages, 2012.

[37] R. Finn, Some theorems on discontinuity of plane fluid motion, *J. Anal. Math.*, **4**, 246-291, 1954-1956.

[38] A. Friedman, S. Huang, The inhomogeneous dam problem with discontinuous permeability, *Annali Scuola Normale Superiore di Pisa*, **14**, 49-77, 1987.

[39] D. Gilbarg, Jets and cavities, *Handbuch der Physik*, **9**, 311-445, 1960.

[40] Y. Kan-on, K. Namkawa, Y. Teramoto, On the equations of biconvective flow, *J. Math. Kyoto Univ.*, **32**, 135-153, 1994.

[41] M. Levandovsky, W. S. Childress, S. H. Hunter, E. A. Spiegel, A mathematical model of pattern formation by swimming microorganisms, *J. Protozoology*, **22**, 296-306, 1975.

[42] J. L. Lions, *Contrôle optimal de systèmes gouvernés par des équations aux dérivées partielles*, Dunod, Paris, 1968.

[43] J. L. Lions, *Contrôle de systèmes distribués singuliers*, Dunod, Paris, 1983.

[44] I. Malakhova-Ziablova, G. Panasenko, **R. Stavre**, Asymptotic analysis of a thin rigid stratified elastic plate-viscous fluid interaction problem, *Appl. Anal.*, **95**, 1467-1507, 2016.

[45] S. Martin, C. Vázquez, Homogenization of the layer-structured dam problem with isotropic permeability, *Nonlin. Anal. RWA*, **14**, 2133-2151, 2013.

[46] A. Mohammadi, J. M. Floryan, Groove optimization for drag reduction, *Phys. Fluids*, **25**, 113601, 2013.

[47] P. Neittaanmaki, D. Tiba, *Optimal control of nonlinear parabolic systems*, Marcel Dekker, New York, 1994.

[48] G. P. Panasenko, Method of asymptotic partial decomposition of the domain, *Math. Mod. Meth. Appl. Sci.*, **8**, 139-156, 1998.

[49] G. P. Panasenko, Asymptotic partial decomposition of variational problems, *C. R. Acad. Sci. Paris Série IIb*, **327**, 1185-1190, 1999.

[50] G. P. Panasenko, Y. Sirakov, **R. Stavre**, Asymptotic and numerical modeling of a flow in a thin channel with visco-elastic wall, *Int. J. Multiscale Comput. Engng.*, **5**, 473-482, 2007.

[51] G. P. Panasenko, **R. Stavre**, Asymptotic analysis of a periodic flow in a thin channel with visco-elastic wall, *J. Math. Pures Appl.*, **85**, 558-579, 2006.

[52] G. P. Panasenko, **R. Stavre**, Asymptotic analysis of a non-periodic flow in a thin channel with visco-elastic wall, *Netw. Heterog. Media*, **3**, 651-673, 2008.

[53] G. P. Panasenko, **R. Stavre**, Asymptotic analysis of the Stokes flow with variable viscosity in a thin elastic channel, *Netw. Heterog. Media*, **5**, 783-812, 2010.

[54] G. P. Panasenko, **R. Stavre**, Well posedness and asymptotic expansion of solution of Stokes equation set in a thin cylindrical elastic tube, Around the Research of Vladimir Maz'ya II/International Mathematical Series, Volume 12, editors: Ari Laptev, Springer 2010 (book chapter).

[55] G. P. Panasenko, **R. Stavre**, Asymptotic analysis of the Stokes flow in a thin cylindrical elastic tube, *Appl. Anal.*, **91**, 1999-2027, 2012.

[56] G. P. Panasenko, **R. Stavre**, Asymptotic analysis of a viscous fluid-thin plate interaction: periodic flow, *C. R. Mécanique*, **340**, 590-595, 2012.

[57] G. P. Panasenko, **R. Stavre**, Asymptotic Analysis of a Viscous Fluid-Thin Plate Interaction: Periodic Flow, *Math. Mod. Meth. Appl. Sci. (M3AS)*, **24**, 1781-1822, 2014.

[58] G. P. Panasenko, **R. Stavre**, Viscous fluid-thin cylindrical elastic layer interaction: asymptotic analysis, *Appl. Anal.*, **93**, 2032-2056, 2014.

[59] G. Panasenko, **R. Stavre**, Asymptotic analysis for the Kelvin-Voigt model for a thin laminate, *C. R. Mécanique*, **343**, 365-370, 2015.

[60] M. L. Pastor, X. Balandraud, M. Grédiac, J. L. Robert, Applying infrared thermography to study the heating of 2024-T3 aluminium specimens under fatigue loading, *Infrared Physics&Technology*, **51**, 505-515, 2008.

