HABILITATION THESIS

Ecological and epidemiological interactions: characterizing dynamics and influencing outcomes

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ABSTRACT

Domeniul fundamental: **Matematică**
Domeniul de abilitare: **Matematică**
Teză elaborată în vederea obţinerii atestatului de abilitare în scopul conducerii lucrărilor de doctorat în domeniul **Matematică**

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Abstract

The aim of this Habilitation Thesis, which adheres to the idea that the outcomes of most biological interactions can be reliably predicted via the use of analytic methods, is to present a collection of scientific contributions published since I defended my (second) doctoral thesis in 2008, in Budapest, at CEU. The main part of this Thesis is divided into three chapters, each of them corresponding to a biological theme to be pursued, with the intention of not only presenting analytical results, but also outlining their biological relevance and, whenever possible, practical implications. An additional fourth chapter is dedicated to presenting directions of further study and current research projects, along with further avenues of professional development which are intended to be pursued.

Chapter 1 is concerned with the analysis of models describing integrated pest management (IPM) strategies. Sections 1.2 to 1.4 discuss the dynamics of certain models with impulsive perturbations occurring at prescribed times, with a view towards steering the behavior of the biological system of concern to a desirable practical outcome. In this regard, the occurrence of a supercritical bifurcation when the amount of infective pests released each time reaches a critical value is shown in Section 1.2, while sufficient conditions for the local and global stability, respectively, of the so-called susceptible pest-eradication periodic solution and for the permanence of the system, amounting to various degrees of success of the IPM strategy, are established in Sections 1.3 and 1.4. The influence of the functional response of the predator upon the possibility to derive threshold stability conditions is also analyzed. Section 1.5 presents further stability and bifurcation results for a delayed model describing the control of the *Aedes Aegypti* mosquito in terms of biologically relevant parameters. A discrete control problem associated to a delay-free discretization of the initial model via a nonstandard difference scheme is also investigated. In Section 1.6, sufficient conditions for the existence and orbital stability of a positive order-1 periodic solution of a model with state-dependent impulsive perturbations, also describing mosquito control, are determined via a geometric approach.

Chapter 2 is devoted to the analysis of certain mutualistic and commensalistic interactions. Sufficient conditions for the global stability of the coexistence equilibrium associated to an abstract model of mutualism are obtained in Section 2.2 using nonstandard Lyapunov functionals, under various trade-offs between monotonicity and sign conditions imposed upon the functional coefficients of the model. It is shown in Sections 2.2 and 2.3 that these functionals can be used for the analysis of several concrete models in widespread use, as well as of certain generalizations. A procedure to adapt these functionals for the study of certain commensalistic models is indicated in

Section 2.4. It is also investigated how a commensalistic interaction may prevent population extinction if an Allee effect is incorporated into the intrinsic growth rate of the positively influenced species.

In Sections 2.5 and 2.6, a different path is taken. Using mild assumptions on the growth and self-limiting functions, necessary and sufficient conditions for the boundedness of solutions are established in terms of reproductive ratios (idea which is in itself somewhat alien to Ecology) which, although similar in scope to the basic reproduction number of Mathematical Epidemiology, are computed at large population densities, rather than in near-extinction conditions. Section 2.5 is additionally dedicated to a global stability analysis, using, in a certain sense, a reduced number of threshold parameters, the possibilities to overcome Allee effects via mutualistic contributions being analyzed. Section 2.6 is further concerned with extending the framework to the case of an arbitrary number of species, being observed that for a representative class of models, the boundedness condition can be expressed in terms of the stability of a certain limiting matrix.

Chapter 3 is devoted to an analysis of two disease propagation models. In Section 3.1, the transmission of schistosomiasis is modeled as a 12-dimensional multi-scale system of ODEs that includes vector-host and within-host dynamics of infection. An explicit expression for the basic reproduction number R_0 is obtained via the next generation method and then interpreted in biological terms, as well as in terms of sub-reproductive numbers. A bifurcation result is then used to characterize the stability of the endemic equilibrium. A sensitivity analysis indicates the parameter with respect to which R_0 is the most sensitive, this finding being used to investigate the most effective strategy for the control of disease propagation.

In Section 3.2, a model of HIV transmission which keeps track of two interacting high-risk groups, female sex workers (FSW) and male injecting drug users (IDU), along with a third "bridge" group of male drug-free clients (DFC), is formulated and analyzed. First, the stability of a multi-group model featuring abstract forces of infection is studied by means of the graph theoretic approach of Li and Shuai. It is again determined that R_0 is a threshold parameter for the stability of the equilibria. Global stability results for the initial model are then obtained via suitable particularisations. Sub-reproductive numbers for each disease transmission route are obtained, adequate courses of action depending on the goal of the intervention measures being indicated based on these findings.