THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 12, Number 3/2011, pp. 213–220

STRUCTURAL AND BEHAVIORAL OPTIMIZATION OF THE NONLINEAR HILL MODEL

Tudor SIRETEANU *, Silviu NĂSTAC**, Mihaela PICU**

* Institute of Solid Mechanics, Bucharest, Romania
** Dunărea de Jos" University, Galati, Romania
E-mail: mihaelapicu@yahoo.com

The paper is based on the nonlinear biomechanical model developed by Hill. The central idea of this study is the behavioural and structural optimization of this model so that it emphasizes the auto adaptive capacity of the real biomechanical system. The paper analyses and compares the responses to harmonic and step signals. A version of the Hill model which contains nonlinear elastic and dissipative elements was developed based on observed results. It has also been established the constitutive law of these additional elements so that the initial objective of the study to be met. The evaluation of characteristic measurements and completing the final expression of the constitutive equation have been established taking into account a series of experimental results developed on the actual model of the hand-arm system subjected to dynamic vibration actions. The results of this research provide the necessary foundation and support for future implementations of this version of Hill's model in some virtual tools to analyze the dynamics of the human body subjected to vibration or shock.

Key words: Biodynamic, Optimization, Hill model, Nonlinear characteristics, Autoadaptive behaviour.

1. INTRODUCTION

One of the well known model in biomechanics was proposed by A.V. Hill in 1938, based on a series of practical observations dated since 1922 [5]. Thus, Hill was the first who noticed that an activated muscle produces a higher force in isometric conditions (when both ends are fixed) than in normal conditions (when its length decreases) [6, 9]. This is equivalent to the loss of a quantity of energy to overcome an internal resistance force. It is obvious that this resistance force can not be simulated by the elastic element of the model [7]. However, Hill noted that the total force is much less developed as the muscle contraction speed is higher. Considering that active force (the force introduced by the active element) has a constant value, Hill stated that a high-speed contraction leads to a high resistive force. For this reason one of the components of the model is a viscous damper. It simulates very well the real evolution: the higher the speed applied to the ends of the element, the higher value the developed force reaches, and the general behaviour of the model is a direct consequence of the fluid viscosity value [8, 11, 12].

2. HILL MODEL DIAGRAM

Hill proposed a model (Fig. 1) with a series structure, consisting of an elastic element (further noted as a serial element) and a parallel type viscoelastic configuration, like Voight model (further noted as a parallel element) [3]. The difference from classic viscoelastic model Voight type consists of an additional element mounted in parallel which simulates the muscle action. In Fig. 1 this additional element is noted with Ψ and along with the viscous element forms the active component of Hill's model [2, 8].

The constitutive equation of this type of model is obtained by considering a linear feature for the two elastic elements [4].

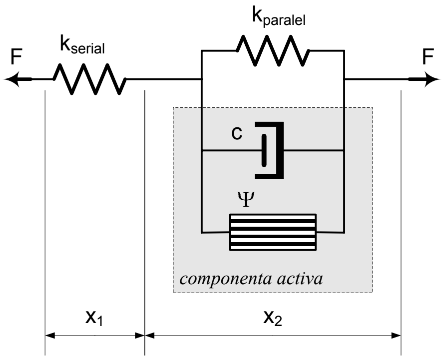


Fig. 1 – Hill model diagram.

For the serial component of the model the force value is:

$$F = k_{serial}(x_1 - x_{10}) , \tag{1}$$

and for the parallel component is:

$$F = k_{parallel}\left(x_2 - x_{20}\right) + c\dot{x}_2 + \psi , \tag{2}$$

where x and x_0 are the lengths of each component in free and strained state.

The force developed in the parallel component will be:

$$F = k_{parallel}\left| x - x_0 - \frac{F}{k_{serial}} \right| + c\left(\dot{x} - \frac{\dot{F}}{k_{serial}} \right) + \psi , \tag{3}$$

where:

$$F = \frac{k_{serial}}{k_{serial} + k_{parallel}} \cdot \left[k_{parallel}\left(x - x_0\right) + c\left(\dot{x} - \frac{\dot{F}}{k_{serial}} \right) + \psi \right] , \tag{4}$$

where F, $\dot{F}$ are the force developed by the model and respectively its time variation; x, $\dot{x}$ are the strain and respectively the strain speed, considered for the entire model; Ψ is the force from the active component of the model; k_{serial}, $k_{parallel}$ are the rigidities of the serial an parallel elements and c is the damping coefficient of the active component of the model. It must be mentioned that the F, $\dot{F}$, x, $\dot{x}$, Ψ parameters are time functions.

