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ON THE DOUBLE COUPLE RADIATION PATTERN FOR A SLAB-TRACK SYSTEM

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The present work discusses the radiation of mechanical energy which occurs in modern slab-track railway systems. The track is modeled by a beam on elastic half-space, whereas a uniformly traveling constant load has been used to represent the axle load. The steady-state response of this system is studied by taking into account the mechanical radiation. An understanding of the radiation attenuation is important in the design of devices used for suppressing vibrations.

Key Words: Mechanical radiation, Slab-track railway system, Waves.

1. INTRODUCTION

Mechanical waves generated by body forces distributed over a finite region B of interior V and surface S of an unbounded medium are transmitted through the medium. These waves carry along the kinetic and potential energies over considerable distances. The result of this transmission of energy is that a part of this energy is lost for the body B . The energy leaves the body through its surface S . We name this lost energy the *radiation energy.* We also point out that the loss of energy by waves propagating to the exterior of the body is said to be the mechanical radiation (Chiroiu, Nicolae and Munteanu [1], Chiroiu and Nicolae [2]). The estimation of this radiation energy is crucial to the understanding of the physical structure-soil interaction. Our goal is to find this energy. This will be achieved through the modeling of the radiation force as a nonlocal residual. The nonlocal theory of elastodynamics is used here in order to include the existence of the radiation force within the domain of continuum mechanics. In this theory the balance laws contain nonlocal residuals of fields which are determined from the global statements. The nonlocal residuals are sufficient to account for the interaction of all parts of the body with the state of any material point in the body. The local form of the balance law is derived from the basic global balance law by including the nonlocal residual whose contribution to the global law is nil (Eringen and Edelen [3], Eringen [4],[5]).

Different aspects of dynamics of railway tracks were discussed by Dieterman and Metrikine [6], [7], Metrikine and Dieterman [8], Sheng, Jones and Thompson [9], [10]. Many researchers applied the 3D-model of a beam on elastic half-space, mainly to study different aspects of dynamics of railway track (Kononov and Wolfert [11], Takemiya [12], Takemiya and Bian [13], Picoux and Le Houédec [14], Chiroiu *et al.* [15] and Munteanu *et al.* [16]). Nonlinear aspects of wave focusing were studied by Dumitriu *et al*. [17] and Mailat *et al.* [18]. The dynamic response of a slab–track railway system, loaded by a running train axle has been analkyzed by Steenbergen and Metrikine [19]. The track has been modelled by a beam on elastic and viscoelastic half-space, whereas a uniformly traveling constant or harmonic load has been used to represent the axle load and its spectral components.

In considering the mechanical radiation generated in an unbounded medium by a nonlocal body force distribution, a couple-radiation pattern is introduced in this paper to describe the interface between the body and the half-space on the track response to dynamic loading.

2. NONLOCAL MODEL

Consider an open, 3D region of 3D Euclidean point space E^3 referred to the Cartesian coordinate x_k, $k=1,2,3$. Let B denote an initially undisturbed body with interior B of interior V and boundary S, which occupies at $t=0$ this region. Consider the displacement field $u(x,t)$ and the nonlocal residual body force per unit mass $\hat{f}(x,t)$ which satisfy the nonlocal motion equation for $t\in[0,T]$

$$\mu\Delta u+(\lambda+\mu)\nabla\nabla\cdot u=\rho(u_{,tt}+\hat{f}),\ \int_V\rho\hat{f}\,\mathrm{d}V=0,\ u=0, u_{,t}=0 \ \text{ for } t=0, \tag{2.1}$$

where ρ is the mass density, λ and μ are Lamé elastic constants, and comma represents the differentiation with respect to specified variable. The restriction $(2.1)_2$ is the statement of the fact that there is not net production of the body forces in the body. Using the Helmholtz decomposition of $\hat{f}(x,t)$ we have

$$\hat{f}=v_p^2\nabla f+v_s^2\nabla\times g, \tag{2.2}$$

where f and g are the scalar and the vector-valued nonlocal residuals, and $v_p=\sqrt{\dfrac{\lambda+2\mu}{\rho}}$, $v_s=\sqrt{\dfrac{\mu}{\rho}}$.

