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KINEMATICS MODELLING OF A 3-PRC PARALLEL KINEMATIC MACHINE

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Recursive matrix relations for kinematics of a three-prismatic-revolute-cylindrical (3-PRC) parallel kinematic machine (PKM) are established in this paper. Knowing the translational motion of the platform, we develop the inverse kinematical problem and determine the positions, velocities and accelerations of the robot's elements. . Finally, compact matrix equations and graphs of simulation for displacements, velocities and accelerations of each of three legs are obtained.

Key words: Kinematics; Parallel kinematic machine; Platform.

LIST OF SYMBOLS

$a_{k,k-1}$, $b_{k,k-1}$, $c_{k,k-1}$ – orthogonal transformation matrices
$\vec{u}_1, \vec{u}_2, \vec{u}_3$ – three right-handed orthogonal unit vectors
$l = 2r\sqrt{3}$ – length of the side of moving platform
θ – angle of inclination of three sliders
λ_{10}^{A}, λ_{10}^{B}, λ_{10}^{C} – displacements of three prismatic actuators
$\varphi_{k,k-1}$ – relative rotation angle of T_k rigid body
$\vec{\omega}_{k,k-1}$ – relative angular velocity of T_k
$\vec{\omega}_{k0}$ – absolute angular velocity of T_k
$\tilde{\omega}_{k,k-1}$ – skew-symmetric matrix associated to the angular velocity $\vec{\omega}_{k,k-1}$
$\vec{\varepsilon}_{k,k-1}$ – relative angular acceleration of T_k
$\vec{\varepsilon}_{k0}$ – absolute angular acceleration of T_k
$\tilde{\varepsilon}_{k,k-1}$ – skew-symmetric matrix associated to the angular acceleration $\vec{\varepsilon}_{k,k-1}$

1. INTRODUCTION

The research activities on parallel mechanisms have recently gained greater attention for engineers within the robotic community as their advantages become better known. Parallel kinematic machines (PKM) are closed-loop structures presenting very good potential in terms of accuracy, rigidity and ability to manipulate large loads. In general, these mechanisms consist of two main bodies coupled via numerous legs acting in parallel. One body is designated as fixed and is called *base*, while the other is regarded as movable and hence is called *moving platform* of the manipulator. The number of actuators is typically equal to the number of degrees of freedom such that each leg is controlled at or near the fixed base [1].

Compared with traditional serial manipulators, the followings are the potential advantages of parallel architectures: higher kinematical accuracy, lighter weight and better structural rigidity, stable capacity and suitable position of arrangement of actuators, low manufacturing cost and better payload carrying ability. But, from an application point of view, a limited workspace and complicated singularities are two major drawbacks of parallel manipulators. Thus, it is more suitable for situations where high precision, stiffness, velocity and heavy load-carrying are required within a restricted workspace [2].

Most three DOF mechanisms exist in one of three of following classes: spherical, translational or spatial. A translation device is capable of moving in Cartesian coordinates and is suitable for pick-and-place applications. Spherical mechanisms are strictly for pointing applications such as orienting a camera. The final class is the class of spatial mechanisms, which include a combination of both rotational and translational degrees of freedom.

Over the past two decades, parallel manipulators have received more attention from researches and industries. Important companies such as Giddings & Lewis, Ingersoll, Hexel and others have developed them as high precision machine tools. Accuracy and precision in the direction of the tasks are essential since the positioning errors of the tool could end in costly damage. Considerable efforts have been devoted to the kinematics and dynamic investigations of fully parallel manipulators. Among these, the class of manipulators known as Stewart-Gough platform focused great attention (Stewart [3]; Di Gregorio and Parenti Castelli [4]). They are used in flight simulators and more recently for PKMs. The prototype of Delta parallel robot (Clavel [5]; Tsai and Stamper [6]; Staicu [7]); developed by Clavel at the Federal Polytechnic Institute of Lausanne and by Tsai and Stamper at the University of Maryland as well as the Star parallel manipulator (Hervé and Sparacino [8], Staicu [9]) are equipped with three motors, which train on the mobile platform in a three-degree-of-freedom general translation motion. Angeles [10], Wang and Gosselin [11], Staicu [12] analysed the kinematics, dynamics and singularity loci of Agile Wrist spherical robot with three actuators.