Equation (4) is the constitutive equation of the Hill model presented in Fig. 1. Taking into account the time variation of the force applied to the model the characteristic equation (6.35) becomes:

$$\dot{F} = \frac{k_{serial}}{c}\left[k_{parallel}\left(x - x_0\right) + c\dot{x} - \left(1 + \frac{k_{parallel}}{k_{serial}} \right) F + \psi \right] . \tag{5}$$

The active force from the parallel component is:

$$\psi = F\left(1 + \frac{k_{parallel}}{k_{serial}} \right) - k_{parallel}\left(x - x_0\right) - c\left(\dot{x} - \frac{\dot{F}}{k_{serial}} \right) . \tag{6}$$

Fenn and Marsh in 1935, then Hill in 1938 discovered that the relation between the force and the contraction speed is nonlinear in certain strain conditions [1, 10]. This phenomenon increases the complexity of the model without a significant increase of the performance obtained by direct stimulation. Taking all this into account the nonlinear model will be further studied in the initial configuration presented in Fig. 1.

Integrating the characteristically equation (6.35) results the capable force formula $F(t)$:

$$F(t) = \left[\frac{k_{serial}}{c} \int_0^t \exp\left(\frac{k_{serial} + k_{parallel}}{c} u \right) \left[k_{parallel}\left(x_u - x_0\right) + c\frac{\mathrm{d}x(u)}{\mathrm{d}u} + \Psi(u) \right] \mathrm{d}u + F_0 \right] e\left(-\frac{k_{serial} + k_{parallel}}{c} t \right) , \tag{7}$$

where $\Psi(u)$ is the active force specific to the muscular fibre, $x(u)$ is the compulsory transit law, and the inceptive force value is $F_0 = F(0)$.

If the integration is made considering the independent variable $x(t)$ it results:

$$x(t) = \left[\frac{1}{c\, k_{serial}} \int_0^t \exp\left(\frac{k_{parallel}}{c} u \right) \left[k_{parallel} k_{serial} x_0 + F(u)\left(k_{serial} + k_{parallel} \right) + c\frac{\mathrm{d}F(u)}{\mathrm{d}u} - \Psi(u) k_{serial} \right] \mathrm{d}u + x_0 \right] \cdot \\ \cdot \exp\left(-\frac{k_{parallel}}{c} t \right) , \tag{8}$$

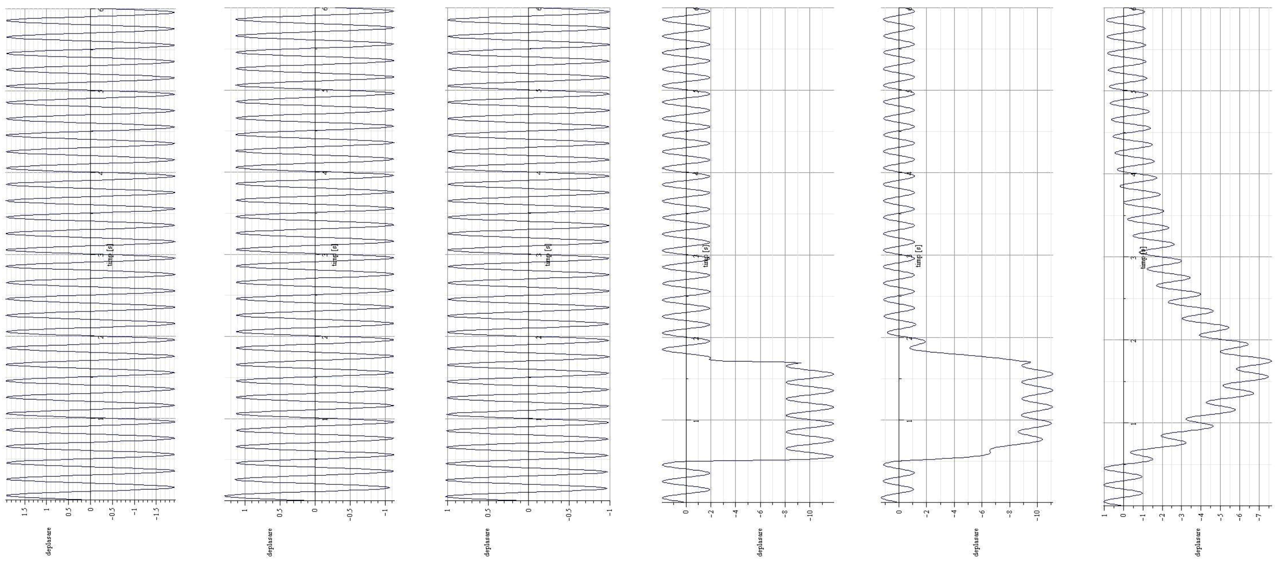
where $\Psi(u)$ is the active force, $F(u)$ is the capable (compulsory) force for the whole model and $x_0 = x(0)$ is the initial length of the muscle fibre.