The problem we consider here is the elastodynamic radiation motion generated in an unbounded medium by nonlocal residual forces distributed over a finite region V of the medium. According to the completeness theorem there exists a scalar function $\varphi(x,t)$ and a vector-valued function $\psi(x,t)$ such that $u(x,t)$ is represented by (Achenbach [20]) as $u=\nabla\varphi+\nabla\times\psi$, $\nabla\cdot\psi=0$. The potentials φ and ψ satisfy the motion equations

$$\nabla^2\varphi=\frac{1}{v_p^2}\varphi_{,tt}+f,\ \nabla^2\psi=\frac{1}{v_s^2}\psi_{,tt}+g, \tag{2.3}$$

with homogeneous initial conditions

$$\varphi(x,0)=0,\ \varphi_{,t}(x,0)=0,\ \psi(x,0)=0,\ \psi_{,t}(x,0)=0. \tag{2.4}$$

The solutions of (2.3) and (2.4) can be expressed by the Green's function for the unbounded medium

$$\varphi(x,t)=\frac{1}{4\pi}\int_{B_p}\frac{f(\xi,\tau_p)}{|x-\xi|}\mathrm{d}V_\xi,\quad \psi(x,t)=\frac{1}{4\pi}\int_{B_s}\frac{g(\xi,\tau_s)}{|x-\xi|}\mathrm{d}V_\xi, \tag{2.5}$$

where $\mathrm{d}V_\xi=\mathrm{d}\xi_1\mathrm{d}\xi_2\mathrm{d}\xi_3$, $B_{p;s}$ are the spheres with center at x and radii $v_{p;s}t$, and

$$\tau_{p;s}=t-\frac{|x-\xi|}{v_{p;s}}. \tag{2.6}$$

The solutions (2.5) are called the *retarded potentials* [20]. A local disturbance of the medium is not instantaneously detected at positions that are at a distance from the region of sources. It takes time for a disturbance to propagate from its source to other positions.

Thus, the residuals f and g are calculated at the moment $\tau_{p;s}$ given by (2.6). The quantity $|x-\xi|/v_{p;s}$ represents the necessary time that the effect of a disturbance at ξ to reach the position x.

The solution of (2.1) and (2.2) is given by

$$4\pi u(x,t)=\int_{B_p}\nabla_x\times\frac{f(\xi,\tau_p)}{|x-\xi|}\mathrm{d}V_\xi+\int_{B_s}\nabla_x\times\frac{g(\xi,\tau_s)}{|x-\xi|}\mathrm{d}V_\xi, \tag{2.7}$$

where the operator ∇_x is calculated with respect to x.

Without loss of generality we may consider at $x=\xi$ a concentrated nonlocal force of magnitude $h(t)$ directed along the constant unit vector α

$$f(x,t)=\frac{1}{v_p^2}\nabla\cdot\left(\frac{h(t)}{|x-\xi|}\cdot\alpha\right),\quad g(x,t)=-\frac{1}{v_s^2}\nabla\times\left(\frac{h(t)}{|x-\xi|}\cdot\alpha\right). \tag{2.8}$$

Writing

$$\varphi=\nabla\cdot(\gamma_p\alpha),\quad \psi=-\nabla\times(\gamma_s\alpha),$$

equations (2.3) become

$$\nabla^2\gamma_{p;s}=\frac{1}{v_{p,s}^2}\left(\gamma_{p;s,tt}+\frac{h(t)}{|x-\xi|}\right). \tag{2.9}$$