In the present paper, a recursive matrix method, already implemented in the inverse kinematics of parallel robots, is applied to the analysis of the spatial three-degrees-of-freedom mechanism. It has been proved to reduce the number of equations and computation operations significantly by using a set of matrices for kinematics modelling.

2. KINEMATICS ANALYSIS

In the previous works of Li and Xu [13], [14], the 3-PRS parallel manipulator and the 3-PRC PKM with relative simple structure was presented with its kinematics problems solved in details. The potential application as a positioning device of the tool in a new parallel kinematics machine for high precision blasting attracted a scientific and practical interest to this manipulator type/

Having a closed-loop structure, the spatial 3-PRC parallel kinematic machine is a special symmetrical mechanism composed of three kinematical chains with identical topology, all connecting the fixed base to the moving platform (Fig. 1). Each limb connects the moving platform to the base by a prismatic joint, axe of which being inclined from the base platform, a revolute joint and a passive cylindrical joint in sequence, where the prismatic joints are driven by three linear actuators assembled to the fixed base. Since the kinematics and mobility problems have already resolved, we only review some matrix useful results below which provide a base for the dynamic modelling. The manipulator consists of the upper fixed base and the moving platform $A_3B_3C_3$ that are two equilateral triangles with L and $l = 2r\sqrt{3}$ the lengths of the sides, respectively. Overall, there are seven moving links, three prismatic joints, three revolute joints and three cylindrical joints. Grübler mobility equation predicts that the device has certainly three degrees of freedom.

In the configuration $(\underline{P}RC)$ with all actuators installed on the fixed base, we consider the moving platform as the output link and the sliders A_1, B_1, C_1 as the input links (Fig. 2). Having a common point of intersection O, the lines of action of each of three prismatic joints may be inclined from the fixed base by a constant angle θ as architectural parameter. These rails and the revolute joint axes are mutually perpendicular. Finally, the passive revolute joint of each leg is separated from the cylindrical joint connecting the platform's edge by a fixed-length limb. In this configuration, any translation along the vertical axis is limited by the constraints and therefore only finite displacements along this axis are obtained.

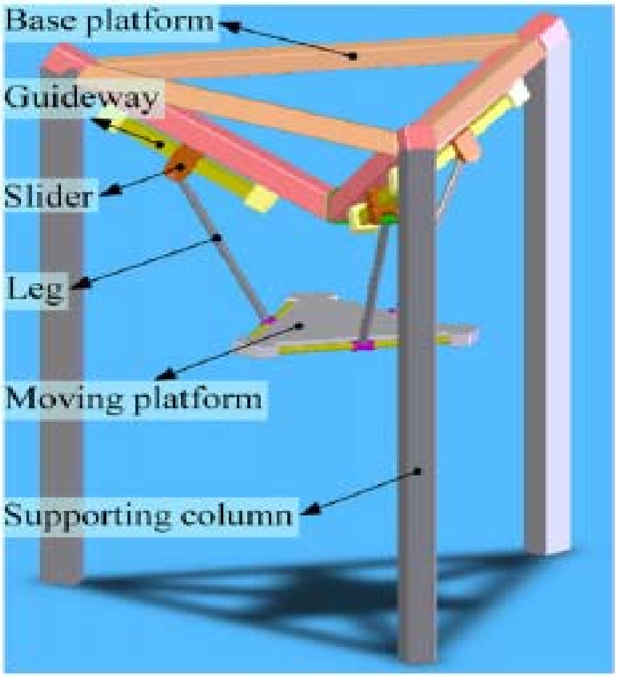

Fig. 1 – Virtual prototype for the 3-PRC PKM.

For the purpose of analysis, a Cartesian frame $Ox_0y_0z_0\,(T_0)$ is attached to the fixed base with its origin located just at the common point O of three rails, the z_0 axis perpendicular to the base. Another mobile reference frame $Gx_Gy_Gz_G$ is attached to the moving platform. The origin of this coordinate central system is fixed at the centre G of the moving triangle. The kinematical constraints of each leg limit the *absolute motion* of cylindrical joint of each limb to a plane perpendicular to the axis of the revolute joint and containing the rail. In the symmetrical version of the manipulator, the first leg A is typically contained within the Ox_0z_0 vertical plane, whereas the remaining two legs B, C make angles $\alpha_B = 120^0, \alpha_C = -120^0$ respectively, with the first leg. It is noted that the relative rotation with $\varphi_{k,k-1}$ angle or relative translation of T_k body with $\lambda_{\,k,k-1}$ displacement must be always pointing about or along the direction of z_k axis.