3. RESPONSE ANALYSIS OF THE HILL MODEL

Hereinafter the response of this model will be analyzed in the following cases:

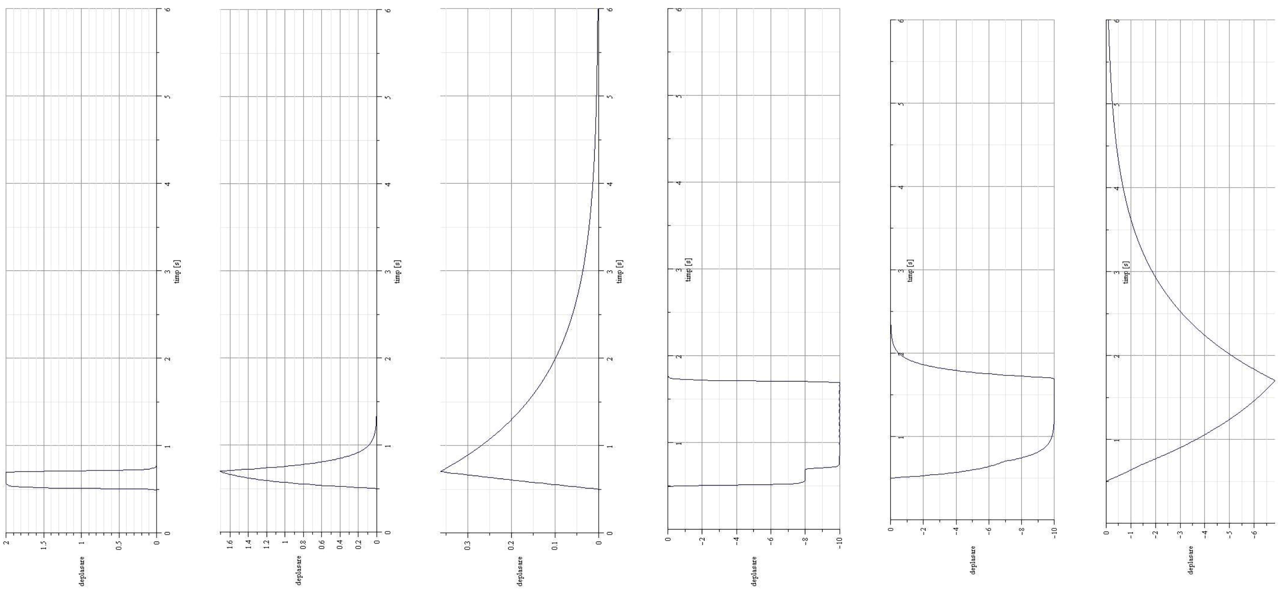
	Compulsory harmonic force	Compulsory force step-type
Displacement response	Fig. 2	Fig. 3
	compulsory displacement step-type	compulsory unitary displacement
Force response	Fig. 4	Fig. 5

For each case, the active force Ψ is null or has a unit value, the analyse was made by comparing the two conditions of the active component and for three values of the time constant of the model: $\tau_1 = 0.01$; $\tau_2 = 0.10$; $\tau_3 = 1.00$ which we considered to be representative.



