THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 9, Number 2/2008, pp. 000–000

ON THE ARHITECTURE OF A NEW CELLULAR ELASTIC MATERIAL

Veturia CHIROIU, Ligia MUNTEANU, Dan DUMITRIU, Miruna BELDIMAN, Cornel SECARĂ

Institute of Solid Mechanics, Romanian Academy, Ctin Mille 15, 010141 Bucharest
Corresponding author Veturia CHIROIU, e-mail : veturiachiroiu@yahoo.com

The present work discusses a new architecture for a cellular elastic solid with a chess board structure, consisted from positive and negative stiffness materials. The negative stiffness mechanism is defined in terms of the Eshelby's steps: 1) remove the black cells BC from the composite and allow them to undergo a stress-free biaxial deformation in the [111] direction, becoming trigonal; 2) apply a surface biaxial traction to the black cells BC and the white cells WC to bring the black cells BC back into the matrix, by assuring the continuity of displacements and normal stresses across the boundaries. The properties of such materials are discussed.

Key Words: Composites, Cellular materials, Negative stiffness material, Chess board st5ructure.

1. INTRODUCTION

The negative stiffness exerts an opposite force which amplifies the deformations and cancels out the stiffness into a material. As a result, an instable material is obtained. When the instability is tensile, the instability is associated with cracks. When the instability is in shear, the instability is associated with the formation of shear bands, observed in metals, rock, soils, being an important failure mode. It is of interest that in the 1970's, considerable controversy arose as to whether unstable materials should ever be used in composites. The advocates of stable material models (Sandler and Wright [1]) argued that computer models with unstable material models are essentially ill-posed according to classical definitions such as given by the Lax equivalence theorem. Though a material of negative stiffness is unstable, if the negative stiffness material is included into a composite, the surrounding matrix will stabilized it (Chiroiu, Munteanu, Dumitriu, Beldiman si Secara [2]). For this reason, some authors (Lakes [3], [4], Lakes, Lee, Bersie and Wang [5], Lakes and Drugan [6]) show theoretically and experimentally, that composites with negative stiffness inclusions are characterized by a greater deformation in and near inclusions than the composite as a whole, and consequently exhibit interesting properties which exceed conventional bounds having higher stiffness and damping than that of either composites.

Using certain results formulated in Teodorescu, Badea, Munteanu and Onisoru [7] and Chiroiu, Munteanu, Dumitriu, Beldiman si Secara [2], in this paper we present a model for a cellular elastic solid with a chess board structure, consisted from positive and negative stiffness materials. To do this, after a formulation of the problem in Section 2, we show in Section 3 that the new composite exhibits interesting mechanical properties.

2. THE MODEL

To calculate the elastic constants of a single metallic layer subject to an arbitrary small initial deformations, the Jankowski and Tsakalakos model material [8], [9] is adopted. According to this model the total crystal energy of metal may be written as

$$E = E_{es} + E_{fe} + E_{bs} + E_r \,, \tag{2.1}$$

where E_{es} represents the electrostatic Coulomb energy of positive point charges in the uniform negative-charge background (the Madelung energy), E_{fe} is the free-electron energy, which depends on the crystal volume, E_{bs} is the band-structure energy and, E_r the ion-core (Born-Mayer) repulsive energy term.

Jankowski and Tsakalakos conclude that E_r is the predominant term for calculations of elastic constants. Following Delsanto, Provenzano and Uberall [10], the energy term E_r can be expressed as

$$E_r = \tfrac{1}{2}\alpha \sum_R \exp(-\beta R) \tag{2.2}$$

where α, β are the repulsive energy parameter and respectively, the repulsive range parameter. The sum is extended to the nearest neighbors which are located at distances $R^{(n)}$. The second-order elastic constants (stiffness constants) are defined as

$$C_{ijkl} = \frac{\partial^2 \tilde{V}}{\partial \varepsilon_{ij} \partial \varepsilon_{kl}}, \ \tilde{V} = \frac{E}{\Omega}, \tag{2.3}$$

where $\tilde{V}$ is the potential energy of deformation per unit volume (or elastic potential), Ω is the cell volume and ε_{ij} is the Lagrangian strain tensor. Starting with (1)-(3), the elastic constants are determined with

$$\frac{\partial}{\partial \varepsilon_{ij}} = \frac{1}{2}\left(X_i \frac{\partial}{\partial x_j} + X_j \frac{\partial}{\partial x_i}\right). \tag{2.4}$$

In (2.4), X_i are the Lagrangian coordinates corresponding to an initial state which may be subject to an initial finite deformation, x_i are the final Eulerian coordinates, differing from X_i by an infinitesimal deformation. It is important to note that, in applying (2.4) the Lagrangian coordinates X_i must be considered as constants since they refer to a predefined initial state. Using (2.4) it is straightforward to prove that, for a differentiable function $f(r)$, it follows

$$\left(\frac{\partial^2 f(r)}{\partial \varepsilon_{kl} \partial \varepsilon_{ij}}\right)_{r=R} = \frac{1}{R^3}[Rf''(R) - f'(R)]Y_{ijkl} + \frac{1}{4R} f'(R) Z_{ijkl},$$

where

$$Y_{ijkl} = X_i X_j X_k X_l, \ R = \sqrt{X_1^2 + X_2^2 + X_3^2}, \tag{2.5}$$

$$Z_{ijkl} = X_i X_k \delta_{jl} + X_i X_l \delta_{jk} + X_j X_l \delta_{ik} + X_j X_k \delta_{il}. \tag{2.6}$$

From (2.1)-(2.4) it follows that

$$C_{ijkl} = A_{ijkl} - B_{ijkl}, \quad A_{ijkl} = \sum_n f^{(n)} Y_{ijkl}^{(n)}, \ B_{ijkl} = \sum_n g^{(n)} Z_{ijkl}^{(n)}, \ f^{(n)} = f(R^{(n)}) = 4g^{(n)}\left(1 + \frac{\beta R^{(n)}}{(R^{(n)})^2}\right),$$

$$g^{(n)} = g(R^{(n)}) = \frac{K}{R^{(n)}} \exp(-\beta R^{(n)}), \ K = \frac{\alpha\beta}{8\Omega}.$$

Expressions $Y^{(n)}$, $Z^{(n)}$ and $R^{(n)}$ refer to the corresponding quantities defined in (2.5), (2.6) as calculated for the nth nearest neighbor. In order to simplify the calculation, it is useful to observe that the number of terms to be evaluated explicitly may be greater reduced due to some special symmetries, in addition to the usual symmetries of C_{ijkl}.