3. RADIATIVE EQUATION

Consider the case when the position of the point P is lying at the distance $|x|$ larger than the linear dimension $l\approx V^{1/3}$ of the region over which the distribution of residuals is defined. Supposing that $|\xi|\ll l$ and $l\ll|x|$ the expression $\dfrac{1}{|x-\xi|}$ can be expanded in power series with respect to ξ/x

$$\begin{gathered}|x-\xi|=\left(|x|^2-2x\cdot\xi+|\xi|^2\right)^{1/2}=|x|\left(1+\left(\frac{\xi^2}{|x|^2}-2\frac{x\cdot\xi}{|x|^2}\right)\right)^{1/2},\\ \frac{1}{|x-\xi|}=\frac{1}{|x|}+\frac{x\cdot\xi}{|x|^3}+\frac{1}{2}\xi_\alpha\xi_\beta\left(\frac{3x_\alpha x_\beta}{|x|^5}-\frac{\delta_{\alpha\beta}}{|x|^3}\right),\end{gathered} \tag{3.1}$$

where ξ_α and x_α are the components of ξ and x. In $(3.1)_2$ we employed the usual summation over the repeated indices that is α and β, $\alpha,\beta=1,2,3$. We calculate the potentials in $\tau_{p,s}$ and obtain

$$\gamma_{p;s}(x,t)=\frac{x\cdot d(\tau_{p;s})}{4\pi|x|^2},\ d(\tau_{p,s})=\int_{B_{p;s}}\xi\frac{h(t)}{|x-\xi|}\mathrm{d}V_\xi, \tag{3.2}$$

where $d(\tau_{p,s}^0)$ is the double couple of the residual force which begins to act at $t=0$. We name the residual body force per unit mass the radiation force per unit mass. The radiative energy that leaves the body in the interval of time $\mathrm{d}t$ is

$$\mathrm{d}W_{rad}=-\frac{\beta}{6\pi|x|^2\xi v_s}a^2\mathrm{d}t, \tag{3.3}$$

where $a=u_{,tt}$ is the acceleration and β a constant. The nonlocal residual body force per unit mass $\hat{f}$ is the radiative force defined as the time variation of the acceleration field

$$\hat{f}(x,t)=f_{rad}(x,t)=\tau u_{,ttt}(x,t),\ \tau=\frac{\beta}{6\pi|x|^2\xi v_s}. \tag{3.4}$$

In (3.4) τ is the radiative time. The radiative force acts upon the body likewise a friction force. The rate of work of this force equals the radiation energy that leaves the body per unit time. The loss of energy of the body by mechanical radiation can be interpreted by the action upon this body of a radiation force f_{rad}.

By including the double couple radiation pattern into the motion equation (2.1) we obtain

$$\mu\Delta u + (\lambda+\mu)\nabla\nabla\cdot u = \rho(u_{,tt} + \tau u_{,ttt}) \,. \qquad (3.5)$$

4. A SLAB-TRACK RAILWAY SYSTEM

We apply the theory to investigate the dynamic response of a slab–track railway system to an axle of a train (fig. 1). The system consists of a rectangular beam of length l, width $2b$ and thickness h, subject to a moving load with a constant velocity V along the beam's surface, and lying on an elastic half-space at $z=0$. The beam cross-section is infinitely rigid and homogeneity of the model is assumed in the longitudinal direction. Let E_b, I_b, A_b and ρ_b be the Young's modulus, moment of inertia, cross section area and respectively the density of the beam. The transverse displacement is denoted by $w(x,t)$. The load P is assumed to be normal to the half-space surface and to act on the centre-line of the beam. Because of the symmetry with respect to the plane $y=0$, the beam is moving only vertically. The motion equation for the half space is given by (3.5) where $u(u_x,u_y,u_z)$ is the displacement vector in the Cartesian coordinate system, λ and μ are Lamé elastic constants and ρ the mass density of the half-space.