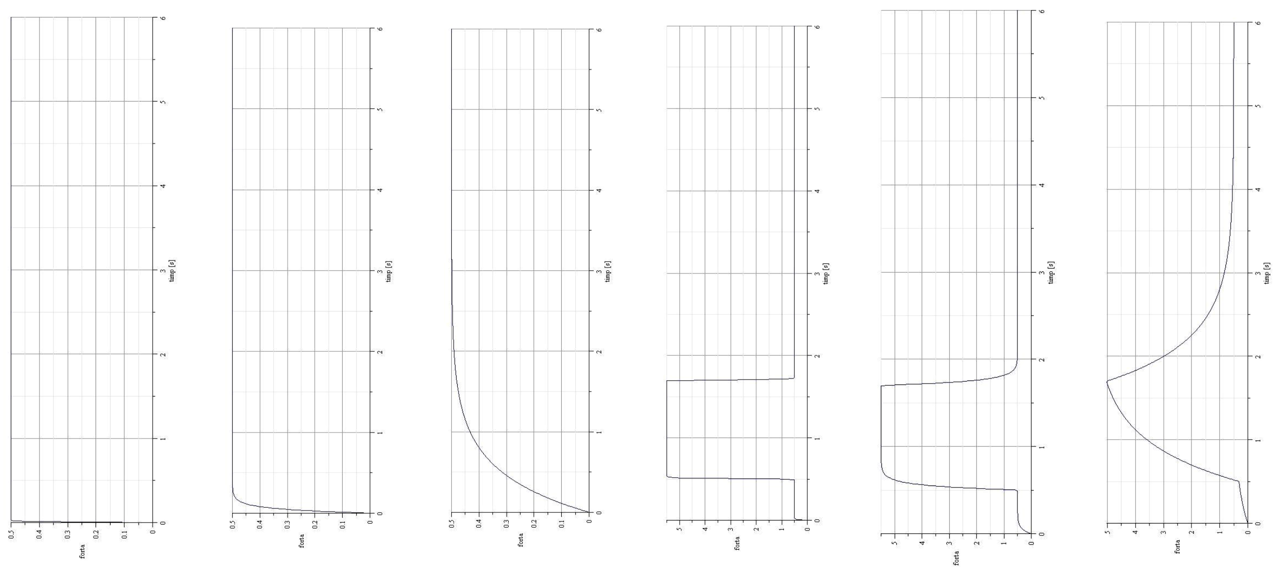
a b c d e f

Fig. 2 – Displacement response for the compulsory harmonic force and the null active force: a) $\tau_1 = 0.01$; b) $\tau_2 = 0.10$; c) $\tau_3 = 1.00$; unit active force: d) $\tau_1 = 0.01$; e) $\tau_2 = 0.10$; f) $\tau_3 = 1.00$.



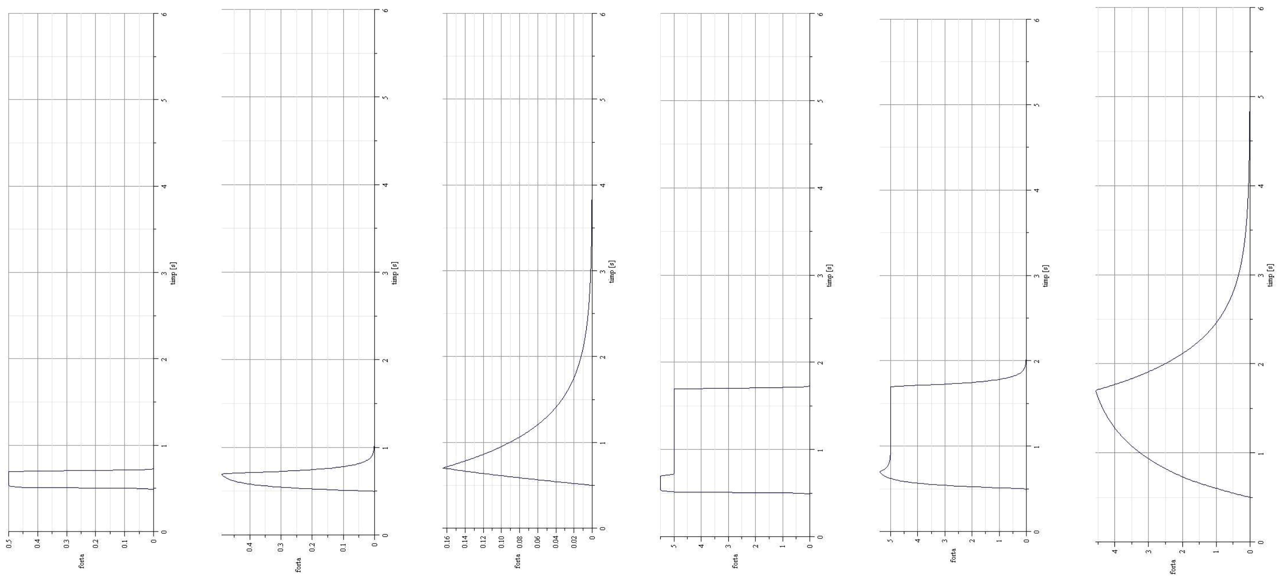
a b c d e f

Fig. 3 – Displacement response for the compulsory step type force and for the null active force: a) $\tau_1 = 0.01$; b) $\tau_2 = 0.10$; c) $\tau_3 = 1.00$; unit active force: d) $\tau_1 = 0.01$; e) $\tau_2 = 0.10$; f) $\tau_3 = 1.00$.



