APPLICATION OF DUAL RECIPROCITY METHOD TO LAMINAR FORCED HEAT CONVECTION IN CONCENTRIC ANNULAR TUBE

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This paper presents a study of the dual reciprocity boundary element method (DRBEM) for the laminar heat convection problem between two coaxial cylinders with constant heat flux boundary condition. DRBEM is one of the most successful techniques used to transform the domain integrals arising from the nonhomogeneous term of the Poisson equation into equivalent boundary only integrals. This recently developed and highly efficient numerical method is tested for the solution accuracy of the fluid flow and heat transfer study between two coaxial cylinders. Since their exact solutions are available, DRBEM solutions are verified with different number of boundary element discretizations and internal points. The results obtained in this study are discussed with the relative error percentage of velocity and temperature solutions, and potential applicability of the method for the more complicated heat convection problems with arbitrary duct geometries.

Key words: Concentric annular tube; Laminar heat convection; Heat flux boundary condition; Numerical analysis; Dual reciprocity boundary element method.

1. INTRODUCTION

Among the various numerical methods, the boundary element method (BEM) becomes one of the favorite analysis tools ever since its introduction to the solution of heat transfer problems. Its advantage over the finite difference or the finite element methods comes from the fact that instead of full domain discretization, only the boundary is discretized into elements and internal point position can be freely defined. Therefore the quantity of data necessary to solve the problems can be greatly reduced [5].

Until recent years the main area of the BEM application has been limited to the conduction heat transfer problems among different heat transfer modes and therefore, with various research efforts, BEM for the solution of heat conduction direct or inverse problem is now well established [9, 13]. However BEM study for the application of heat convection problems can be considered as insufficient and in still developing stage. Since the convection effects are of considerable importance in many heat transfer phenomena, they need much more research focus. The main difficulties of the BEM application to such problems are due to the facts that the fundamental solutions are only available for the few governing equation types and, except Laplace equation, additional domain discretization is required to account source type domain integral terms [25].

The dual reciprocity boundary element method (DRBEM) which was introduced by Nardini and Brebbia [15] is thus far the most successful technique for dealing with above mentioned lack of fundamental solution types and domain integral problems. Since its introduction DRBEM has been applied in many fields of engineering problems [18, 20, 27]. In the DRBEM, available fundamental soluion is used for the complete governing equation, and domain integral arising from the heat source–like term is transferred to the boundary by using the radial basis interpolation functions [19, 28].

This paper presents the application of DRBEM to the Poisson type equations, and fully developed laminar convection heat transfer problems between two coaxial cylinders are illustrated as their applicable examples. The concentric annulus is chosen because of its simplicity and available exact solutions, so that basic nature of the proposed method for the convection problems can be analyzed and revealed in a detailed manner [12, 14]. Therefore present research efforts are confirmed within basic study aiming at the establishment of DRBEM's applicability for the heat convection analysis to be eventually extended in the future study of various heat transfer system.

In this paper, hydrodynamically and thermally fully developed laminar flow with uniform heat flux through thermal boundary between two coaxial cylinders is studied by using the DRBEM. To verify the methods on heat convection problems, numerical solutions with different number of boundary element discretizations and internal points are compared with the exact solutions for its convergence and accuracy.

2. FORMULATION OF THE PROBLEM

Consider an incompressible Newtonian fluid flow in a concentric annular tube as shown in Fig. 1. In the system to be analyzed, z coordinate represents the axial direction and x–y coordinates are attached to the cross–sectional surface. The inner and outer cylinder radii are taken as R_i and R_o.
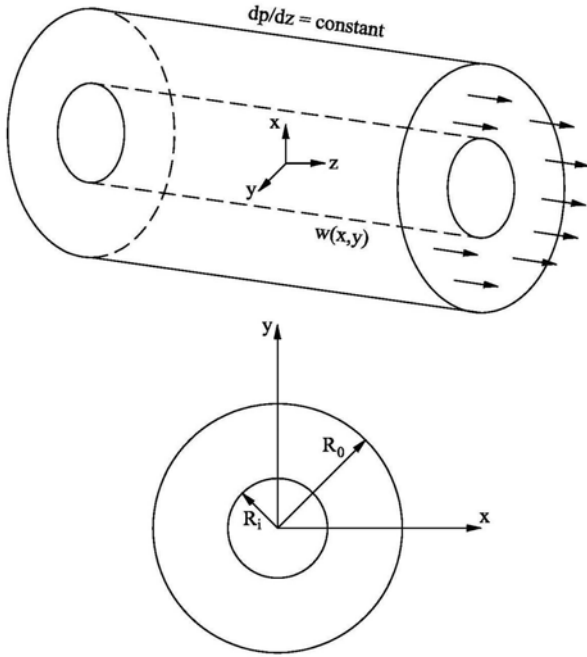

Fig. 1 – Geometry of the concentric annular tube.