In what follows we consider that the moving platform is initially located at a *central configuration*, where the platform is not translated with respect to the fixed base and the origin O of fixed frame is located at an elevation $OG = h$ above the mass centre G. A complete description of the absolute position of the translational moving platform with respect to the reference frame requires generally three variables: the coordinates x_0^G, y_0^G, z_0^G of the mass center G.

One of three active legs (for example leg A) consists of a prismatic joint linked at the $A_1x_1^Ay_1^Az_1^A$ moving frame, having a translation with the displacement $\lambda_{\,10}^{\,A}$, the velocity $v_{10}^A = \dot{\lambda}_{\,10}^{\,A}$ and the acceleration $\gamma_{10}^A = \ddot{\lambda}_{\,10}^{\,A}$. An intermediate link of length $A_2A_3 = l_2$ has a relative rotation about z_2^A axis with the angle φ_{21}^A, the angular velocity $\omega_{21}^A = \dot{\varphi}_{21}^A$ and the angular acceleration $\varepsilon_{21}^A = \ddot{\varphi}_{21}^A$. Finally, a cylindrical joint is introduced at the edge of a planar moving platform, which is schematised as an equilateral triangle.

At the central configuration, we also consider that the three sliders are initially starting from same position $OA_1 = l_1 = r\cos\theta - h\sin\theta + l_2\sin(\theta+\beta)$ and that the angles of orientation of the legs are given by

$$\theta = \frac{\pi}{6},\ \alpha_A = 0,\ \alpha_B = \frac{2\pi}{3},\ \alpha_C = -\frac{2\pi}{3},\ l_2\cos(\theta+\beta) = r\sin\theta + h\cos\theta\,. \tag{1}$$

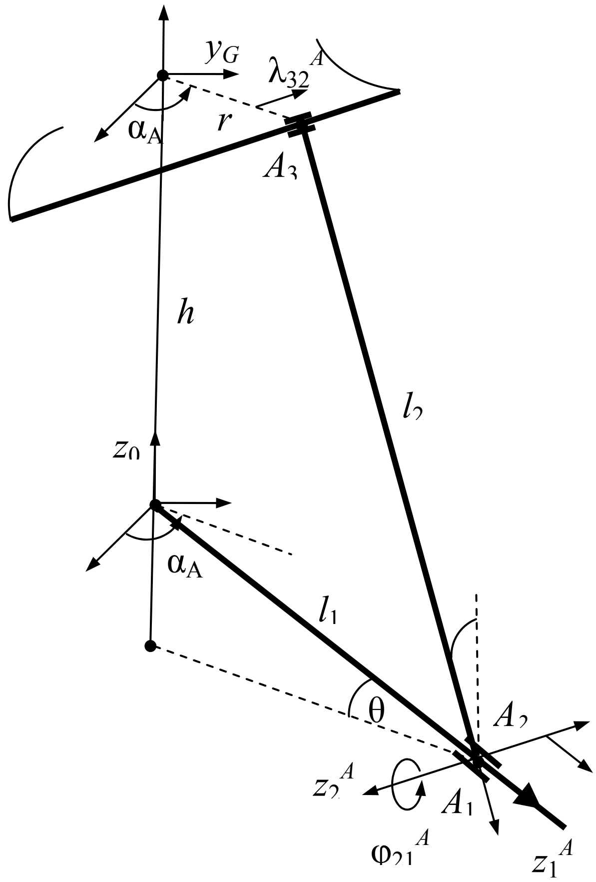

Fig. 2 – Kinematical scheme of first leg A of upside-down mechanism.