Let assume that an FCC crystal cell belonging to the cubic system with three independent elastic constants, that is deformed in the [111] direction, which is represented in fig.2.1, by an axis X parallel to OD (diagonal of the cube) and normal to ABC. The deformation can be analyzed by considering three sets

of axes, all centered in O: a set of axes (x,y,z) parallel to three sides of the cube, an auxiliary set of axes (x',y',z') obtained from (x,y,z) with a 45^0 rotation around $z \equiv z'$, a third set of axes (X,Y,Z) obtained from (x',y',z') with a rotation around $y' \equiv Y$ of an angle

$$\theta = \arcsin(1/\sqrt{3}) . \tag{2.7}$$

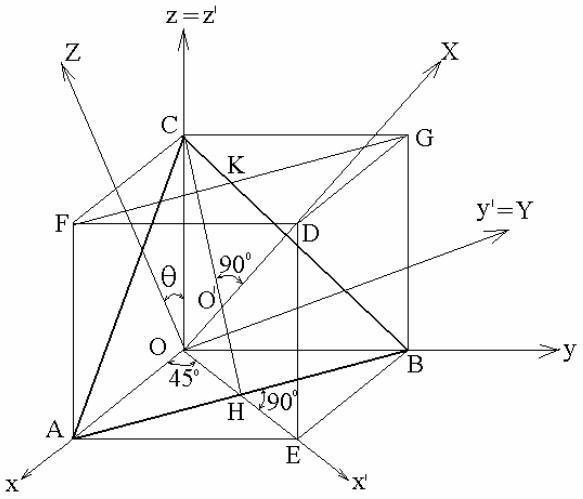

Fig.2.1. Representation of the sets of axes (x,y,z) and (X,Y,Z).
The later is used to study the crystal in a biaxially deformed state.

It can be easily shown that the transformation equations from (x,y,z) to (X,Y,Z) are

$$X = (x+y+z)/\sqrt{3}\ ,\ Y = (-x+y)/\sqrt{2}\ ,\ \ Z = (-x-y+2z)/\sqrt{6}\ . \tag{2.8}$$

All allowed positions for atoms in the underformed FCC crystal (fig.2.2) may be specified by coordinates x,y,z which are integer or half-integer multiples of the lattice constant a, the only restriction being that $x+y+z$ must be an integer multiplier of a. Applying (2.8) we find that the corresponding coordinates in the (X,Y,Z) reference system are

$$X = la/\sqrt{3}\ ,\ Y = ma/\sqrt{8}\ ,\ Z = na/\sqrt{24}\ , \tag{2.9}$$

where l,m,n are integers, the only restrictions being that m and n must have the same parity. Fig. 2.3 shows the lattice arrangement, as obtained from (2.9), for the plane ABC, $X = a/\sqrt{3}$ and two neighboring planes $X = 0$ and $X = 2a/\sqrt{3}$. Fig.2.4 shows all lattice positions of fig.2.2, except for the lattice point D, which lies in the plane $X = 3a/\sqrt{3}$. Using (2.9), the eight nearest neighbors to any atom (say the atom at O) are represented. Fig.2.4 shows them schematically both in the (x,y,z) and (X,Y,Z) systems of axes. In the underformed crystal they are located at $a/\sqrt{2}$. A biaxial deformation in the [111] direction is described by

$$X_1 = X(1+\varepsilon')\ ,\ X_2 = Y(1+\varepsilon)\ ,\ X_3 = Z(1+\varepsilon)\ , \tag{2.10}$$

where X_i are the coordonates of lattice positions in the initial deformed state, after the biaxial deformation, but before the infinitesimal deformation, and $\varepsilon' = -2\varepsilon C_{12}/C_{11}$. The Voigt's convention is used to denote each pair of indexes of the elastic constants defined by (3) by a single index

$$C_{1111} \to C_{11}\ ,\ C_{1122} \to C_{12}\ ,\ C_{1133} \to C_{13},\ \ C_{3333} \to C_{33}\ ,\ C_{2323} \to C_{44}\ ,\ C_{1212} \to C_{66}\ .$$

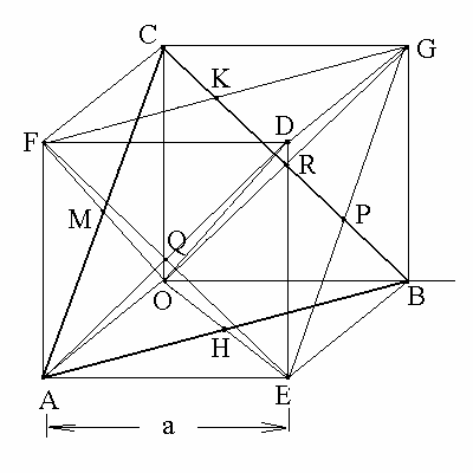

Fig. 2.2. The FCC unit crystal cell

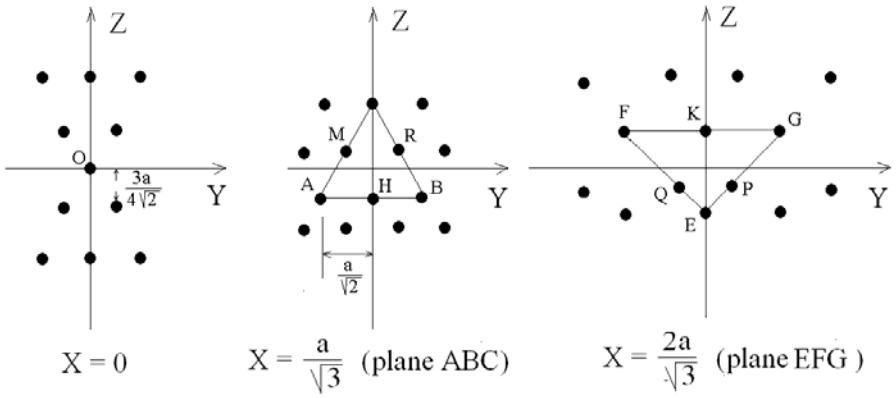

Fig.2.3. Lattice representation in the (X,Y,Z) reference system.

