



## ON ZALMAI'S SEMIPARAMETRIC DUALITY MODEL FOR MULTIOBJECTIVE FRACTIONAL PROGRAMMING WITH $n$ -SET FUNCTIONS

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New duality results for a semiparametric duality model are given for a fractional programming problem involving  $n$ -set functions.

*Key words:* multiobjective programming,  $n$ -set function, duality, generalized convexity.

### 1. INTRODUCTION AND PRELIMINARIES

We consider the frame of optimization theory for  $n$ -set [2,5,8]. For formulating and proving various duality results, we use the class of generalized convex  $n$ -set functions called  $(F, b, \varphi, \rho, \theta)$ -univex functions, which were defined in Zalmai [11]. Until now,  $F$  was assumed to be a sublinear function in the third argument. In our approach, we suppose that  $F$  is a convex function in the third argument, as in Preda *et al.* [7,8] and Bățătorescu *et al.* [1].

Let  $(X, A, \mu)$  be a finite atomless measure space with  $L_1(X, A, \mu)$  separable, and let  $d$  be the pseudometric on  $A^n$  defined by

$$d(R, S) := \left[ \sum_{k=1}^n \mu^2(R_k \Delta S_k) \right]^{1/2}$$

where  $R = (R_1, \dots, R_n)$ ,  $S = (S_1, \dots, S_n) \in A^n$  and  $\Delta$  stands for symmetric difference. Thus,  $(A^n, d)$  is a pseudometric space.

For  $h \in L_1(X, A, \mu)$  and  $T \in A$  with indicator (characteristic) function  $\chi_T \in L_\infty(X, A, \mu)$ , the integral  $\int h d\mu$  is denoted by  $\langle h, \chi_T \rangle$ .

**Definition 1.1.** [4] A function  $f: A \rightarrow \mathbb{R}$  is said to be differentiable at  $S^* \in A$  if there exist  $Df(S^*) \in L_1(X, A, \mu)$ , called the derivative of  $f$  at  $S^*$ , and  $V_f: A \times A \rightarrow \mathbb{R}$  such that

$$f(S) = F(S^*) + \langle Df(S^*), \chi_S - \chi_{S^*} \rangle + V_f(S, S^*)$$

for each  $S \in A$ , where  $V_f(S, S^*)$  is  $o(d(S, S^*))$ , that is,

$$\lim_{d(S, S^*) \rightarrow 0} \frac{V_f(S, S^*)}{d(S, S^*)} = 0.$$

**Definition 1.2.** [2] A function  $g : A^n \rightarrow \mathbb{R}$  is said to have a partial derivative at  $S^* = (S_1^*, \dots, S_n^*) \in A^n$  with respect to its  $i$ -th argument if the function  $f(S_i) = g(S_1^*, \dots, S_{i-1}^*, S_i, S_{i+1}^*, \dots, S_n^*)$  has derivative  $Df(S_i^*)$ ,  $i \in \underline{n} = \{1, 2, \dots, n\}$ .

We define  $D_i g(S^*) = Df(S_i^*)$  and write  $Dg(S^*) = (D_1 g(S^*), \dots, D_n g(S^*))$ .

**Definition 1.3.** [2] A function  $g : A^n \rightarrow \mathbb{R}$  is said to be differentiable at  $S^*$  if there exist  $Dg(S^*)$  and  $W_g : A^n \times A^n \rightarrow \mathbb{R}$  such that

$$G(S) = G(S^*) + \sum_{i=1}^n \left\langle D_i G(S^*), \chi_{S_i} - \chi_{S_i^*} \right\rangle + W_G(S, S^*),$$

where  $W_G(S, S^*)$  is  $o(d(S, S^*))$  for all  $S \in A^n$ .

Let  $\mathbb{R}^q$  be the  $q$ -dimensional Euclidean space and  $\mathbb{R}_+^q$  its positive orthant, i.e.

$$\mathbb{R}_+^q = \{x = (x_1, \dots, x_q) \in \mathbb{R}^q : x_j \geq 0, j = 1, \dots, q\}.$$

For any vectors  $x = (x_1, \dots, x_n) \in \mathbb{R}^n$  and  $y = (y_1, \dots, y_n) \in \mathbb{R}^n$ , we put  $x \leq y$  iff  $x_i \leq y_i$ , for each  $i \in \underline{n} = \{1, 2, \dots, n\}$ ;  $x \leq y$  iff  $x \leq y$ , with  $x \neq y$ ;  $x < y$  iff  $x_i < y_i$ , for each  $i \in \underline{n} = \{1, 2, \dots, n\}$ ;  $x \ll y$  means the negation of  $x \leq y$ . Clearly,  $x \in \mathbb{R}_+^n$  iff  $x \geq 0$ .