a b c d e f

Fig. 4 – Force response for the compulsory displacement step type and null active force: a) $\tau_1 = 0.01$; b) $\tau_2 = 0.10$; c) $\tau_3 = 1.00$; unit active force: d) $\tau_1 = 0.01$; e) $\tau_2 = 0.10$; f) $\tau_3 = 1.00$.



a b c d e f

Fig. 5 – Force response for the compulsory unitary displacement and null active force: a) τ_1=0.01; b) τ_2=0.10; c) τ_3=1.00; unit active force: d) τ_1=0.01; e) τ_2=0.10; (f) τ_3=1.00.

Comparing the diagrams from in Figs. 2–5, which are related to the Hill model behaviour to the compulsory excitation signals, in the presence or absence of the active force, it was determined:

- In the presence of the active force, this is an unitary step type with a period from $t_1 = 0{,}5$ s. up to $t_2 = 1{,}7$s with the origin being the starting point of the analysis $t_0 = 0{,}0$s. these two temporal coordinates were established in relation with the response time of the nervous stimulation on the muscle fibre and with the step type excitation period (cases presented in Fig. 3 and Fig. 4). For the other two analysed cases (harmonic displacement response – Fig. 2, respectively response to the initial unitary displacement – Fig. 5) the same time values were maintained to ease the comparative analysis.
- Regarding the displacement responses (Fig. 2 and Fig. 3) it is noted that in the absence of the active component (a, b, c diagrams) the model follows the assertion of the excitation signal as being exclusively influenced by the specific time constant value. So we have an amplitude decrease of the

harmonic signal response (Fig. 2), respectively a more and more pronounced highlight of the integration response of the initial signal (Fig. 3) by increasing the value of the time constant of the model. The active component leads to contractions of the muscle fibre length which is highlighted by the decrease of the absolute value recorded by the model under the external excitation action (d, e, f diagrams). The value of the time constant assess directly the way in which the "translation" of the transit takes place, meaning: once the τ_1 value increases the "translation" of the transit assumes a nature continuous in time, so is highlighted the natural transit way from an active state to an inactive one specific to the muscular structure.

- Regarding the responses in force (Fig. 4 and Fig. 5) it is noticed that in the inactive state (a, b, c diagrams) the model regards the exterior compulsory restriction (compulsory displacement step-type in the initial moment or at two arbitrary time moments) and the evolution is in greater accordance with the value of the time constant (the highlight of the viscous component of the model). The inception of the active state of the muscular fibre leads to an increase of the immediate value of the force during the entire active state (d, e, f diagrams). It is obvious that the passing from the inactive state to the active one and backwards happens with a direct influence on the value of the time constant of the model, highlighting once again the natural transition between the two states specific to the muscular fibre.
- The diagrams corresponding to the force responses (Fig. 4 and Fig. 5) also show the seeming pointlessness of initiating an active state when the model is subjected to external compulsory displacement. The pointless comes from the fact that the only "visible" consequence is the increase of the value of the immediate force. The speciosity is justified by the fact that by setting a fix value (immediate) of displacement at the end of the model, according to the initial theory suggested and studied by Hill, the capable force from the muscle fibre should increase to maximum values which can be seen in the evolution of the analysed model (Fig. 4 and Fig. 5, d, e, f diagrams).

4. THE ACTIVE AUTOADAPTIVE COMPONENT OF THE NONLINEAR MODEL

An upgrade to the Hill model is suggested by taking into account of some nonlinear material characteristics. So the serial element will be still linear, while the parallel element will have a nonlinear characteristic:

$$k_{serial}=\text{ct.}\ ;\quad k_{parallel}=f_k({}^{i}k_p,\ t,\ x,\ \dot{x})\quad \text{with } i=1, 2, ...\quad ;\quad c_{parallel}=f_c({}^{i}c_p,\ t,\ x,\ \dot{x})\quad \text{with } i=1, 2, ... \tag{9}$$

Thus the model built-up (Fig. 6) is active because it keeps the active element $\Psi=\Psi(t)$ in its structure, but acquires an autoadaptive characteristic due to the modification of the essential parameters – *rigidity, damping* – with the parameters of the disturbing movement – *strain*, respectively the *strain fluctuation in time*.