Equation (2.10) can be used to obtain the coordinates of twelve nearest neighbors after the biaxial deformation (fig.2.4). Alternatively, fig 2.4(b) may be used directly to read their coordinates, with the change that, in the equations of the planes, a becomes $a(1+\varepsilon')$ and, in the planes themselves, $a(1+\varepsilon)$. It then becomes straightforward, to calculate all the sums over the nearest neighbors and obtain explicit expressions for elastic constants. Although one might assume a priori that in the deformed state the crystal loses all its symmetries, becoming triclinic, it can be proved either by crystallographic arguments or directly through our calculations, that crystal retains trigonal symmetry (three planes of mirror symmetry).

The independent non-zero elastic constants after the biaxial deformation are then given by

$$C_{11}=A_{11}-B_{11},\ C_{22}=C_{33}=A_{22}-B_{22},\ C_{44}=1/2(C_{22}-C_{23}),\ C_{55}=C_{66}=A_{13}-B_{55},$$

$$C_{12}=C_{13}=A_{13},\ C_{23}=(1/3)A_{22},\ C_{25}=C_{46}=A_{25},\ C_{35}=A_{25},$$

where

$$A_{11}=(2/3)f_c\eta'^4,\ A_{22}=(1/16)(9f_a+f_c)\eta^4,\ A_{13}=(1/6)f_c\eta^2\eta'^2,\ A_{25}=(1/12\sqrt{2})f_c\eta^3\eta',$$

$$.\ B_{11}=8g_c\eta'^2,\ B_{22}=(6g_a+2g_c)\eta^2,\ B_{55}=0.5[3g_a\eta^2+g_c(\eta^2+4\eta'^2)],$$

$$f_a=f(R_a),\ f_c=f(R_c),\ g_a=g(R_a),\ g_c=g(R_c),$$

$$R_a = (1/\sqrt{2})\eta a\ ,\ R_c = a\sqrt{(1/3)\eta'^2 + (1/6)\eta^2}\ ,$$

$$\eta = 1+\varepsilon\ ,\ \eta' = 1+\varepsilon'\ .$$

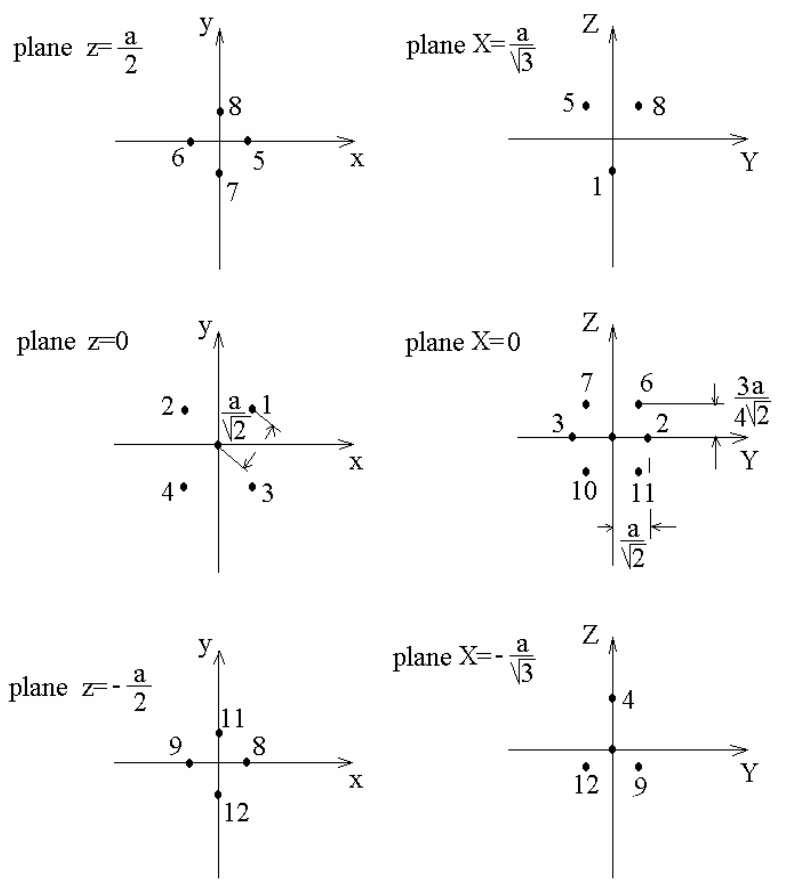

Fig.2.4. Representation of the twelve nearest neighbours (a) in (x, y, z) system; (b) in (X, Y, Z) system.

The biaxial modulus is defined as the stress-strain ratio $Y_b = \sigma/\varepsilon$, where assuming symmetry in the X_2, X_3 plane yields $\sigma_2 = \sigma_3 = \sigma$, and $\sigma_1 = \sigma_4 = \sigma_5 = \sigma_6 = 0$, $\varepsilon_2 = \varepsilon_3 = \varepsilon$. The Y_b can be measured with a bulge test in a thin structure: it represents the stress over strain ratio for stretching without shear $(\sigma_4 = 0)$.

