ACADEMIA ROMÂNĂ
SCOSAAR

HABILITATION THESIS

Stochastic numerical approach for branching-fragmentation processes

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ABSTRACT

Domeniul fundamental: Matematică și științe ale naturii
Domeniul de abilitare: Matematică

Teză elaborată în vederea obţinerii atestatului de abilitare în scopul conducerii lucrărilor de doctorat în domeniul Matematică

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Abstract

The thesis presents stochastic and numerical models for the time evolution of several real-life phenomena, involving branching and fragmentation time continuous processes, with applications to avalanches. We also consider related nonlinear boundary value problems. We study fragmentation-branching processes and their stochastic differential equations for continuous and discontinuous fragmentation kernels with numerical probabilistic approaches, we obtain a numerical solution for a nonlinear Dirichlet problem associated with a non-local, discrete, branching processes in a bounded Euclidean domain, we investigate Rosenblatt subordinate processes in the second Wiener chaos space, motivated by applications in hydraulic conductivity, and finally, we introduce a geometric optimisation method for the nonlinear domain functionals, modelling the flow onset of the dense avalanches, we emphasise a numerical approach.

The work is organised as follows.

The first chapter develops branching and fragmentation processes, involving stochastic differential equations with applications to avalanches. We investigate branching properties of the solution of a fragmentation equation for the mass distribution and we properly associate a continuous time càdlàg Markov process on the space of all fragmentation sizes, introduced by J. Bertoin. A binary fragmentation kernel induces a specific class of integral type branching kernels and taking as base process the solution of the initial fragmentation equation for the mass distribution, we construct a branching process corresponding to a rate of loss of mass greater than a given strictly positive threshold d. It turns out that this branching process takes values in the set of all finite configurations of sizes greater than d. The process on the space of all fragmentation sizes is then obtained by letting d tend to zero. A key argument for the convergence of the branching processes is given by the Bochner-Kolmogorov theorem. The construction and the proof of the path regularity of the Markov processes are based on several newly developed potential theoretical tools, in terms of excessive functions and measures, compact Lyapunov functions, and some appropriate absorbing sets. We present further a stochastic model for the fragmentation phase of an avalanche, by constructing a related fragmentation-branching process on the set of all fragmentation sizes. A fractal property of this process is emphasized. We also establish the associate stochastic differential equation of fragmentation. The results are obtained by combining analytic and probabilistic potential theoretical tools.

In Chapter 2 we build and develop an unifying method for the construction of a continuous time

fragmentation-branching processes on the space of all fragmentation sizes, induced either by continuous fragmentation kernels or by discontinuous ones. We introduce an approximation scheme for the process which solves the corresponding stochastic differential equations of fragmentation. One of the main achievements is to compute the distributions of the branching processes approximating the forthcoming branching-fragmentation process. We present numerical results that confirm the validity of the fractal property which was emphasized by our model for an avalanche, presented in Section 1.2.

In Chapter 3 we give a probabilistic numerical approach for the nonlinear Dirichlet problem associated with a branching process. Main tools are the probabilistic representation of the solution with the measure-valued branching process, as well as specific techniques for the numerical solution of linear partial differential equations, introduced and developed by Milstein and Tretyakov, and Monte Carlo methods.

Chapter 4 introduces the Rosenblatt Laplace motion, by subordinating the Rosenblatt process to an independent Gamma process, motivated by several works on the modelization of hydraulic conductivity. We derive the basic properties of this new fractal-type stochastic process and we also make a numerical analysis of it. In particular, we compute numerically its moments and cumulants and we provide a method to simulate its sample paths.

Chapter 5 deals with a new numerical boundary variation method for solving the generalized Cheeger problem. This problem (also called weighted Cheeger problem, modeling landslides, snow avalanches and other geophysical flows) aims to find the safety factor and the collapse domain (onset flow region). It is formulated in the space of bounded variations functions and rewritten in terms of a shape optimization problem, involving only boundary valued functions. We propose here a numerical scheme, based on a boundary variation method (without changing the topology). For that, we have computed the shape derivative of the Cheeger functional and derived a surface divergence formula for surface defined functions. For the spatial discretization we only use a shape boundary discretization. Even if sometimes the choice of the initial shape of the algorithm had to be done using a global optimization method, the boundary variation method is very attractive. Finally, we illustrate the proposed method with numerical computations of the onset velocity regions for certain physical sound problems (in two and three dimensions).

Chapter 6 completes the work with possible continuations of this research in the future.
All the issues from the List of references are cited in the text.