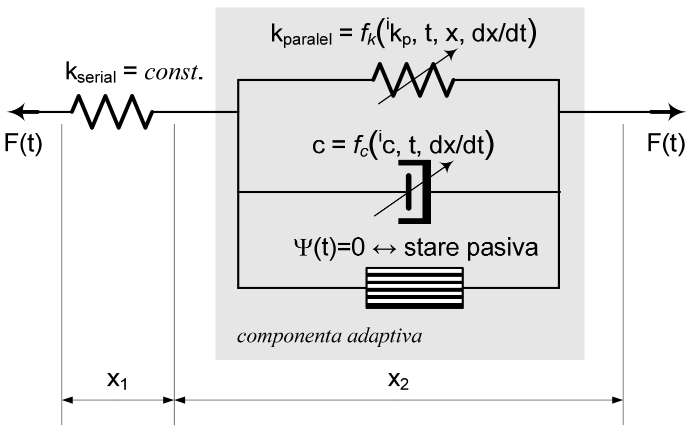


Fig. 6 – Active autoadaptive model of the muscular fibre.

This comes to support the real behaviour of the human body (or of a body part) under the action of the dynamic external factors namely: when the "grasp" value of these parameters exceeds a limit threshold an "arrest" of the body from the vibration source is tried by reducing the transmissibility of the propagation way.

Furthermore, the lab test showed that the body's adaptability is a continuous value and it is directly proportional with the stress level, so that an increase of the level picked up by the organism asserts a decrease in transmissibility performances of the propagation way of the dynamic perturbation.

In mathematical formulae, according to its use in a lucrative model all the above can be put into practice using different functions. In this paper it is proposed the following functional dependence:

$$k_{parallel}=f_k({}^{i}k_p,\ t,\ x,\ \dot{x})={}^{lin}k_{parallel}\cdot\frac{\beta_1}{1+\left|\sum\limits_{i=2\ldots n}(\beta_i\ x)^i\right|}, \tag{10}$$

where ${}^{lin}k_{parallel}$ is the known constant value of the stiffness of the parallel element, β_1 is a dimensionless non-zero constant, β_i [m^{-i}], with i =1, 2, ... are the summation polynominal coefficients (can have any real value).

It can be seen that when all the polynominal coefficient are null, the rigidity function becomes linear, and the stiffness value is multiplicative influenced with β_1:

$$k_{parallel}=f_k({}^{i}k_p,\ t,\ x,\ \dot{x})={}^{lin}k_{parallel}\cdot\frac{\beta_1}{1+0}$$

$$k_{parallel}=f_k({}^{i}k_p,\ t,\ x,\ \dot{x})={}^{lin}k_{parallel}\cdot\beta_1\,. \tag{11}$$

If β_1 = 1, for the linear case we will obtain:

$$k_{paralel}={}^{lin}k_{paralel}\,. \tag{12}$$

Fig. 7 presents a simulation of the elastic characteristic of an autoadaptive element according to the relation (10), for: ${}^{lin}k_{parallel}$ =1 000 N/m, β_1 = 1, $\beta_2 = \beta_5 = \beta_{12}$ = 10, and the rest of the polynominal coefficients were considered null.