In this paper, we consider the multiobjective fractional subset programming problem

$$(P) \quad \min \Phi(S) = \left( \frac{f_1(S)}{g_1(S)}, \frac{f_2(S)}{g_2(S)}, \dots, \frac{f_p(S)}{g_p(S)} \right)$$

subject to

$$h_j(S) \leq 0, \quad j \in \underline{q} = \{1, 2, \dots, q\}, \quad S \in A^n,$$

where  $A^n$  is the  $n$ -fold product of the  $\sigma$ -algebra  $A$  of subsets of a given set  $X$ ,  $f_i : A^n \rightarrow \mathbb{R}$ ,  $g_i : A^n \rightarrow \mathbb{R}$ ,  $i \in \underline{p} = \{1, 2, \dots, p\}$ , and  $h_j : A^n \rightarrow \mathbb{R}$ ,  $j \in \underline{q}$ , such that  $g_i(S) > 0$  for each  $i \in \underline{p}$  and all  $S \in \mathcal{P}$ . We denote by  $\mathcal{P} = \{S \in A^n : h_j(S) \leq 0, j \in \underline{q}\}$  the set of all feasible solutions to (P).

**Definition 1.4.** A feasible solution  $S^0 \in \mathcal{P}$  is said to be an efficient solution to (P) if there exists no other feasible solution  $S \in \mathcal{P}$  such that

$$\left( \frac{f_1(S)}{g_1(S)}, \frac{f_2(S)}{g_2(S)}, \dots, \frac{f_p(S)}{g_p(S)} \right) \leq \left( \frac{f_1(S^0)}{g_1(S^0)}, \frac{f_2(S^0)}{g_2(S^0)}, \dots, \frac{f_p(S^0)}{g_p(S^0)} \right)$$

In the following we consider  $F : A^n \times A^n \times \mathbb{R} \rightarrow \mathbb{R}$  and a differentiable function  $f : A^n \rightarrow \mathbb{R}$ . The definitions below unify the concepts of  $(F, \rho)$ -convexity,  $(F, \rho)$ -pseudoconvexity,  $(F, \rho)$ -quasiconvexity from Preda [6] and univexity, pseudounivexity, quasiunivexity from Mishra [3].

Let  $b : A^n \times A^n \rightarrow \mathbb{R}_+$ ,  $\theta : A^n \times A^n \rightarrow A^n \times A^n$  such that  $S \neq S^* \Rightarrow \theta(S, S^*) \neq (0, 0)$ ,  $\varphi : \mathbb{R} \rightarrow \mathbb{R}$ , and a real number  $\rho$ .

**Definition 1.5.** [11] A function  $f$  is said to be (strictly)  $(F, b, \varphi, \rho, \theta)$ -univex at  $S^*$  if

$$\varphi(F(S) - F(S^*))(>) \geq F(S, S^*; b(S, S^*)DF(S^*)) + \rho d^2(\theta(S, S^*))$$

for each  $S \in A^n$ .

**Definition 1.6.** [11] A function  $f$  is said to be (strictly)  $(F, b, \varphi, \rho, \theta)$ -pseudounivex at  $S^*$  if

$$F(S, S^*; b(S, S^*)Df(S^*)) \geq -\rho d^2(\theta(S, S^*)) \Rightarrow \varphi(f(S) - f(S^*)) (>) \geq 0$$

for each  $S \in A^n$ ,  $S \neq S^*$ .

**Definition 1.7.** [11] A function  $f$  is said to be (prestrictly)  $(F, b, \varphi, \rho, \theta)$ -quasiunivex at  $S^*$  if

$$\varphi(f(S) - f(S^*)) (<) \leq 0 \Rightarrow F(S, S^*; b(S, S^*)Df(S^*)) \leq -\rho d^2(\theta(S, S^*))$$

for each  $S \in A^n$ .

For problem (P), Zalmai [10] gave the necessary conditions for efficiency below.