For the fully developed steady laminar flow with constant transport properties and negligible body forces, Navier-Stokes equation becomes simple pressure–driven Poiseuille flow equation. Since the flow is fully developed, axial flow velocity is a function of only x–y coordinates, and axial pressure gradient is constant. In the energy equation, the viscous dissipation and axial heat conduction effects are neglected. Therefore the governing equation can be expressed in the form of a Poisson equation as follows:

– momentum equation:

$$\nabla^2 w = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{1}{\mu}\frac{dp}{dz};$$ (1)

– energy equation:

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{w}{a}\frac{dT}{dz},$$ (2)

in which: w is the axial flow velocity; μ – coefficient of viscosity; p – pressure; T – temperature; $a=\lambda/\rho c$ – thermal diffusivity.

For the thermally fully developed flow with constant heat flux boundary condition, equation (2) can be rewritten by using the mixed mean temperature T_m [14] as:

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{w}{a}\frac{dT_m}{dz},$$ (3)

where $\partial T/\partial z = dT_m/dz = $ const. from the given conditions. The boundary conditions associated with the equations (1) and (3) are:

$$w = 0 \text{ at } R = R_i, \quad w = 0 \text{ at } R = R_o$$ (4)

$$T = T_i \text{ at } R = R_i, \quad T = T_o \text{ at } R = R_o$$ (5)

where subscripts "i" and "o" represent for the inner and outer surfaces.

For the solution of temperatures, velocity from equation (1) is obtained first and then equation (3) can be solved in sequence since the assumption of constant viscosity uncoupled the momentum and energy equations.

3. DUAL RECIPROCITY BOUNDARY ELEMENT EQUATION

For the BEM solution, equations (1) and (3) subject to equations (4) and (5) can be generalized as the following type of Poisson equation [18]:

$$\nabla^2 u(x, y) = b(x, y), \qquad (x, y) \in \Omega, \tag{6}$$

with the boundary conditions:

$$u(x, y) = \bar{u}, \qquad (x, y) \in \Gamma_1, \tag{7}$$

$$q(x, y) = \frac{\partial u(x, y)}{\partial n} = \bar{q}, \quad (x, y) \in \Gamma_2, \tag{8}$$

and to represent convective heat transfer problems:

$$u(x, y) = w, \; b(x, y) = \frac{1}{\mu} \frac{\mathrm{d}p}{\mathrm{d}z} = \text{const.},$$

$$u(x, y) = T, \; b(x, y) = \frac{w}{a} \frac{\mathrm{d}T_m}{\mathrm{d}z}, \tag{9}$$

where: $\Gamma_1 + \Gamma_2 = \Gamma$ is the total boundary of solution domain Ω; n – normal to the boundary; $\bar{u}$ and $\bar{q}$ – specified values at each boundary.

Applying the usual boundary element technique to equation (6), an integral equation can be deduced as follows:

$$c_i u_i + \int_\Gamma u q^* \mathrm{d}\Gamma - \int_\Gamma q u^* \mathrm{d}\Gamma = \int_\Omega b u^* \mathrm{d}\Omega, \tag{10}$$

where the constant c_i depends on the geometry at point i as:

$$c_i = \begin{cases} 1 & \text{for } (x_i, y_i) \in \Omega \\ \dfrac{\theta}{2\pi} & \text{for } (x_i, y_i) \in \Gamma, \end{cases} \tag{11}$$

where θ is the angle between the tangent to Γ on either side of point i.