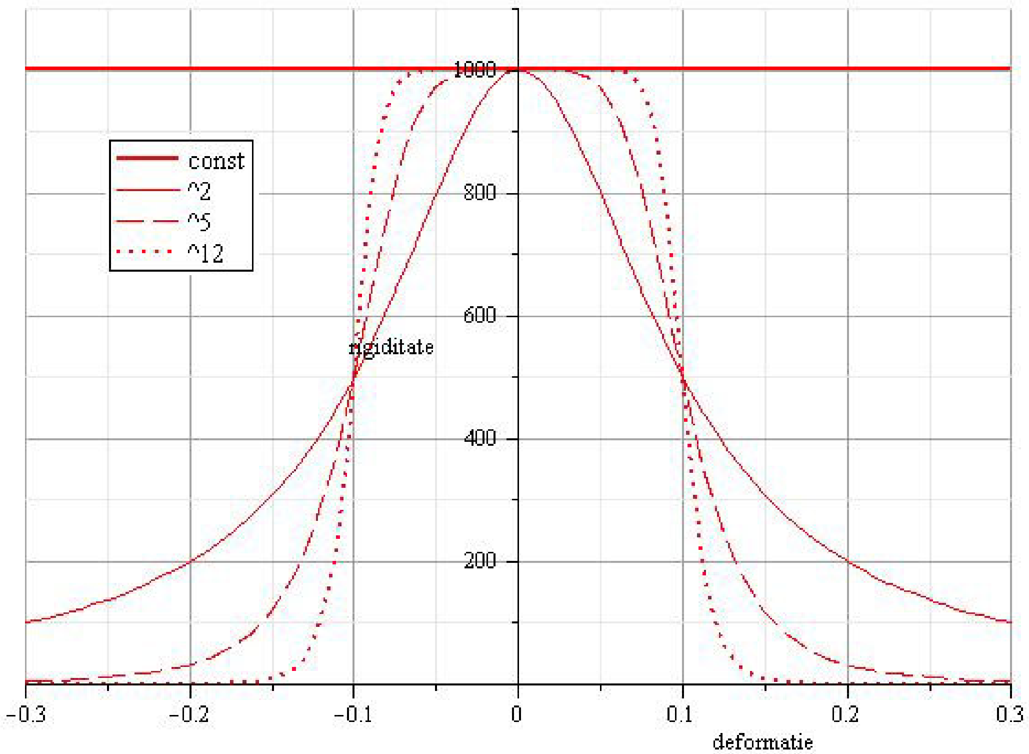


Fig. 7 – Non-linear elastic characteristic of the autoadaptive model.

The simulation was realised separately for each case (i = 2; i = 5; i = 12) and the obtained results were presented comparative. The diagram correspondent to the linear case was drawn.

Fig. 8 shows correspondent diagrams of the non-linear elastic force developed by the autoadaptive model in the same conditions previous presented.

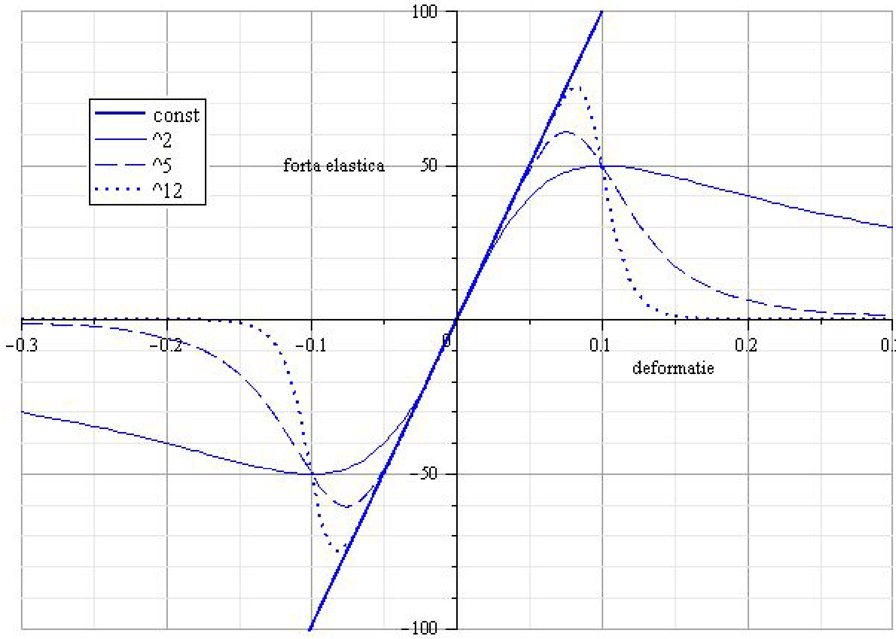


Fig. 8 – Non-linear elastic force developed by the autoadaptive model.

$$^{elastic}F_{parallel} = {}^{lin}k_{parallel} \cdot \frac{\beta_1}{1 + \left| \sum_{i=2...n} \left(\beta_i \;\; x(t)\right)^i \right|} x(t) \,, \tag{13}$$

where we noted with $^{elastic}F_{parallel}$ the elastic force of the parallel component.