**Theorem 1.1.** Assume that  $f_i, g_i, i \in \underline{p}$ , and  $h_j, j \in \underline{q}$ , are differentiable at  $S^* \in A^n$ , and that for each  $i \in \underline{p}$  there exists  $\hat{S}_i \in A^n$ , such that

$$h_j(S^*) + \sum_{k=1}^n \left\langle D_k h_j(S^*), \chi_{\hat{S}_k} - \chi_{S_k^*} \right\rangle < 0, j \in \underline{q},$$

and for each  $l \in \underline{p} \setminus \{i\}$  we have

$$\sum_{k=1}^n \left\langle g_i(S^*)D_k f_l(S^*) - f_l(S^*)D_k g_i(S^*), \chi_{\hat{S}_k} - \chi_{S_k^*} \right\rangle < 0.$$

If  $S^*$  is an efficient solution to (P), then there exists  $u^* \in U = \{u \in \mathbb{R}^p : u > 0, \sum_{i=1}^p u_i = 1\}$  and  $v^* \in \mathbb{R}_+^q$

such that

$$\sum_{k=1}^n \left\langle \sum_{i=1}^p u_i^* [g_i(S^*)D_k f_i(S^*) - f_i(S^*)D_k g_i(S^*)] + \sum_{j=1}^q v_j^* D_k h_j(S^*), \chi_{S_k} - \chi_{S_k^*} \right\rangle \geq 0, \quad (1)$$

for all  $S \in A^n$ ,  $v_j^* h_j(S^*) = 0, j \in \underline{q}$ .

We shall refer to an efficient solution  $S^*$  to (P) satisfying the first two conditions in Theorem 1.1 for some  $\hat{S}_i, i \in \underline{p}$ , as a *normal* efficient solution.

## 2. THE DUALITY MODEL AND DUALITY RESULTS

In this section we present a general duality model for (P). Here we use two partitions of the index sets  $\underline{q}$  and  $\underline{p}$ , respectively.

Let  $\{I_0, I_1, \dots, I_k\}$  be a partition of the index set  $\underline{p}$  and  $\{J_0, J_1, \dots, J_m\}$  a partition of the index set  $\underline{q}$  such that  $K = \{0, 1, \dots, k\} \subset M = \{0, 1, \dots, m\}$ , and, for fixed  $S, u$  and  $v$ , and  $t \in K$  let the function  $\Omega_t(S; \cdot, u, v): A^n \rightarrow \mathbb{R}$  be defined by

$$\Omega_t(S, T, u, v) = \sum_{i \in I_t} u_i [f_i(S)g_i(T) - f_i(T)g_i(S)] + \sum v_j h_j(T)$$

We associate with problem (P) the dual problem

$$(D) \quad \max \delta(T, u, v) = \left( \frac{f_1(T)}{g_1(T)}, \frac{f_2(T)}{g_2(T)}, \dots, \frac{f_p(T)}{g_p(T)} \right)$$

subject to

$$F \left( S, T; b(S, T) \sum_{i=1}^p u_i [G_i(T) DF_i(T) - F_i(T) DG_i(T)] + \sum_{j=1}^q v_j Dh_j(T) \right) x \geq 0 \quad \forall S \in A^n$$

$$\sum_{j \in J_t} v_j h_j(T) \geq 0, \quad t \in M$$

$$T \in A^n, \quad u \in U, \quad v \in \mathbb{R}_+^q.$$

In the following we consider a convex function  $F(S, T; \cdot): L_1(X, A, \mu) \rightarrow \mathbb{R}$  and  $\Lambda_t(\cdot, v^*): A^n \rightarrow \mathbb{R}$ ,  $\Lambda_t(T, v^*) = \sum_{j \in J_t} v_j^* h_j(T)$ ,  $t \in M$ .

The result below establishes several versions of weak duality related to problems (P) and (D).

**Theorem 2.1** (Weak duality). *Let  $S$  and  $(T, u, v)$  be arbitrary feasible solutions to (P) and (D), respectively, and assume that any one of the following sets of hypotheses is satisfied:*

(a) (i)  $2k\Omega_t(\cdot, T, u, v)$  is strictly  $(F, b, \varphi_t, \rho_t, \theta)$ -pseudounivex at  $T$ ,  $\varphi_t$  is increasing, and  $\varphi_t(0) = 0$  for each  $t \in K$ ;

(ii)  $2(m-k)\Lambda_t(\cdot, v)$  is  $(F, b, \varphi_t, \rho_t, \theta)$ -quasiunivex at  $T$ ,  $\varphi_t$  is increasing, and  $\varphi_t(0) = 0$  for each  $t \in M \setminus K$ ;

$$(iii) \frac{1}{k} \sum_{t \in K} \rho_t + \sum_{t \in M \setminus K} \frac{\rho_t}{m-k} \geq 0;$$

(b) (i)  $2k\Omega_t(\cdot, T, u, v)$  is prestrictly  $(F, b, \varphi_t, \rho_t, \theta)$ -quasiunivex at  $T$ ,  $\varphi_t$  is increasing, and  $\varphi_t(0) = 0$  for each  $t \in K$ ;