The key method of DRM is to take the domain integral of equation (10) to the boundary and remove the needs of complicated domain discretization. To accomplish this idea, the source term $b(x, y)$ is expanded as its values at each node j and a set of interpolating functions f_j are used as [18, 20, 27]:

$$b(x, y) \cong \sum_{j=1}^{N+L} \alpha_j f_j, \tag{12}$$

where: α_j is a set of initially unknown coefficients; $N+L$ – the number of boundary nodes plus internal points.

If the function $\bar{u}_j$ can be found such that:

$$\nabla^2 \bar{u}_j = f_j, \tag{13}$$

then the domain integral can be transferred to the boundary.

Substituting equation (13) into equation (12), and applying integration by parts twice for the domain integral term of equation (10) leads to:

$$c_i u_i + \int_\Gamma u q^* \mathrm{d}\Gamma - \int_\Gamma q u^* \mathrm{d}\Gamma = \sum_{j=1}^{N+L} \alpha_j \times \left(c_i \hat{u}_{ij} + \int_\Gamma \hat{u}_j q^* \mathrm{d}\Gamma - \int_\Gamma \hat{q}_j u^* \mathrm{d}\Gamma \right). \tag{14}$$

For the two–dimensional domain of interest in this study, u^*, q^* and $\hat{u}$, $\hat{q}$ can be derived as:

$$u^* = \frac{1}{2\pi} \ln(\frac{1}{r})$$

$$q^* = \frac{-1}{2\pi r} \nabla r \cdot \vec{n} \tag{15}$$

$$\hat{u} = \frac{r^2}{4} + \frac{r^3}{9}$$

$$\hat{q} = (\frac{r}{2} + \frac{r^2}{3})\nabla r \cdot \vec{n}$$

(16)

where r stands for the distance from a source point i or a DRM collocation point j to a field point (x, y). As for the equation (13), a radial basis function $f = 1 + r$ is chosen as an interpolating function which was shown to be generally sufficient [19, 20, 28].

In the numerical solution of the integral equation (14), u, q, $\hat{u}$ and $\hat{q}$ in the integrals are modelled using the linear interpolation functions as:

$$\int_{\Gamma_k} u q^* \mathrm{d}\Gamma = u_k h_{ik}^1 + u_{k+1} h_{ik}^2 ,$$

(17)

$$\int_{\Gamma_k} q u^* \mathrm{d}\Gamma = q_k g_{ik}^1 + q_{k+1} g_{ik}^2 ,$$

(18)

$$\int_{\Gamma_k} \hat{u}_j q^* \mathrm{d}\Gamma = \hat{u}_{kj} h_{ik}^1 + \hat{u}_{(k+1)j} h_{ik}^2 ,$$

(19)

$$\int_{\Gamma} \hat{q}_j u^* \mathrm{d}\Gamma = \hat{q}_{kj} g_{ik}^1 + \hat{q}_{(k+1)j} g_{ik}^2 ,$$

(20)

where:

$$h_{ik}^1 = \int_{\Gamma_k} \Phi_1 q^* \mathrm{d}\Gamma, \quad h_{ik}^2 = \int_{\Gamma_k} \Phi_2 q^* \mathrm{d}\Gamma ,$$

(21)

$$g_{ik}^1 = \int_{\Gamma_k} \Phi_1 u^* \mathrm{d}\Gamma, \quad g_{ik}^2 = \int_{\Gamma_k} \Phi_2 u^* \mathrm{d}\Gamma .$$

(22)

Here the first subscript of Eq. (21) and (22) refers to the specific position of the point where the flow velocity or temperature is evaluated; the second subscript refers to the boundary element over which the contour integral is carried out. The superscript 1 and 2 designate the linear interpolation function Φ_1 and Φ_2 respectively, with which the u^* and q^* functions are weighted in the integrals in equation (17) through (20).

For the boundary $\Gamma = \Gamma_1 \bigcup \Gamma_2$ discretized into N elements, integral terms in equation (14) can be re-written as:

$$\int_{\Gamma} u q^* \mathrm{d}\Gamma = \sum_{k=1}^{N} \int_{\Gamma_k} u q^* \mathrm{d}\Gamma = \sum_{k=1}^{N} \left[h_{i(k-1)}^2 + h_{ik}^1 \right] u_k = \sum_{k=1}^{N} H_{ik} u_k \quad \text{or} \quad = \sum_{j=1}^{N_n} H_{ik} \hat{u}_{kj} \quad \text{for } \hat{u}_j ,$$