In the followings, we apply the method of successive displacements to geometric analysis of closed-loop chains So, starting from the reference origin O and pursuing the independent legs $OA_1A_2A_3$, $OB_1B_2B_3$, $OC_1C_2C_3$, we obtain following transformation matrices

$$p_{10} = a_\theta \theta_1 a_\alpha^i,\ p_{21} = p_{21}^\varphi a_{\beta\theta} \theta_1 \theta_2^{\mathrm{T}},\ p_{20} = p_{21} p_{10}\quad (p = a,\ b,\ c\quad i = A,\ B,\ C), \tag{2}$$

where we denote [15]:

$$a_\alpha^i = \begin{bmatrix} \cos\alpha_i & \sin\alpha_i & 0 \\ -\sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 1 \end{bmatrix},\ a_{\beta\theta} = \begin{bmatrix} \cos(\theta+\beta) & \sin(\theta+\beta) & 0 \\ -\sin(\theta+\beta) & \cos(\theta+\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix},\ a_\theta = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix},$$

$$p_{21}^\varphi = \begin{bmatrix} \cos\varphi_{21}^i & \sin\varphi_{21}^i & 0 \\ -\sin\varphi_{21}^i & \cos\varphi_{21}^i & 0 \\ 0 & 0 & 1 \end{bmatrix}, \theta_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \theta_2 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

Three independent displacements $\lambda_{10}^A, \lambda_{10}^B, \lambda_{10}^C$ of the active links are the joint variables that give the input vector $\vec{\lambda}_{10} = [\lambda_{10}^A \quad \lambda_{10}^B \quad \lambda_{10}^C]^{\mathrm{T}}$ of the instantaneous pose of the mechanism. But, in the inverse geometric problem, it can be considered that the position of the mechanism is completely given through the coordinate x_0^G, y_0^G, z_0^G of the mass centre G. Further, we suppose that following three analytical functions can describe a *helical* translation of the moving platform [16]:

$$x_0^G = d\sin(\omega_1 t)\cos(\omega_2 t)\,,\, y_0^G = d\sin(\omega_1 t)\sin(\omega_2 t)\,,\; z_0^G = h + d\,e[1-\cos(\omega_1 t)]\,, \tag{4}$$

where $d = 0.5,\ e = 0.5,\ \omega_1 = \dfrac{\pi}{2},\ \omega_2 = 3\omega_1$, t is the time variable in unit of second and x_0^G, y_0^G, z_0^G are the coordinates of the centre G in units of meters.

Nine independent variables $\lambda_{10}^A,\ \varphi_{21}^{AA}, \lambda_{32}^A, \lambda_{10}^B,\ \varphi_{21}^B, \lambda_{32}^B, \lambda_{10}^C,\ \varphi_{21}^C, \lambda_{32}^C$ will be determined by several vector-loop equations as follows

$$\vec{r}_{10}^{\,A} + \sum_{k=1}^{2} a_{k0}^T \vec{r}_{k+1,k}^{\,A} - \vec{r}_G^{\,A_3} = \vec{r}_{10}^{\,B} + \sum_{k=1}^{2} b_{k0}^T \vec{r}_{k+1,k}^{\,B} - \vec{r}_G^{\,B_3} = \vec{r}_{10}^{\,C} + \sum_{k=1}^{2} c_{k0}^T \vec{r}_{k+1,k}^{\,C} - \vec{r}_{\mathrm{G}}^{\,C_3} = \vec{r}_0^{\,G}\,, \tag{5}$$

where

$$\vec{u}_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}, \vec{u}_2 = \begin{bmatrix}0\\1\\0\end{bmatrix}, \vec{u}_3 = \begin{bmatrix}0\\0\\1\end{bmatrix},\ \tilde{u}_3 = \begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$
$$\vec{r}_{10}^{\,i} = (l_1 + \lambda_{\ 10}^{\,i}) p_{10}^{\mathrm{T}} \vec{u}_3,\ \vec{r}_{21}^{\,i} = \vec{0},\ \vec{r}_{32}^{\,i} = -l_2 \vec{u}_2$$
$$\vec{r}_G^{\,i_3} = [r\cos\alpha_i - \lambda_{\ 32}^{\,i}\sin\alpha_i \quad r\sin\alpha_i - \lambda_{\ 32}^{\,i}\cos\alpha_i \quad 0]^{\mathrm{T}} \quad (i = A,\ B,\ C). \tag{6}$$

Actually, these vector equations mean that there is only one inverse geometric solution for the spatial manipulator:

$$l_2\cos(\varphi_{21}^i + \theta + \beta) = (r + x_0^G\cos\alpha_i + y_0^G\sin\alpha_i)\sin\theta + z_0^G\cos\theta\,,$$
$$\lambda_{\ 10}^{\,i} = l_2\sin(\varphi_{21}^i + \theta + \beta) + (r + x_0^G\cos\alpha_i + y_0^G\sin\alpha_i)\cos\theta - z_0^G\sin\theta - l_1\,, \tag{7}$$
$$\lambda_{\ 32}^{\,i} = x_0^G\sin\alpha_i - y_0^G\cos\alpha_i\,.$$