The condition $\sigma_1 = \sigma_5 = \sigma_6 = 0$ gives the natural boundary conditions, since there cannot be any stress component outside the X_2, X_3 plane. Due to the assumed symmetry in the X_2, X_3 plane, it follows

$$\varepsilon_2 = \varepsilon_3 = (S_{22} + S_{23})\sigma\ ,\quad Y_b = 1/(S_{22} + S_{23})\ , \tag{2.11}$$

where S_{ij} is the elastic compliance tensor, which is the inverse of the stiffness tensor $S_{ij}C_{jk} = \delta_{ik}$. For special symmetries, e.g. trigonal in our case, it is possible to find simple relationships between the components of the stiffness and compliance tensors

$$\varepsilon' = \varepsilon_1 = -2\varepsilon C_{12}/C_{11}\ ,\quad Y_b = C_{22} + C_{23} - 2C_{12}^2/C_{11}\ .$$

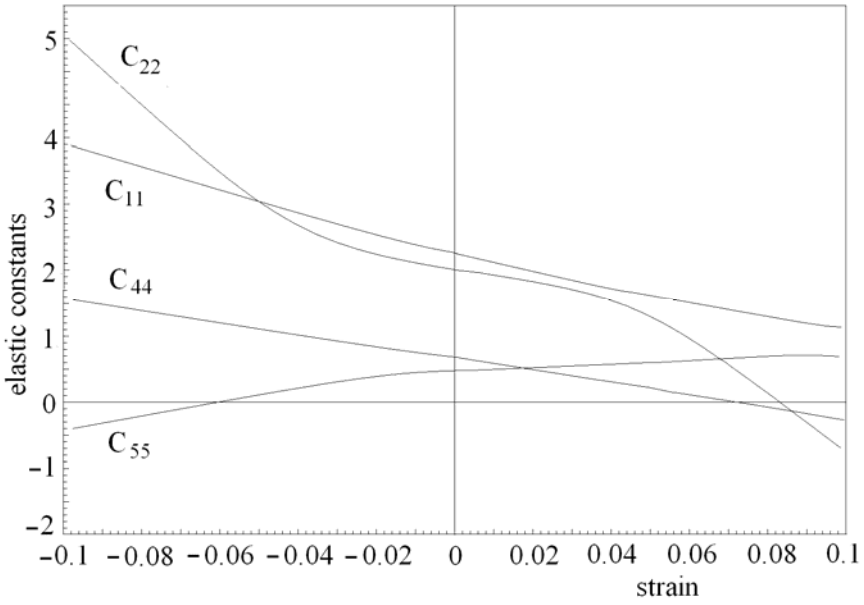

Fig. 2.5. Plots of elastic constants as functions of strain for Cu (units are 10^3 GPa).

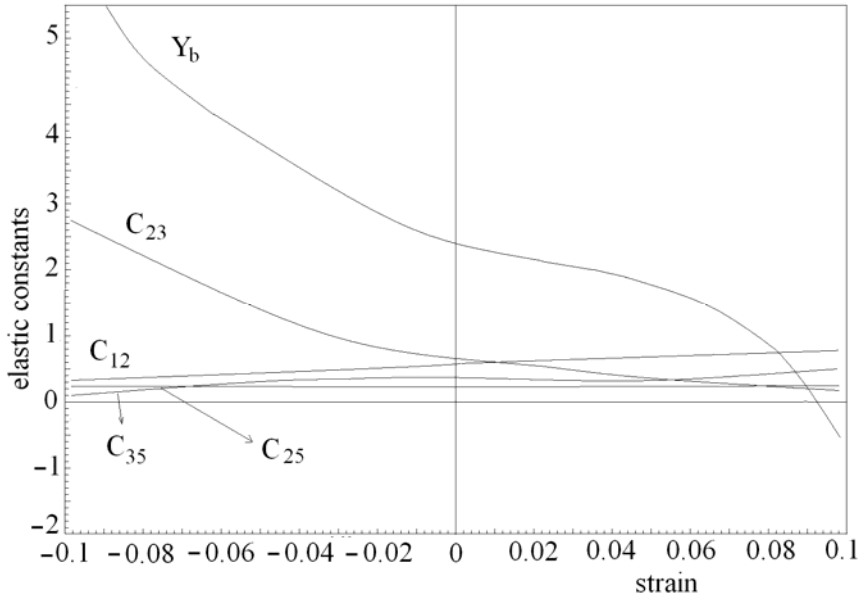

Fig. 2.6. Plots of elastic constants as functions of strain for Cu (units are 10^3 GPa).

In a similar fashion we find the other moduli. There are two parameters in our formulae that need to be specified: α (units Ryd (Rydberg) 1 Ryd = 13.6 eV = 2.092×10^{-21} J) and β (units ua in units of a^{-1} where a is the lattice constant). The chosen of α is irrelevant since it affects only the absolute values of the elastic constants and moduli as a multiplicative constant whereas we are interested in their relative change. To give it value, we need it as a free parameter to fit the experimental value of C_{11} in the case of zero deformation in (x, y, z) reference system. For Cu it is obtained $\alpha = 0.3415\times 10^6$ Ryd, and for Au and Ag $\alpha = 0.36\times 10^6$ Ryd (Jankowski and Tsakalakos [8]).

From the same source we have β = 13.84 ua for Cu, β = 12 ua for Au and β = 12.24ua for Ag. All independent elastic constants and the corresponding biaxial modulus and shear modulus are evaluated for strains between -10% and 10%. The numerical results are plotted in figs.2.5 and 2.6 for Cu. The reported

values refer to the system of axes (X,Y,Z). Figs. 2.5 and 2.6 show that a relatively small deformation may lead to a large change in Y_b due mainly to the contribution of C_{22}. For large positive deformations, C_{22} is negative. The $C_{11}, C_{23}, C_{12}, C_{25}$ and C_{35} are positive, but C_{44} is negative for large positive deformations, and C_{55} is negative for large negative deformations.

In a cubic system we have three independent constants C_{11}, C_{44}, C_{12} ($C_{11}=C_{22}=C_{33}$, $C_{44}=C_{55}=C_{66}$, $C_{12}=C_{13}=C_{23}$) and the shear modulus is given by $\mu = G = C_{44} + \frac{1}{5}(C_{11} - C_{12} + 3C_{44})$.