(ii)  $2(m-k)\Lambda_t(\cdot, v)$  is strictly  $(F, b, \varphi_t, \rho_t, \theta)$ -pseudounivex at  $T$ ,  $\varphi_t$  is increasing, and  $\varphi_t(0) = 0$  for each  $t \in M \setminus K$ ;

$$(iii) \frac{1}{k} \sum_{t \in K} \rho_t + \sum_{t \in M \setminus K} \frac{\rho_t}{m-k} \geq 0;$$

(c) (i)  $2k\Omega_t(\cdot, T, u, v)$  is prestrictly  $(F, b, \varphi_t, \rho_t, \theta)$ -quasiunivex at  $T$ ,  $\varphi_t$  is increasing, and  $\varphi_t(0) = 0$  for each  $t \in K$ ;

(ii)  $2(m-k)\Lambda_t(\cdot, v)$  is  $(F, b, \varphi_t, \rho_t, \theta)$ -quasiunivex at  $T$ ,  $\varphi_t$  is increasing, and  $\varphi_t(0) = 0$  for each  $t \in M \setminus K$ ;

$$(iii) \frac{1}{k} \sum_{t \in K} \rho_t + \sum_{t \in M \setminus K} \frac{\rho_t}{m-k} \geq 0;$$

(d) (i)  $3k_1\Omega_t(\cdot, T, u, v)$  is strictly  $(F, b, \bar{\varphi}_t, \bar{\rho}_t, \theta)$ -pseudounivex at  $T$  for each  $t \in K_1$ ,  $\bar{\varphi}_t$  is increasing, and  $\bar{\varphi}_t(0) = 0$  for each  $t \in K_1$ ,  $3k_2\Omega_t(\cdot, T, u, v)$  is prestrictly  $(F, b, \bar{\varphi}_t, \bar{\rho}_t, \theta)$ -quasiunivex at  $T$  for each  $t \in K_2$ ,  $\bar{\varphi}_t$  is increasing, and  $\bar{\varphi}_t(0) = 0$  for each  $t \in K_2$ , where  $\{K_1, K_2\}$  is a partition of  $K$ , with  $K_1 \neq \emptyset$ ,  $K_2 \neq \emptyset$ ,  $k_1 = |K_1|$ ,  $k_2 = |K_2|$ ;

(ii)  $3(m-k)\Lambda_t(\cdot, v)$  is  $(F, b, \varphi_t, \rho_t, \theta)$ -quasiunivex at  $T$ ,  $\varphi_t$  is increasing, and  $\varphi_t(0) = 0$  for each  $t \in M \setminus K$ ;

$$(iii) \frac{1}{k_1} \sum_{t \in K_1} \rho_t + \frac{1}{k_2} \sum_{t \in K_2} \rho_t + \sum_{t \in M \setminus K} \frac{\rho_t}{m-k} \geq 0;$$

(e) (i)  $3k\Omega_t(\cdot, T, u, v)$  is prestrictly  $(F, b, \varphi_t, \rho_t, \theta)$ -quasiunivex at  $T$ ,  $\varphi_t$  is increasing, and  $\varphi_t(0) = 0$  for each  $t \in K$ ;

(ii)  $3(m_1 - k_1)\Lambda_t(\cdot, v)$  is strictly  $(F, b, \tilde{\varphi}_t, \tilde{\rho}_t, \theta)$ -pseudounivex at  $T$  for each  $t \in (M \setminus K)_1$ ,  $\tilde{\varphi}_t$  is increasing, and  $\tilde{\varphi}_t(0) = 0$  for each  $t \in (M \setminus K)_1$ ,  $3(m_2 - k_2)\Lambda_t(\cdot, v)$  is  $(F, b, \tilde{\varphi}_t, \tilde{\rho}_t, \theta)$ -quasiunivex at  $T$  for each  $t \in (M \setminus K)_2$ ,  $\tilde{\varphi}_t$  is increasing, and  $\tilde{\varphi}_t(0) = 0$  for each  $t \in (M \setminus K)_2$ , where  $\{(M \setminus K)_1, (M \setminus K)_2\}$  is a partition of  $M \setminus K$ , with  $(M \setminus K)_1 \neq \emptyset$ ,  $m_1 = |(M \setminus K)_1|$ ,  $(M \setminus K)_2 \neq \emptyset$ ,  $m_2 = |(M \setminus K)_2|$ ;

$$(iii) \frac{1}{k} \sum_{t \in K} \rho_t + \sum_{t \in (M \setminus K)_1} \frac{\rho_t}{m_1 - k_1} + \sum_{t \in (M \setminus K)_2} \frac{\rho_t}{m_2 - k_2} \geq 0;$$