It must be mentioned that the simulations were made exclusively for the adaptive component of the proposed model, considering the total absence of the active element $\Psi = \Psi(t) = 0$. It was highlighted the specific behaviour for the action of some summation polynominal coefficients (2, 5, 12).

So, we can model a differentiate active characteristic on adaptability levels, through the fact that every active element will bring into the model an action compulsory by the given summation polynominal coefficient. The difficult modelling of this type of element and the necessity for a lot of data explains this strictly theoretical approach, without practical illustrations.

5. CONCLUSIONS

This paper presented a shortly theoretical approach of the Hill's basic model with a set of an additional improvements regarding behavioural and/or structural optimization. Additional charges of the Hill's basic model leads to better behavioural approaches, even those charges not involved main structural modifications.

One of the first charges enjoined to the Hill's basic model supposed the elimination of the damper component. This modification leads to a structural simplification, but have not the capacity to dignify the natural transitions between the active and inactive states or due to the external stimulus. In the same time, these simplified variants of the Hill's model help to evaluate the natural frequencies of biomechanics systems and also the resonance area taking into account the active or inactive state of the special component.

The last and complex charges applied to the Hill's model, which was briefly presented in this paper, not involve main structural changes. But through the nonlinear characteristics imposed to elastic and viscous elements, the muscular fibre responses on external dynamic actions respect more precisely the experimental observations.

The set of diagrams presented in this paper reveal the major importance of the proper time constant for each model. Hereby, the jumping evolutions of the numerical model versus natural transitions of the real system are the two states between the simulations can be evolved.

The main advantage of these approaches of the Hill's model is the active auto-adaptive capacity which dignifies the natural ability of the muscular system to respond on dynamic external charge like shock and vibration. This study derived from human biodynamic analysis requirements, with the basic application framed by the hand-arm dynamic behaviour simulation.

REFERENCES

1. ABDULLAH, H.A., TARRY, C., DATTA, R., MITTAL, G.S. and ABDERRAHIM, M., *Dynamic biomechanical model for assessing and monitoring robot-assisted upper-limb therapy*, J. of Rehabilitation Research & Development, **44**, *1*, pp. 43–62, 2007.
2. ALDIEN, Y., MARCOTTE, P., RAKHEJA, S. and BOILEAU, P.É., *Mechanical Impedance and Absorbed Power of Hand-Arm under xh–Axis Vibration and Role of Hand Forces and Posture*, Industrial Health, **43**, pp. 495–508, 2005.
3. BRATOSIN, D., BĂLAN, F.S., CIOFLAN, C.O., *Multiple resonance of the site oscillating systems*, Proceedings of the Romanian Academy, Series A, **11**, *3*, pp. 261–268, 2010.
4. DRĂGĂNESCU, M., KAFATOS, M., *Categories and Functors for the Structural Phenomenological Modeling*, Proceedings of the Romanian Academy, Series A, **1**, *2*, pp. 111–115, 2000.
5. HILL, A.V., *The heat of shortening and the dynamic constants of muscle*, Proc. R. Soc. Lond. B Biol. Sci., **126**, pp. 136–195, 1938.
6. KNUDSON, D., *Fundamentals of Biomechanics,* Second Edition, Springer Science+Business Media, LLC, 2007.
7. KYRIACOS A.A. and ROMAN, M.N., *Introduction to Continuum Biomechanics*, Synthesis Lectures on Biomedical Engineering, **19**, Series Editor: John D. Enderle, Morgan & Claypool Publishers, 2008.
8. LUNDSTRÖM R. and HOLMLUND P., *Absorption of energy during whole-body vibration exposure*, Journal of Sound and Vibration, **215**, pp. 801–812, 2007.
9. MANSFIELD, N.J., *Human response to vibration*, CRC Press, 2005.
10. PICU, A., NASTAC, S., *Advanced Simulations of Human Protective Devices Against Technological Vibrations,* Proceedings of the ACOUSTICS High Tatras 2009 "34th International Acoustical Conference-EAA Symposium", 2009.
11. SIRETEANU, T., GUGLIELMINO, E., STAMMERS, C.W., GHITA, G. and GIUCLEA, M., *Semi-active Suspension Control: Improved Vehicle Ride and Road Friendliness*, Springer, 06-03, 2009.
12. SOUTH, T., Managing Noise and Vibration at Work. *A practical guide to assessment, measurement and control*, Elsevier Butterworth-Heinemann, Linacre House, Jordan Hill, Oxford, 2004.

Received November 25, 2010