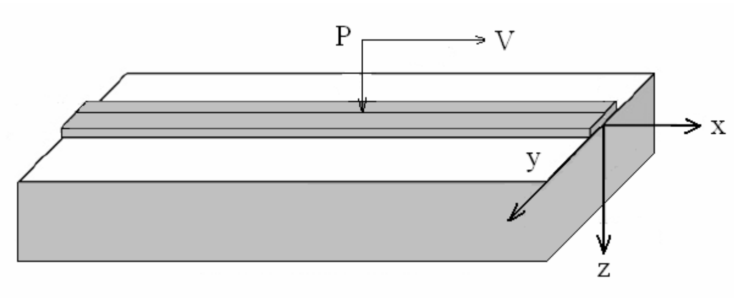

Fig.1. Scheme of a slab-track railway system.

We add the following conditions applied on the interface $z=0$ [19]

$$E_b I_b w_{,xxxx} + \rho_b A_b w_{,tt} - \int_{-b/2}^{b/2} \sigma_{zz}\mathrm{d}y = -P\delta(\zeta)\,,\ \ \zeta = x - Vt\,,\ \ u_z = w\,,\ u_x = u_y = 0 \ \text{ for } \ |y| < b/2\,,$$

$$\sigma_{zz} = \sigma_{zx} = \sigma_{zy} = 0 \ \text{ for } |y| > b/2\,, \qquad (4.1)$$

where σ denotes the stresses. Numerical solutions of (3.5) and (4.1) for the steady-state response of the beam were simulated for the beam parameters: $P = 200\times10^3$ N, $\beta = 0.3$, $2b = 6.4$m, $A_b = 1.1\ \mathrm{m}^2$, $I_b = 0.011\ \mathrm{m}^4$, $\rho_b = 2400\mathrm{kg\,m}^{-3}$, $E_b = 2\times10^{10}\ \mathrm{Nm}^{-2}$, and for the half-space parameters: $E = 2.33\times10^9\ \mathrm{Nm}^{-2}$, $\nu = 0.31$, $\rho = 2100\ \mathrm{kgm}^{-3}$.

The steady-state response of the beam to traveling load velocities $V = 0.5\,v_{cr}$, $1\,v_R$, $1.5\,v_R$ and $2\,v_R$ (v_R is the Rayleigh velocity, $v_R = 420$km/h) are displayed in figs. 2 and 3 as a function of variable ζ. From these figures we observe that deflections reach a maximum value for $V = 0.5\,v_{cr}$, $1\,v_R$ under the loading point. For velocities higher than v_R, the deflections become asymmetric with respect to the loading point. When the radiation of elastic waves into the half-space and the beam is taking into account the deflections decreased in comparison with no radiation. The load has to pump energy to the system while traveling and a part of this energy is lost by mechanical radiation.