(f) (i)  $4k_1\Omega_t(\cdot, T, u, v)$  is strictly  $(F, b, \bar{\varphi}_t, \bar{\rho}_t, \theta)$ -pseudounivex at  $T$ ,  $\bar{\varphi}_t$  is increasing, and  $\bar{\varphi}_t(0) = 0$  for each  $t \in K_1$ ,  $4k_2\Omega_t(\cdot, T, u, v)$  is prestrictly  $(F, b, \bar{\varphi}_t, \bar{\rho}_t, \theta)$ -quasiunivex at  $T$ ,  $\bar{\varphi}_t$  is increasing and  $\bar{\varphi}_t(0) = 0$  for each  $t \in K_2$ , where  $\{K_1, K_2\}$  is a partition of  $K$ , with  $K_1 \neq \emptyset$ ,  $K_2 \neq \emptyset$ ,  $k_1 = |K_1|$ ,  $k_2 = |K_2|$ ;

(ii)  $4(m_1 - k_1)\Lambda_t(\cdot, v)$  is strictly  $(F, b, \tilde{\varphi}_t, \tilde{\rho}_t, \theta)$ -pseudounivex at  $T$ ,  $\tilde{\varphi}_t$  is increasing, and  $\tilde{\varphi}_t(0) = 0$  for each  $t \in (M \setminus K)_1$ ,  $4(m_2 - k_2)\Lambda_t(\cdot, v)$  is  $(F, b, \tilde{\varphi}_t, \tilde{\rho}_t, \theta)$ -quasiunivex at  $T$ ,  $\tilde{\varphi}_t$  is increasing, and  $\tilde{\varphi}_t(0) = 0$  for each  $t \in (M \setminus K)_2$ , where  $\{(M \setminus K)_1, (M \setminus K)_2\}$  is a partition of  $M \setminus K$ , with  $(M \setminus K)_1 \neq \emptyset$ ,  $m_1 = |(M \setminus K)_1|$ ,  $(M \setminus K)_2 \neq \emptyset$ ,  $m_2 = |(M \setminus K)_2|$ ;

$$(iii) \frac{1}{k_1} \sum_{t \in K_1} \rho_t + \frac{1}{k_2} \sum_{t \in K_2} \rho_t + \sum_{t \in (M \setminus K)_1} \frac{\rho_t}{m_1 - k_1} + \sum_{t \in (M \setminus K)_2} \frac{\rho_t}{m_2 - k_2} \geq 0;$$

(iv)  $K_1 \neq \emptyset$  or  $(M \setminus K)_1 \neq \emptyset$  or

$$\frac{1}{k_1} \sum_{t \in K_1} \rho_t + \frac{1}{k_2} \sum_{t \in K_2} \rho_t + \sum_{t \in (M \setminus K)_1} \frac{\rho_t}{m_1 - k_1} + \sum_{t \in (M \setminus K)_2} \frac{\rho_t}{m_2 - k_2} > 0;$$

Then  $\Phi(S) \leq \delta(T, u, v)$ .

**Theorem 2.2.** (Strong duality). Let  $S^* \in \mathcal{P}$  be a normal efficient solution to (P), let  $F(S, S^*; Df(S^*)) = \sum_{k=1}^n \langle D_k f(S^*), \mathcal{X}_{S_k} - \mathcal{X}_{S_k^*} \rangle$  for any differentiable function  $f: A^n \rightarrow \mathbb{R}$  and  $S \in A^n$ , and assume that any one of the sets of hypotheses specified in Theorem 2.1. holds for all feasible solutions to (D). Then there exist  $u^* \in U$  and  $v^* \in \mathbb{R}_+^q$  such that  $(S^*, u^*, v^*)$  is an efficient solution of (D) and  $\Phi(S^*) = \delta(S^*, u^*, v^*)$ .

**Remark 2.1.** Using Theorems 2.1 and 2.2, and techniques from [5] and [10], we can also obtain a strict converse duality result.

For a detailed presentation of these results, the reader is referred to [9].

### 3. CONCLUSIONS

We have obtained duality results for a dual model of Zalmai [10], replacing the assumption of sublinearity by that of convexity. Similar results can be obtained for the other dual models from [10]. Also, almost all results of this type present in the literature can be extended to the case where  $F$  is not necessarily sublinear.

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