(23)

$$\int_{\Gamma} q u^* \mathrm{d}\Gamma = \sum_{k=1}^{N} \int_{\Gamma_k} q u^* \mathrm{d}\Gamma = \sum_{k=1}^{N} \left[g_{i(k-1)}^2 + g_{ik}^1 \right] q_k = \sum_{k=1}^{N} G_{ik} q_k \quad \text{or} \quad = \sum_{j=1}^{N_n} G_{ik} \hat{q}_{kj} \quad \text{for } \hat{q}_j ,$$

(24)

where $h_{i0}^2 = h_{iN}^2$ and $g_{i0}^2 = g_{iN}^2$. Introducing equation (23) and (24) into equation (14) and manipulating results yields a dual reciprocity boundary element equation as:

$$c_i u_i + \sum_{k=1}^{N} H_{ik} u_k - \sum_{k=1}^{N} G_{ik} q_k = \sum_{j=1}^{N+L} \alpha_j \times \left(c_i \hat{u}_{ij} + \sum_{k=1}^{N} H_{ik} \hat{u}_{kj} - \sum_{k=1}^{N} G_{ik} \hat{q}_{kj} \right) .$$

(25)

4. NUMERICAL SOLUTION

For the computer implementation of numerical solution, equation (25) can now be written in a matrix form as:

$$\mathbf{HU} - \mathbf{GQ} = (\mathbf{H\hat{U}} - \mathbf{G\hat{Q}})\boldsymbol{\alpha} ,$$

(26)

where **H** and **G** are matrices of their elements being H_{ik} and G_{ik}, with c_i being incoroporated into the principal diagonal element, respectively. **U**, **Q** and their terms with hat of equation (26) correspond to vectors of u_k, q_k and matrices with jth column vectors of hat u_{kj}, q_{kj}. It is noted that vector α of unknown coefficients j can be evaluated from equation (12) with chosen interpolating function f_j and the function $b(x, y)$ of governing equation. Therefore introducing the boundary conditions into the nodes of u_k and q_k vectors and rearranging by taking known quantities to the right hand side and unknowns to the left hand side leads to a set of simultaneous linear equations of the form:

$$\mathbf{AX} = \mathbf{B} . \qquad (27)$$

Using the DRBEM matrix equation, the numerical solution of laminar convection heat transfer problem in a concentric annulus can be readily obtained as x being the flow velocity w for momentum equation and also temperature T for energy equation or their normal derivatives, respectively.

Consider the geometry illustrated in Figure 2. For the sake of simplification, the surface temperatures of two cylinders are assumed to be equal. Thus, the solution satisfies the following boundary conditions:

$$w(x, y)\big|_{R=R_i} = w(x, y)\big|_{R=R_o} = 0,$$
$$T*(x, y)\big|_{R=R_i} = T*(x, y)\big|_{R=R_o} = 0, \qquad (28)$$
$$T* = T_w - T, \ T_w = T_i = T_o.$$

As a note, no slip conditions are applied for the velocity boundary condition.

For the numerical test case, following numerical values in equations (1) and (3) are taken from the paper [24] where the spectral collocation method is used for the exccentric annuli heat convection analysis: $R_o = 0.055$ m, $R_i = 0.030$ m, $(1/\mu)\mathrm{d}p/\mathrm{d}z = -836$ m$^{-1}\cdot$s^{-1}, $a = 1.3418\times10^{-9}$ m^2/s, $\mathrm{d}T_m/\mathrm{d}z = 0.47$ °C/m.