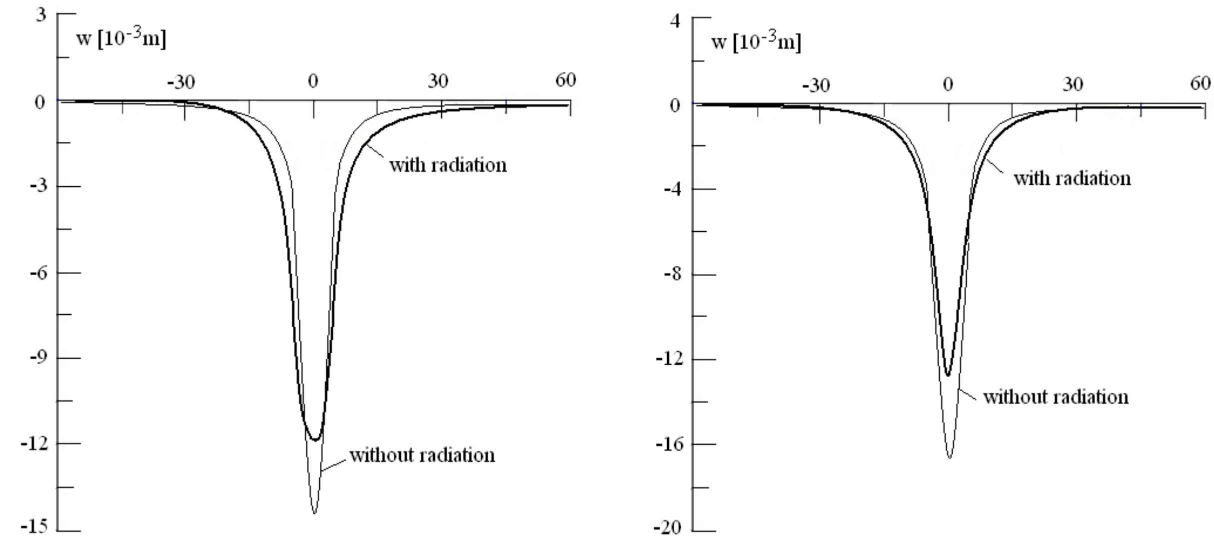

Fig.2. Deflection of beam versus ζ for load velocity 0.5 v_R (left) and 1 v_R (right).

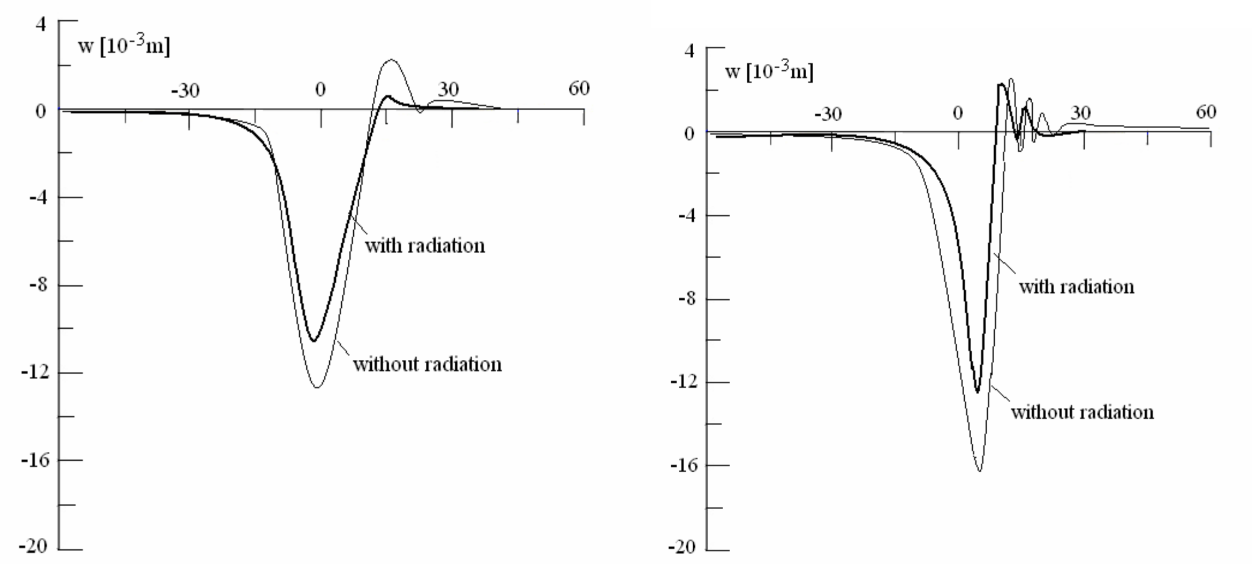

Fig.3. Deflection of beam versus ζ for load velocity 1.5 v_R (left) and 2 v_R (right).

4. CONCLUSIONS

Real materials exhibit two kind of attenuation of mechanical disturbances: mechanical radiation and the viscous behavior. We have simulated in this paper the mechanical radiation studying the dynamic response of a slab–track railway system, loaded by a moving train axle has been considered. The track has been modeled by a beam on elastic half-space, whereas a uniformly traveling constant load has been used to represent the axle load.

The principal consequence of the double couple radiation pattern is that the waves in an unbounded elastic medium are subjected to dispersion and attenuation. The term proportional to the time rate of change of the acceleration field is responsible for the radiative behavior. Evidently, the frequency is complex, which implies that the amplitude decreases with increasing x and t.

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