Now, we develop the inverse kinematics problem and determine the velocities and accelerations of the robot, supposing that the motion of the moving platform is known. The motion of the compounding elements of the leg A, for example, are characterized by the velocities of joints

$$\vec{v}_{10}^{\,A} = \dot{\lambda}_{10}^A \vec{u}_3,\ \vec{v}_{21}^{\,A} = \vec{0},\ \vec{v}_{20}^{\,A} = a_{21}\vec{v}_{10}^{\,A} \tag{8}$$

and by following angular velocities

$$\vec{\omega}_{10}^{\,A} = \vec{0},\ \vec{\omega}_{20}^{\,A} = \vec{\omega}_{21}^{\,A} = \dot{\varphi}_{21}^A \vec{u}_3\,, \tag{9}$$

which are *associated* to skew-symmetric matrices

$$\tilde{\omega}_{10}^A = \tilde{0},\ \tilde{\omega}_{20}^A = \tilde{\omega}_{21}^A = \dot{\varphi}_{21}^A \tilde{u}_3\,. \tag{10}$$

Vector equations of geometrical constraints (5) can be differentiated with respect to time to obtain the following significant *matrix conditions of connectivity* [17]

$$v_{10}^A\ \vec{u}_j^T a_{10}^T \vec{u}_3 + \omega_{21}^A \vec{u}_j^T a_{20}^T \tilde{u}_3 \vec{r}_{32}^{\,A} - v_{32}^A \vec{u}_j^T a_\alpha^{AT} \vec{u}_2 = \vec{u}_j^T \dot{\vec{r}}_0^{\,G} \quad (j = 1,\ 2,\ 3)\,. \tag{11}$$

If the other two kinematical chains of the robot are pursued, analogous relations can be easily obtained.
From these equations, we obtain the *complete* Jacobian matrix of the manipulator. This matrix is fundamental element for the analysis of the robot workspace and the particular configurations of singularities where the spatial manipulator becomes uncontrollable [18].

Rearranging, above constraint equations (7) can immediately written as

$$[(r + x_0^G\cos\alpha_i + y_0^G\sin\alpha_i)\sin\theta + z_0^G\cos\theta]^2 + [\lambda_{10}^i + l_1 - (r + x_0^G\cos\alpha_i + y_0^G\sin\alpha_i)\cos\theta + z_0^G\sin\theta]^2 = l_2^2$$
$$(i = A,\ B,\ C). \tag{12}$$

The derivative with respect to time of conditions (12) leads to the matrix equation

$$J_1 \dot{\lambda}_{10} = J_2 [\dot{x}_0^G \quad \dot{y}_0^G \quad \dot{z}_0^G]^{\mathrm{T}}, \tag{13}$$

where the matrices J_1 and J_2 are, respectively, the inverse and forward Jacobian of the manipulator

$$J_1 = \mathrm{diag} \quad \{\delta_A \quad \delta_B \quad \delta_C\}, J_2 = \begin{bmatrix} \beta_1^A & \beta_2^A & \beta_3^A \\ \beta_1^B & \beta_2^B & \beta_3^B \\ \beta_1^C & \beta_2^C & \beta_3^C \end{bmatrix}, \tag{14}$$

with the notations

$$\begin{gathered} \delta_i = \sin(\varphi_{21}^i + \theta + \beta) \\ \beta_1^i = \sin(\varphi_{21}^i + \beta)\cos\alpha_i, \beta_2^i = \sin(\varphi_{21}^i + \beta)\sin\alpha_i, \beta_3^i = -\cos(\varphi_{21}^i + \beta) \\ (i = A,\ B,\ C). \end{gathered} \tag{15}$$

The three kinds of singularities of the three closed-loop kinematical chains can be determined through the analysis of two Jacobian matrices J_1 and J_2 [19, 20].