[61] S. G. Pyatkov, M. L. Samkov, Solvability of some inverse problems for the nonstationary heat-and-mass-transfer system, *J. Math. Anal. Appl.*, **446**, 1449-1465, 2017.

[62] **R. Stavre-Stăncescu**, The flow of a fluid through a porous medium with variable permeability, *Bull. Math. Soc. Sci. Math. R. S. Roumanie*, **27(75)**, 169-179, 1984.

[63] **R. Stavre-Stăncescu**, Fluid flow through a porous medium with variable permeability in which the discharge is unknown, *Revue Roumaine Sci. Tech., Mec. Appl.*, **29**, 391-402, 1984.

[64] **R. Stavre**, B. Vernescu, Incompressible fluid flow through a non-homogeneous and anisotropic dam, *Nonlin. Anal. TMA*, **9**, 799-810, 1985.

[65] **R. Stavre**, B. Vernescu, The free boundary problem for the anisotropic dam, *Arch. Mech.* **4**, 455-463, 1988.

[66] **R. Stavre**, B. Vernescu, Free boundary properties in non-homogeneous porous media fluid flow, *Int. J. Engng. Sci.*, **27**, 399-409, 1989.

[67] **R. Stavre**, On a free boundary problem in fluid mechanics, *Eur. J. Mech. B/Fluids*, **10**, 75-95, 1991.

[68] **R. Stavre**, The impact of a jet with two fluids on a porous wall, *Int. Ser. Num. Math.*, **107**, 11-23, 1992.

[69] **R. Stavre**, Study of a jet incident on a porous wall in a gravity field, *IMA J. Appl. Math.*, **52**, 93-103, 1994.

[70] **R. Stavre**, Analysis and finite element approximation of a control model for the dam problem, *Math Reports*, **1**, 289-304, 1999.

[71] **R. Stavre**, A distributed control problem for micropolar fluids, *Rev. Roum. Math. Pures Appl.*, **45**, 353-358, 2000.

[72] **R. Stavre**, Distributed control of a heat-conducting, time-dependent, Navier-Stokes fluid, *Glasgow Math. J.*, **44**, 191-200, 2002.

[73] **R. Stavre**, The control of the pressure for a micropolar fluid, *Z. Angew. Math. Phys. (ZAMP)*, **53**, 912-922, 2002.

[74] **R. Stavre**, Optimization and numerical approximation for micropolar fluids, *Num. Funct. Anal. Optimiz.*, **24**, 223-241, 2003.

[75] **R. Stavre**, Optimization of a non stationary flow in a magnetic field, Current Topics in Continuum Mechanics, vol. III, general editor Lazar Dragoş, Editura Academiei Române, 181-211, 2006 (book chapter).

[76] **R. Stavre**, Incompressible flow of the molten powder in meniscus zone of continuous casting mold, *Ann. Univ. Buc.*, **1**, 121-128, 2006.

[77] **R. Stavre**, Viscous flow of the molten powder in meniscus zone with elastic boundary, *Bulletin of the Transilvania University of Brasov*, **13**, 379-389, 2006.

[78] **R. Stavre**, Optimization of the blood flow in venous insufficiency, *Ann. Univ. Buc.*, **5(LXIII)**, 383-402, 2014.

[79] **R. Stavre**, A Distributed Control Problem for Two Coupled Fluids in a Porous Medium, *SIAM J. Control Optim. (SICON)*, **53**, 313-335, 2015.

[80] **R. Stavre**, A boundary control problem for the blood flow in venous insfficiency. The general case, *Nonlin. Anal. RWA,* **29**, 98-116, 2016.

[81] G. P. Panasenko, **R. Stavre**, Viscous fluid-thin elastic plate interaction: asymptotic analysis with respect to the rigidity and density of the plate, *Appl. Math. Optim.*, DOI:10.1007/s00245-018-9480-2, published on line February 2018.

[82] G. P. Panasenko, **R. Stavre**, Viscous fluid-thin cylindrical elastic body interaction: asymptotic analysis on contrasting properties, *Appl. Anal.*, DOI:10.1080/000336811.2018.1442000, published on line March 2018.

[83] I. Orlik, G. P. Panasenko, **R. Stavre**, Asymptotic analysis of a viscous fluid layer separated by a thin rigid stratified elastic plate, *in preparation*

[84] G. P. Panasenko, **R. Stavre**, Asymptotic solution for a visco-elastic thin plate; quasi-steady and dynamic cases *in preparation.*

[85] **R. Stavre**, Optimization of the blood pressure with the control in coefficients, *in preparation.*