5. RESULTS AND DISCUSSION

In order to confirm the accuracy of the dual re-ciprocity boundary element method for the present heat convection problem, each boundary of outer and inner surface is equally discretized as 36, 48, 60, 72 and 84 elements respectively. The nodes on every boundary and the unternal points of the analysis domain are located as shown in Fig. 2. Therefore total number of internal points used in the analysis are 90, 120, 150, 180 and 210 for each 36, 48, 60, 72 and 84 boundary element cases respectively.
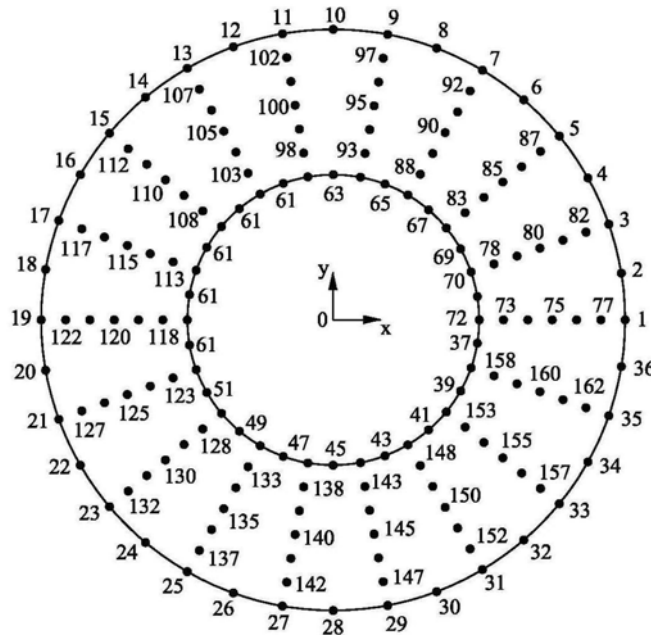
Fig. 2 – Boundary element nodes and internal points for the system to be analyzed.

To obtain the axial flow velocity distribution $w(x,y)$, equation (1) is solved first. Their results for the boundary and internal nodes are shown in Table 1. Here the normal derivative of velocity w at the boundary is listed as well, and all the numerical solutions are compared with the exact solutions [14] for their accuracy.

Table 1

DRBEM results with exact solutions for the boundary and internal locations in flow velocity analysis

Solution variable	Radial location R [m]	DRBEM solution (number of boundary elements case)					Exact solution
		36	48	60	72	84	
$\partial w/\partial n$	0.055	−9.570611	−9.614363	−9.632446	−9.646733	−9.649446	−9.667904
$\partial w/\partial n$	0.030	−11.961390	−11.931682	−11.910987	−11.902295	−11.900702	−11.883840
w	0.0342	0.040336	0.039937	0.039755	0.039656	0.039614	0.039413
w	0.0383	0.061518	0.061112	0.060926	0.060825	0.060760	0.060591
w	0.0425	0.066727	0.066320	0.066133	0.066030	0.065973	0.065803
w	0.0466	0.057611	0.057196	0.057006	0.056908	0.056853	0.056682
w	0.0508	0.035084	0.034883	0.034733	0.034638	0.034606	0.034439

Table 2

DRBEM results with exact solutions for the boundary and internal locations in temperature analysis ($T^*=T_w-T$)

Solution variable	Radial location R [m]	DRBEM solution (number of boundary elements case)					Exact solution
		36	48	60	72	84	
$\partial T^*/\partial n$	0.055	172056.38	170824.22	171362.75	171.600.75	171628.56	170484.20
$\partial T^*/\partial n$	0.030	233420.45	229217.02	230340.65	230698.70	230771.22	229506.90
T^*	0.0342	804.26	828.59	847.40	853.78	855.04	858.72
T^*	0.0383	1268.86	1298.52	1311.65	1315.86	1317.26	1320.13
T^*	0.0425	1383.48	1413.28	1426.46	1429.68	1431.26	1433.69
T^*	0.0466	1180.68	1209.26	1224.48	1230.46	1231.92	1234.96
T^*	0.0508	711.52	730.65	741.78	746.86	747.90	750.34

Figures 3 and 4 show the convergence plot of DRBEM velocity and its normal derivative solutions as the number of boundary elements and internal points increase. DRBEM solutions are in close agreement with the exact solutions and relative errors are within 2.3% for the above 36 element cases.
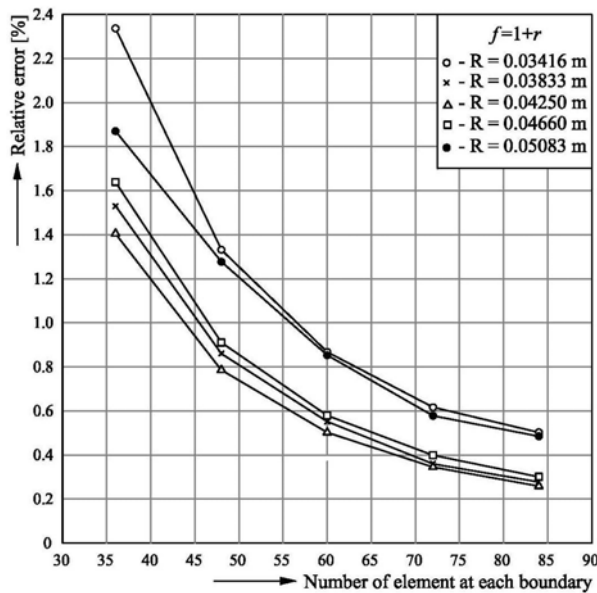