3. A NEW ARHITECTURE OF COMPOSITE

A new architecture for a cellular elastic solid, based on a chess board structure with positive and negative stiffness materials, is advanced in this section. The negative stiffness mechanism is also described. Let us consider a plate with parallel faces, of length L, thickness d and width b, made from a cellular elastic solid with a chess structure. The elastic constants change with the period of the chess board. Such a structure in the isotropic case was studied by Berlyant and Kozlov [11]. We denote by WC the set of points (x,y,z), belonging to the white cubes, and respectively by BC the set of points belonging to the black cubes (fig.3.1). The cube size is l and the angle between two sides is 2α. In the present study, we consider $\alpha = \pi/4$.

To obtain a material with negative stiffness to be included into BC cells, let us introduce the negative stiffness mechanism. This mechanism is based on a fictitious technological procedure using a vapor deposition technique to produce an initial deformed chess material. The negative stiffness mechanism is characterized by: 1) each cell BC is deformed biaxial in the [111] direction, becoming trigonal (consider the case of a large strain ε, negative or positive, so that at least one of the constants of BC to be negative); 2) the deformation assures the continuity of the displacements and normal components of the stress tensor on the boundaries of the cubes. So, to define the negative stiffness mechanism, a set of imaginary cutting, straining and welding operations must be used in the spirit of Eshelby [12].

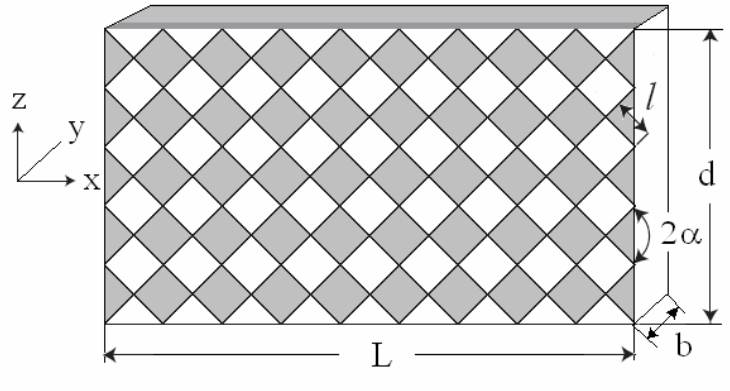

Fig.3.1. The composite plate

The Eshelby's two steps are displayed in fig.3.2: 1) The first step is to remove the black cell BC and allow it to undergo a stress-free biaxial deformation (fig. 3.2a); 2) The second step is to apply a surface biaxial traction to BC and WC cells (fig. 3.2b). This brings the black cells BC back to the matrix, by assuring the continuity of displacements and normal stresses across the boundaries (fig. 3.3).

As result, the composite will contain white cells with elastic constants in a cubic system

$$C_1 = \{C_{11}, C_{44}, C_{12}\} = \{C_{11}, C_{11}, C_{44}, C_{44}, C_{12}, C_{12}, 0, 0\},$$

and black cells with elastic constants in a trigonal system

$$C_2 = \{\tilde{C}_{11}, \tilde{C}_{22}, \tilde{C}_{44}, \tilde{C}_{55}, \tilde{C}_{12}, \tilde{C}_{23}, \tilde{C}_{25}, \tilde{C}_{35}\}.$$

The quantities $\tilde{C}_{ij}$ are given by (Chiroiu [13, [14], Chiroiu and Chiroiu [15, Teodorescu, Baral, Hilliard, Jetterson and Miyano [16])

$$\tilde{C}_{11} = \frac{(1+\varepsilon)^2}{36(1+\varepsilon_3)} C'_{11}, \quad \tilde{C}_{12} = \frac{(1+\varepsilon)^2}{12(1+\varepsilon_3)} C'_{12}, \quad \tilde{C}_{22} = \frac{1}{6}\left(-\frac{(1+\varepsilon)^2}{2(1+\varepsilon_3)} + \frac{1}{3}(1+\varepsilon_3) \right) C'_{22},$$

$$\tilde{C}_{23} = \frac{1}{6}(1+\varepsilon_3) C'_{12} \quad \tilde{C}_{25} = \frac{(1+\varepsilon_3)^3}{9(1+\varepsilon)^2} C'_{33}, \quad \tilde{C}_{44} = \frac{1}{6}\left(-\frac{1}{2}\frac{(1+\varepsilon)^3}{(1+\varepsilon_3)^2} + \frac{1}{3}(1+\varepsilon_3) \right) C'_{44}, \tag{3.1}$$

$$\tilde{C}_{55} = \frac{1}{4}\left(-\frac{(1+\varepsilon_3)^2}{3(1+\varepsilon)} + \frac{1}{2}(1+\varepsilon) \right) C'_{66}, \quad \tilde{C}_{35} = \left(\frac{(1+\varepsilon_3)^3}{7(1+\varepsilon)^2} + \frac{1}{9}(1+\varepsilon) \right) C'_{33},$$

with

$$C'_{11} = (c_{11} + c_{12} + 2c_{44})/2, \quad C'_{22} = (c_{11} + 2c_{12} - c_{44})/6, \quad C'_{44} = (c_{11} + 2c_{12} + 4c_{44})/3,$$

$$`= C'_{12} = (c_{11} + 5c_{12} - 2c_{44}), \quad C'_{33} = (c_{11} + 2c_{12} - 2c_{44})/3, \quad C'_{66} = (c_{11} - c_{12} + c_{44})/6.$$