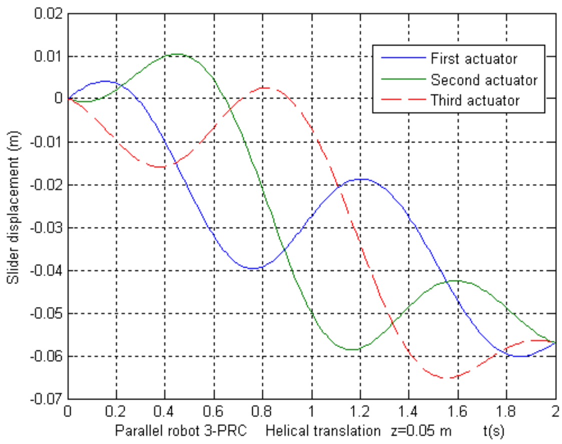

Fig. 3 – Displacements λ_{10}^i of three sliders.

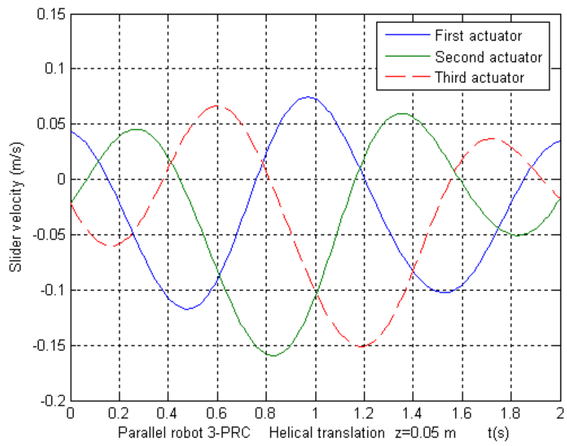

Fig. 4 – Velocities v_{10}^i of three sliders.

As for the linear accelerations γ_{10}^A, γ_{32}^A and the angular acceleration ε_{21}^A of first leg A, the derivatives with respect to time of the equations (11) give other conditions of connectivity

$$\gamma_{10}^A\, \vec{u}_j^{\mathrm{T}} a_{10}^{\mathrm{T}} \vec{u}_3 + \varepsilon_{21}^A \vec{u}_j^{\mathrm{T}} a_{20}^{\mathrm{T}} \tilde{u}_3 \vec{r}_{32}^A - \gamma_{32}^A \vec{u}_j^{\mathrm{T}} a_\alpha^{A\mathrm{T}} \vec{u}_2 = \vec{u}_j^{\mathrm{T}} \dot{\vec{r}}_0^G - \omega_{21}^A \omega_{21}^A \vec{u}_j^{\mathrm{T}} a_{20}^{\mathrm{T}} \tilde{u}_3 \tilde{u}_3 \vec{r}_{32}^A \quad (j = 1,\ 2,\ 3). \tag{16}$$

Following relations give the angular accelerations $\vec{\varepsilon}_{k0}^A$ and the accelerations $\vec{\gamma}_{k0}^A$ of joints A_k

$$\vec{\varepsilon}_{10}^A = \vec{0}, \quad \vec{\varepsilon}_{20}^A = \vec{\varepsilon}_{21}^A = \ddot{\varphi}_{21}^A \vec{u}_3, \quad \vec{\gamma}_{10}^A = \ddot{\lambda}_{10}^A \vec{u}_3, \ \vec{\gamma}_{21}^A = \vec{0}, \quad \vec{\gamma}_{20}^A = a_{21} \vec{\gamma}_{10}^A. \tag{1/}$$

As application let us consider a 3-PRC PKM which has the following characteristics

$$\begin{aligned} & r = 0.152\mathrm{m},\ l_2 = 0.400\mathrm{m},\ h = 0.1612\mathrm{m},\ \theta = \frac{\pi}{6},\ l = 2r\sqrt{3},\ \Delta t = 2\mathrm{s}, \\ & l_2 \cos(\theta + \beta) = r\sin\theta + h\cos\theta,\ l_1 = r\cos\theta - h\sin\theta + l_2\sin(\theta + \beta), \\ & g = 9.807\mathrm{ms}^{-2}. \end{aligned}$$

Using the MATLAB software, a computer program was developed to solve the inverse kinematics of the 3-PRC parallel manipulator. To develop the algorithm, it is assumed that the platform starts at rest from a central configuration and moves pursuing a helical translation. The time-history for displacements λ_{10}^{i} (Fig. 3), velocities v_{10}^{i} (Fig. 4), accelerations γ_{10}^{i} (Fig. 5), angles of rotation φ_{21}^{i} (Fig. 6), angular velocities ω_{21}^{i} (Fig. 7) and angular accelerations ε_{10}^{i} (Fig. 8) of three legs is illustrated for a period of $\Delta t = 2$ second in terms of given analytical equations (4).