Fig. 3 – Accuracy test for the velocity solution at the selected internal points.
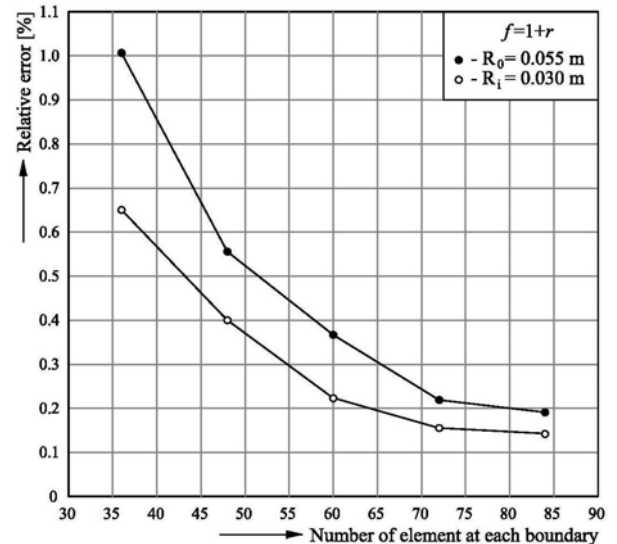

Fig. 4 – Accuracy test for the normal derivative of velocity solution at the inner and outer boundaries.

As noted in Fig. 3, velocity solutions at location of $R = 0.0508$ m and $R = 0.0342$ m are less accurate than the others and, in between, $R = 0.0342$ point gives more inaccurate solution than $R = 0.0508$. And for

the normal derivatives of velocity at boundary $R = 0.055$ m is less accurate than $R = 0.030$ m as shown in Fig. 4. These results are due to the facts that the outer boundary element size is larger than the inner boundary element size and distribution of internal points is getting sparse to the outward direction, whereas rapid change of velocity occurs at inner and outer boundary sides as illustrated in Figs. 2 and 5. Therefore solution's error magnitude regarding to the radial location is closely related to both the physical and the mathematical aspects and nevertheless overall solution accuracy is shown to be fairly acceptable. Thus, 36 element solution case shows maximum 2.34% error at radial position $R = 0.0342$ and later results in accurate temperature solution.

Then these DRM velocity solutions are, in turn, used in the energy equation (3) to solve for the temperature distribution. Table 2 shows the results, and it is found that DRM solutions are in excellent agreement with exact solutions and relative errors are within 5% for the above 36 element cases (Figs. 6, 7 and 8). Although the converging trend in Fig. 7 is not monotonic and radial location effect about error magnitude is not exactly following the previously discussed velocity solution case, solution trends can be considered as indistinguishable within 1% relative error. These test results validate the power of dual reciprocity boundary element method and its solution accuracy, since the numerically solved velocity was used as an input in equation (3) and source – like function $b(x, y)$ of equation (12) in equation (3) is approximated with interpolating function and nodal values of internal points.
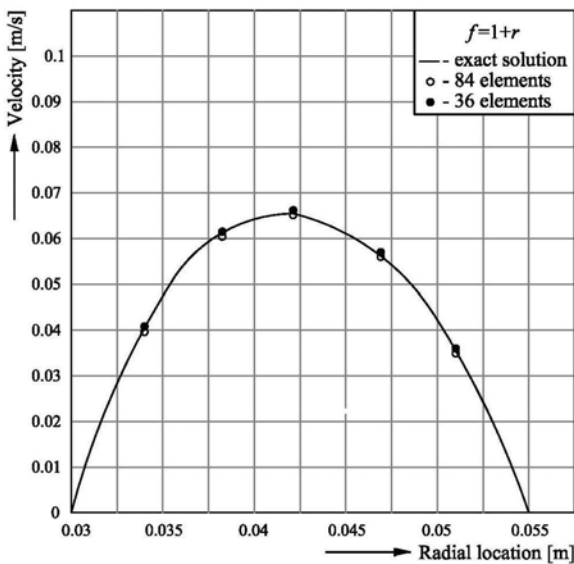