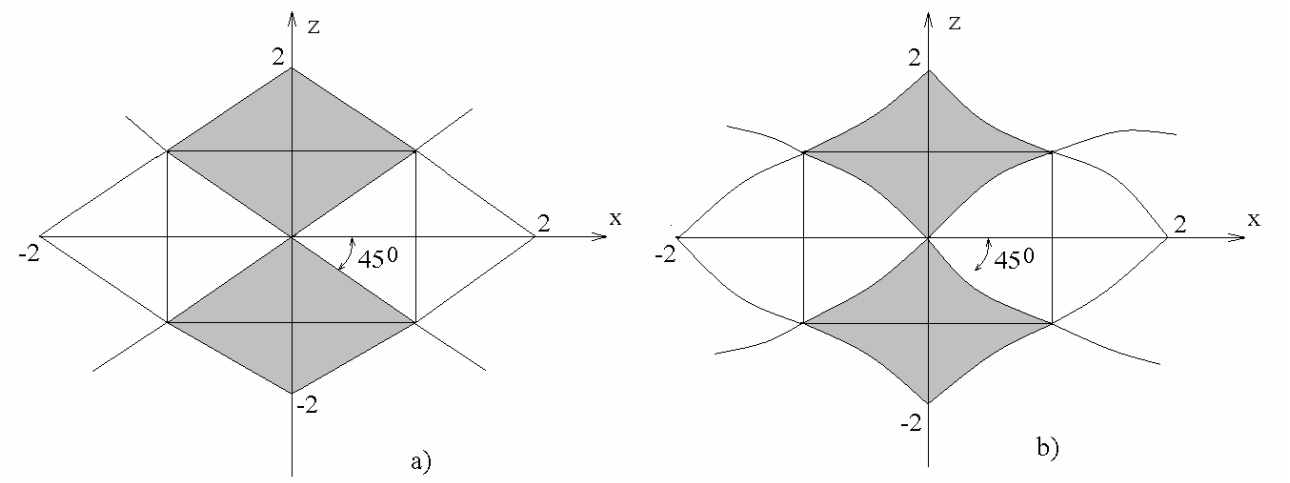

Fig. 3.2. The Eshelby's steps: a) remove the BC and allow it to undergo a stress-free biaxial deformation; b) apply a surface biaxial traction to BC and WC.

The biaxial deformations in the Voigt's notation are

$$\varepsilon_{11} = \varepsilon_1, \quad \varepsilon_{22} = \varepsilon_2, \quad \varepsilon_{33} = \varepsilon_3, \quad 2\varepsilon_{23} = \varepsilon_4, \quad 2\varepsilon_{13} = \varepsilon_5, \quad 2\varepsilon_{12} = \varepsilon_6,$$

$$\varepsilon_1 = \varepsilon_2 = \varepsilon, \quad \varepsilon_3 = -2\frac{C'_{12}}{C'_{33}}\varepsilon, \quad \varepsilon_4 = \varepsilon_5 = \varepsilon_6 = 0$$

It results from (12), that $\tilde{C}_{11}, \tilde{C}_{12}, \tilde{C}_{23}, \tilde{C}_{25}, \tilde{C}_{35} > 0$, $\tilde{C}_{22}, \tilde{C}_{44} < 0$ for $\varepsilon > 0.09$, and $\tilde{C}_{55} < 0$ for $\varepsilon < -0.06$ (figs.2.5, 2.6). In the following, the elastic constants of the composite are defined as

$$\hat{C}(x,y,z) = \{\hat{C}_1, ...\hat{C}_8\} = \begin{cases} C_1, & (x,y,z) \in WC, \\ C_2, & (x,y,z) \in BC. \end{cases} \tag{3.2}$$

Each cell is a cube with diagonals parallel to the axes (x, y, z) of length 2. The black cells are initially deformed, so that on the boundary of the cells there are satisfied the continuity conditions for displacements and normal stress components. The energy density of the medium is

$$W(u)=\frac{1}{2}\hat{C}_1\varepsilon_{11}^2+\frac{1}{2}\hat{C}_2(\varepsilon_{22}^2+\varepsilon_{33}^2)+\frac{1}{2}\hat{C}_3\varepsilon_{23}^2+\frac{1}{2}\hat{C}_4(\varepsilon_{12}^2+\varepsilon_{13}^2)+\hat{C}_5\varepsilon_{11}(\varepsilon_{22}+\varepsilon_{33})+\hat{C}_6\varepsilon_{22}\varepsilon_{33}$$
$$+\hat{C}_7(\varepsilon_{22}\varepsilon_{13}+\varepsilon_{23}\varepsilon_{12})+\hat{C}_8\varepsilon_{33}\varepsilon_{13}, \tag{3.3}$$

where $\varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i})$, $i, j=1,2,3$ is the strain tensor, u is the displacement vector, and $\hat{C}_j$, $j=1,...,8$ is defined by (3.2).

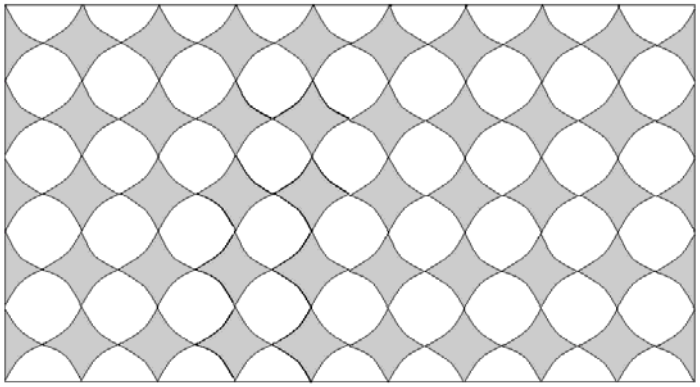

Fig. 3.3. The black cells BC are embedded back into the matrix.