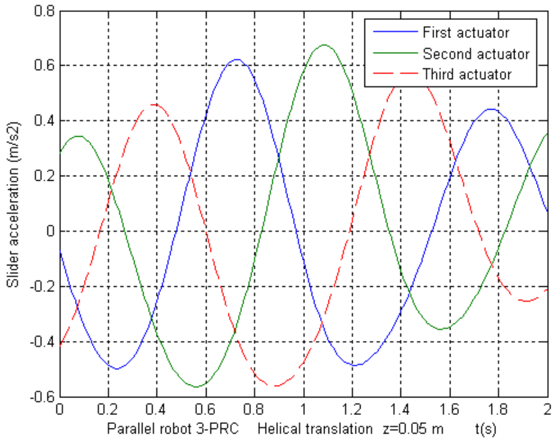

Fig. 5 – Accelerations γ_{10}^{i} of three sliders.

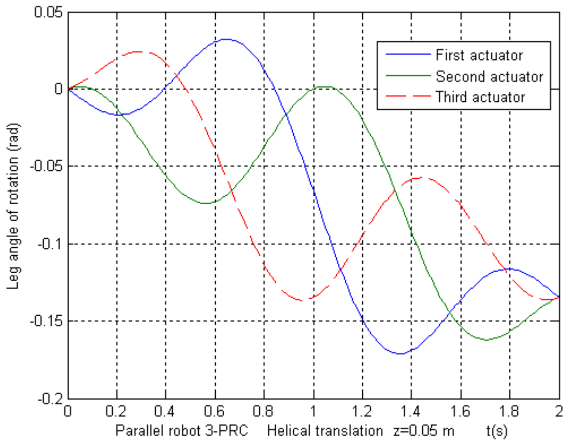

Fig. 6 – Rotation angles φ_{21}^{i} of three legs.

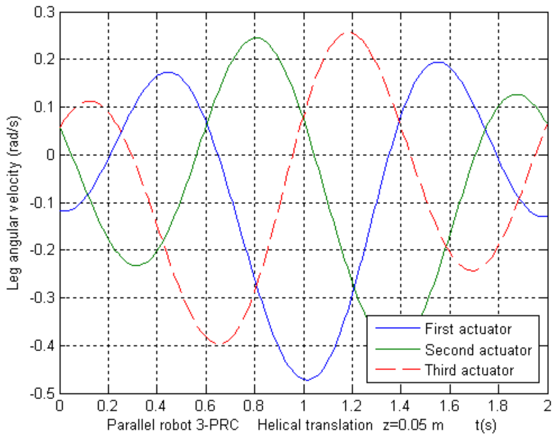

Fig. 7 – Angular velocities ω_{21}^{i} of three legs.

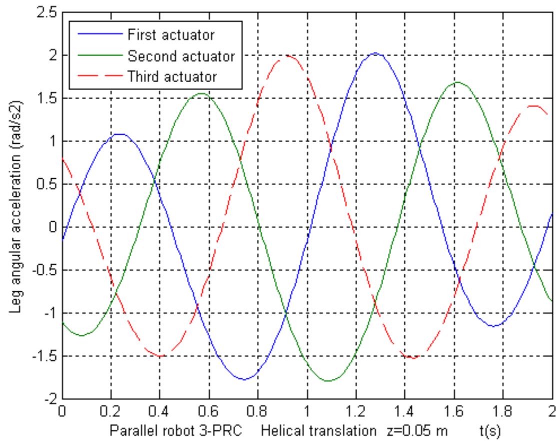

Fig. 8 – Angular accelerations ε_{21}^{i} of three legs.

3. CONCLUSIONS

Within the inverse kinematics analysis some exact relations that give in real-time the position, velocity and acceleration of each element of the parallel robot have been established in the present paper. The simulation through the program certify that one of the major advantages of the current matrix recursive formulation is accuracy and a reduced number of additions or multiplications and consequently a smaller processing time of numerical computation.

Choosing appropriate serial kinematical circuits connecting many moving platforms, the present method can easily be applied in forward and inverse mechanics of various types of parallel mechanisms, complex manipulators of higher degrees of freedom and particularly *hybrid structures*, when the number of components of the mechanisms is increased.

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