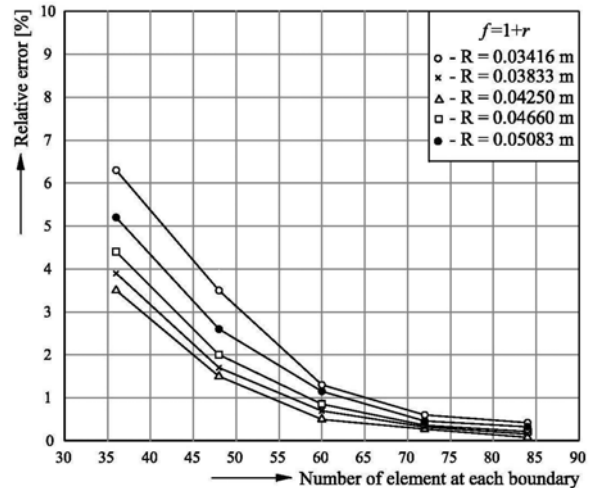Fig. 5 – Velocity profile of exact solution compared with DRBEM results.

Fig. 6 – Accuracy test for the temperature solution at the selected internal points.
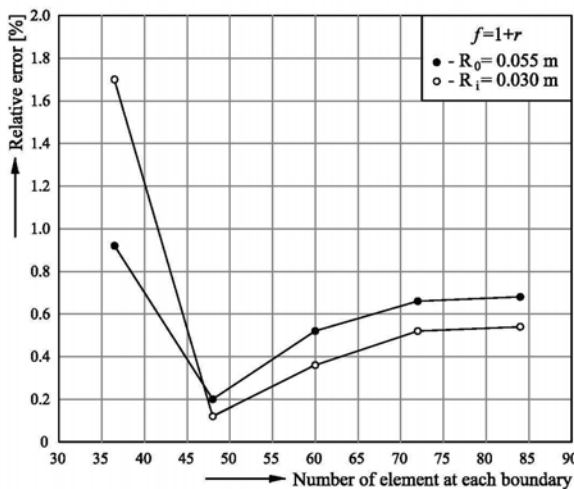

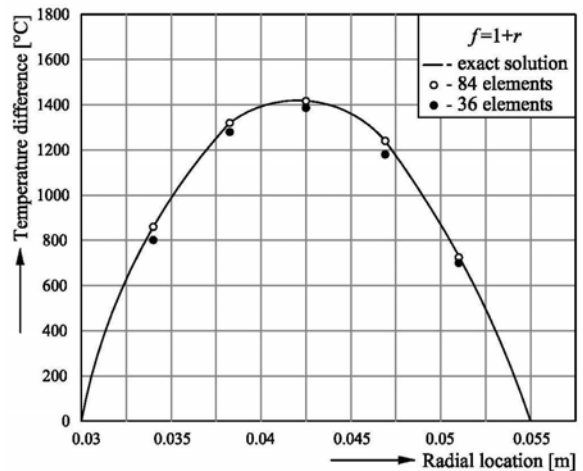Fig. 7 – Accuracy test for the normal derivative of temperature solution at the inner and outer boundaries.

Fig. 8 – Temperature profile T^* of exact solution compared with DRBEM results.

As a final note, all the element cases turns out to be adequate for the solution of this problem. Errors of the velocity and the temperature solution are acceptable.

6. CONCLUSIONS

A dual reciprocity boundary element method has been presented for the solution of laminar heat convection problem in a concentric annulus imposed with constant heat flux. DRBEM matrix is formulated to perform the numerical implementation, and five cases of boundary element discretization are tested with the corresponding number of internal points. Five radial locations are selected to obtain the velocity and temperature solutions. Test results are shown to be in excellent agreement with exact solutions for the above 36 element case.

As a final remark, recently developed dual reciprocity boundary element method is successfully applied to solving the laminar heat convection problem in a concentric annulus, and also current study shows its broad potentiality for further applications.

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