We write $\overline{W}(u)=\frac{1}{8}\int_{\Pi}W(u)\mathrm{d}x\mathrm{d}y\mathrm{d}z$ where $|\Pi|=8$ is the volume of the cell. According to the Voigt notation, we note the strain and stress components by

$$\varepsilon_{11}=S_1,\ \varepsilon_{22}=S_2,\ \varepsilon_{33}=S_3,\ \varepsilon_{23}=S_4,\ \varepsilon_{13}=S_5,\ \varepsilon_{12}=S_6,$$

$$\sigma_{11}=T_1,\ \sigma_{22}=T_2,\ \sigma_{33}=T_3,\ \sigma_{23}=T_4,\ \sigma_{13}=T_5,\ \sigma_{12}=T_6.$$

The strain-displacement relations are

$$S_1=\frac{\partial u_1}{\partial x},\ S_2=\frac{\partial u_2}{\partial y},\ S_3=\frac{\partial u_3}{\partial z},\ S_4=\frac{\partial u_3}{\partial y}+\frac{\partial u_2}{\partial z},\ S_5=\frac{\partial u_1}{\partial z}+\frac{\partial u_3}{\partial x},\ S_6=\frac{\partial u_2}{\partial x}+\frac{\partial u_1}{\partial y},$$

where u_i, $i=1,2,3$ are displacements in the directions x, y and z, respectively, and S_i, $i=1,2,...,6$ are the strain components. The symmetric stress tensor σ_{ij}, $i, j=1,2,3$ is given by

$$\sigma_{ij}=\frac{\partial W}{\partial \varepsilon_{ij}}.$$

The stress-strain relations are

$$T_1=\tilde{C}_{11}S_1+C_{12}(S_2+S_3),\ T_2=\tilde{C}_{12}S_1+\tilde{C}_{22}S_2+\tilde{C}_{23}S_3+\tilde{C}_{25}S_5,\ T_3=\tilde{C}_{13}S_1+\tilde{C}_{33}S_3+\tilde{C}_{35}S_5,$$
$$T_4=\tilde{C}_{44}S_4+\tilde{C}_{22}S_2+\tilde{C}_{23}S_3+\tilde{C}_{25}S_6,\ T_5=\tilde{C}_{55}S_5+\tilde{C}_{35}S_3,\ T_6=\tilde{C}_{55}S_6+\tilde{C}_{25}S_4. \tag{3.4}$$

The equilibrium equations are valid in Π/Γ as

$$T_{1,x}+T_{6,y}+T_{5,z}=0,\ T_{6,x}+T_{2,y}+T_{4,z}=0,\ T_{5,x}+T_{4,y}+T_{3,z}=0, \tag{3.5}$$

$$\Gamma = \{x, y, z \in \Pi, | x | = | y | = | z |\},$$

where ρ is the density of both WC and BC. On Γ we write

$$[u] = 0\,,\ [t(u)] = 0\,, \tag{3..6}$$

where the square brackets mean the discontinuity across Γ, and $t_j(u) = \sigma_{ij} n_i$ where n is the unity normal to Γ. On the faces $z = \pm d/2$ it follows

$$T_5 \mid_{z=\pm d/2} = T_4 \mid_{z=\pm d/2} = T_3 \mid_{z=\pm d/2} = 0\,. \tag{3.7}$$

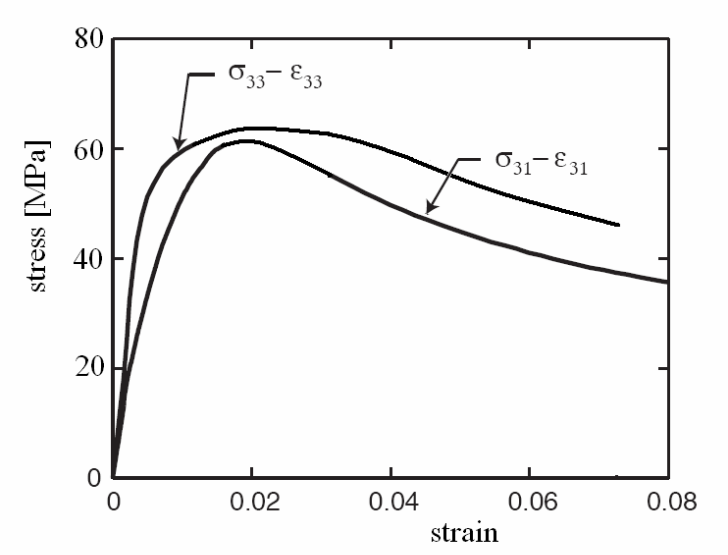

Fig.3.4. The out-of plane compressive and shear responses of the composite plate.

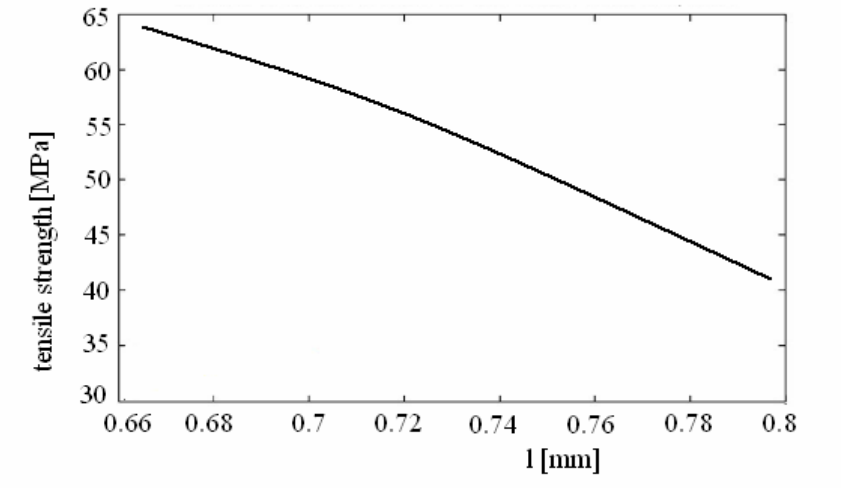

Fig.3.5. The tensile strength as a function of dimension l of the cell..

The solutions u_i of (3.5)-(3.7) are determined analytically. They have singularities at the corner points of the black cells caused by stretching (Berlyand and Kozlov [11]). The surface traction applied to BC and WC is calculated from (3.4) and it is given by

$$\tilde{t}_{22} = \tilde{t}_{33} = \sigma,\ \text{others}\ \tilde{t}_{ij} = 0.$$

The presence of a negative stiffness of the cells BC does not affect the stability of the composite. Negative stiffness entails a reversal of the usual codirectional relationship between force and displacement in deformed media.

For example, the out-of plane compressive and the transverse shear responses are plotted in fig.3.4 for Cu (positive stiffness material) and Cu (negative stiffness material deformed at a strain of 0.1 ($d = 5\text{mm}$, $L = 18\text{mm}$, $b = 3\text{mm}, l = \sqrt{2}/2\text{mm}$). The initial deformation keeps the negative stiffness black cells BC embedded into the matrix assuring the stability of the material. From fig.3.4 we see that the composite exhibits the compressive and shear responses which are higher than that of other cellular composites (Côté, Deshpande, Fleck and Evans [17], Deshpande, Fleck, and Ashby [18], Kaw [19]). Fig. 3.5 represents the tensile strength as a function of the dimension l of the cell. As can be seen, the tensile strength decreases with l ..

4. CONCLUSIONS

In this paper we assumed an FCC crystal cell belonging to the cubic system subjected to a biaxial deformation in the [111] direction. The crystal becomes trigonal, and at least one of the constants is negative for large strains. It is shown that the constants C_{22}, C_{44} and C_{55} are negative for large deformations (for Cu).

The negative stiffness mechanism is introduced next. It exerts an opposite force which amplifies the deformations and cancels out the stiffness into a material. The negative stiffness mechanism is defined in terms of the Eshelby's two steps: 1) remove the black cells BC from the composite and allow them to undergo a stress-free biaxial deformation in the [111] direction, becoming trigonal; 2) apply a surface biaxial traction to the black cells BC and the white cells WC to bring the black cells BC back into the matrix, by assuring the continuity of displacements and normal stresses across the boundaries.

As a consequence, if a negative stiffness material is embedded into a conventional matrix of material, it results a composite characterized by a greater deformation in and near inclusions than the composite as a whole. A new architecture for a cellular elastic solid, based on a chess board structure with positive stiffness material (white cells WC) and negative stiffness material (black cells BC) is advanced.

In this paper, the out-of plane compressive, the transverse shear responses of the composite and the tensile strength as a function of dimension of the cell are calculated. We conclude that the composite exhibits the compressive, the tensile strength and the shear responses which are higher than that of other cellular composites. One motivation for studying the properties of composites with negative stiffness components is the belief that such information might prove useful in finding the new architecture for materials with higher stiffness and damping properties.

ACKNOWLEDGEMENT

The authors acknowledge the financial support of the PNII project-Idei, Grant nr. 106/2007, code ID-247/2007.

REFERENCES

1. SANDLER, S., WRIGHT, J.P., *Theoretical foundation for large scale computations of nonlinear material behavior*, ed by S.Nemat Nasser, R.J.Asaro and G.A. Hegemier, Martinus Nijhoff, pp.285–296, 1984.
2. CHIROIU, V., MUNTEANU, L., DUMITRIU, D., BELDIMAN, M., SECARA, C., *On the theory of materials with negative stiffness components* (in press) 2007.
3. LAKES, R.S., *Extreme damping in compliant composites with a negative stiffness phase*, Philos. Mag. Lett. 81, pp.95–100, 2001.
4. LAKES, R.S., Extreme damping in composite materials with a negative stiffness phase, Phys. Rev. Lett. 86, 13, pp.2897–2900, 2001.
5. LAKES, R.S., LEE, T., BERSIE, A., WANG, Y.C., *Extreme damping in composite materials with negative stiffness inclusions*, Nature 410, pp.565–567, 2001.

6. LAKES, R. S., DRUGAN, W.J., *Dramatically stiffer elastic composite materials due to a negative stiffness phase*?, J. of the Mechanics and Physics of Solids, 50, pp.979–1009, 2002.
7. TEODORESCU, P.P., BADEA, T., MUNTEANU, L., ONISORU, J., *On the wave propagation in composite materials with a negative stiffness phase*, New Trends in Continuum Mechanics, Theta Series in Advanced Math. (ed. M.Mihailescu-Suliciu), pp.295–302, Editura Thetha Foundation, Bucuresti (2005).
8. JANKOWSKI, A.F., TSAKALAKOS, T., *The effect of strain on the elastic constants of noble metals*, J. Phys. F: Met. Phys., **15**, pp.1279–1292, 1985.
9. JANKOWSKI, A.F., *Modelling the supermodulus effect in metallic multilayers*, J. Phys. F: Met. Phys., **18**, pp.413–427, 1988.
10. DELSANTO, P.P., PROVENZANO, V., UBERALL, H., *Coherency strain effects in metallic bilayers*, J. Phys.: Condens. Matter**, 4**, pp.3915–3928, 1992.
11. Berlyand, L.V., Kozlov, S.M., *Asymptotic of the homogenized moduli for the elastic chess-board composite*, Arch. Rational Mech., Anal., 118, pp.95–112, 1992
12. ESHELBY, J.D., *The determination of the elastic foeld of an ellipsoidal inclusion and related problem*, Proc. R. Soc., A241, pp.376–396, 1957.
13. CHIROIU, C., *Ultrasound waves in initial deformed elastic metallic bilayers*, part I : *Theoretical Model*, Rev. Roum. Sci. Techn, **44**, *1*, pp.73–97, 1999.
14. CHIROIU, C., *Ultrasound waves in initial deformed elastic metallic bilayers*, part II : *Numerical Simulations*, Rev. Roum. Sci. Techn, **44**, *2*, pp.219–229, 1999.
15. CHIROIU, V.,CHIROIU, C., *Probleme inverse in mecanica*, Ed. Academiei, 2003.
16. BARAL, D., HILLIARD, J.E., JETTERSON, J.B., MIYANO, K., Determination of the primary elastic constants from thin foils having a strong texture, J. Appl., Phys. **53**, 5, 1982.
17. CÔTÉ, F., DESHPANDE, V.S., FLECK, N.A., EVANS, A.G., *The out-of-plane compressive behavior of metallic honeycombs,* J. Mat. Sci. and Engineering A, **380**, pp.272–280, 2004.
18. DESHPANDE, V.S., FLECK, N.A., ASHBY, M.F.*, Effective properties of the octet-truss lattice material*, J. Mech. Phys. Solids, 49, pp.1747–1769, 2001.
19. KAW, A.K., *Mechanics of composite materials*, CRC Press–Mechanical Engineering series, 2005.

Received November 5, 2007