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NON-OPTIMAL CLONING OF QUBITS AT A DISTANCE

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In contrast to the classical protocol, when one can generate as many copies of a system as one wishes, the no-cloning theorem forbids the perfect cloning of quantum systems. In this paper we present a scheme that enables the imperfect non-optimal copying of a qubit and show how the two non-identical clones can be distributed to two spatially separated receivers with the help of a non-maximally entangled channel.

Key words: Entanglement, quantum cloning.

1. INTRODUCTION

One main difference between the classical and quantum information theory is the *no-cloning theorem*, which imposes that an arbitrary pure quantum state cannot be copied [1]. Because the perfect copying of a pure state is impossible, cloning machines were considered for generating two or more identical mixed output states [2, 3]. Quantum cloning of photons has been demonstrated in two experiments by Lamas-Linares *et al.* [4] and by Fasel *et al.* [5].

Another important process in quantum information theory is *quantum teleportation* introduced by Bennett *et al*. [6]. In this scheme, an observer Alice transmits the information of a quantum system to a distant observer Bob by using a maximally entangled state. When the protocol is completed, Alice's initial state is destroyed.

In Ref. [7], Murao *et al.* proposed a quantum scheme called *telecloning* that combines quantum teleportation and cloning. In this protocol, an observer Alice has to send M identical copies of an unknown qubit to M spatially separated observers. Some generalizations of telecloning were proposed: the $1 \to M$ symmetric telecloning of a d-level system and the asymmetric telecloning for qubits [8]. We have extended the concept of asymmetric telecloning from qubits to d-dimensional quantum systems [9]. Further, we proposed the telecloning of entanglement [10], a scheme that performs the optimal transmission of the two nonlocal optimal clones to two pairs of spatially separated receivers.

Gordon and Rigolin considered a more general quantum symmetric telecloning, which is based on using non-maximally entangled channels and the measurements of the sender are performed using a modified Bell basis [11].

Starting from the proposal of Gordon and Rigolin, we propose here the *asymmetric telecloning* of qubits, which is obtained with the help of a quantum non-maximally entangled channel shared between the sender and receivers. We want to emphasize that our protocol generates two non-identical mixed clones, which are not the optimal one. Due to decoherence, the two states which occur in the expression of the quantum channel are not the states used for the symmetric optimal cloning, and this fact leads to two different clones. We plot the average fidelity of the two non-identical clones versus the parameter a, which characterizes the initial qubit to be cloned, and n, the parameter that defines the non-maximally entangled channel.

2. NON-OPTIMAL CLONING AT A DISTANCE

Let us consider that the initial qubit of Alice is

$$|\Psi\rangle = a|0\rangle + b|1\rangle , \tag{1}$$

where a and b satisfy the normalization condition $|a|^2 + |b|^2 = 1$. Alice wants to clone this state and to transmit this information to two spatially separated receivers Bob and Charlie. Because of the no-cloning theorem, Alice will be able only to send two imperfect copies of the state (1).

Suppose that Alice, Bob, Charlie, and Daniel share a non-maximally entangled channel described by:

$$|\xi\rangle_{ABCD} = \frac{1}{\sqrt{1+n^2}}(|0\rangle_A|\phi_0\rangle_{BCD} + n|1\rangle_A|\phi_1\rangle_{BCD}) , \tag{2}$$

where the states $|\phi_0\rangle$ and $|\phi_1\rangle$ have the following expressions

$$\begin{aligned} |\phi_0\rangle_{BCD} &= \frac{12}{13}\left(|000\rangle + \frac{1}{3}|011\rangle + \frac{1}{4}|101\rangle\right); \\ |\phi_1\rangle_{BCD} &= \frac{12}{13}\left(|111\rangle + \frac{1}{3}|100\rangle + \frac{1}{4}|010\rangle\right). \end{aligned} \tag{3}$$

The fourth observer Daniel has the ancillary qubit and his task is only to help; since he will obtain no information regarding the Alice's qubit.

The total state of the four observers is

$$\begin{aligned} |\psi\rangle_A|\xi\rangle_{ABCD} = \frac{1}{\sqrt{2(1+n^2)}}\Big[&|\Phi^+\rangle_A\left(a|\phi_0\rangle + bn|\phi_1\rangle\right) + |\Phi^-\rangle_A\left(a|\phi_0\rangle - bn|\phi_1\rangle\right) \\ &+ |\Psi^+\rangle_A\left(an|\phi_1\rangle + b|\phi_0\rangle\right) + |\Psi^-\rangle_A\left(an|\phi_1\rangle - b|\phi_0\rangle\right)\Big] \end{aligned} \tag{4}$$

We have denoted the elements of the Bell basis by $|\Phi^{\pm}\rangle, |\Psi^{\pm}\rangle$. Alice performs a measurement in the Bell basis and communicates the outcome to Bob, Charlie, and Daniel. According to the result of Alice's measurement, Bob, Charlie, and Daniel apply a local unitary operator and obtain one of the two states

$$\begin{aligned} |\Pi_0\rangle &= \frac{1}{\sqrt{|a|^2 + n|b|^2}}\left(a|\phi_0\rangle + bn|\phi_1\rangle\right); \\ |\Pi_1\rangle &= \frac{1}{\sqrt{n|a|^2 + |b|^2}}\left(an|\phi_0\rangle + b|\phi_1\rangle\right). \end{aligned} \tag{5}$$

The state $|\Pi_0\rangle$ is obtained with the probability

$$p_0 = \frac{1}{1+n^2}\left(|a|^2 + |b|^2 n\right), \tag{6}$$

while the state $|\Pi_1\rangle$ is generated with the below probability

$$p_1 = \frac{1}{1+n^2}\left(|a|^2 n + |b|^2\right). \tag{7}$$

Let us denote by $|\eta_0\rangle$ and $|\eta_1\rangle$ the following normalized states shared by Bob, Charlie, and Daniel

$$\left|\eta_0\right\rangle_{BCD} = \frac{1}{\sqrt{|a|^2 + |b|^2 n}}\left(a\left|\phi_0\right\rangle + bn\left|\phi_1\right\rangle\right). \tag{8}$$

and

$$\left|\eta_1\right\rangle_{BCD} = \frac{1}{\sqrt{|a|^2 n + |b|^2}}\left(an\left|\phi_0\right\rangle + b\left|\phi_1\right\rangle\right). \tag{9}$$

The reduced density operators of Bob and Charlie are

$$\begin{aligned}\rho_{j,B} &= Tr_{CD}\left|\Pi_j\right\rangle\left\langle\Pi_j\right| = \frac{144}{169}\left[\frac{151}{144}\left|\eta_j\right\rangle\left\langle\eta_j\right| + \frac{1}{16}I\right];\\ \rho_{j,C} &= Tr_{BD}\left|\Pi_j\right\rangle\left\langle\Pi_j\right| = \frac{144}{169}\left[\frac{137}{144}\left|\eta_j\right\rangle\left\langle\eta_j\right| + \frac{1}{9}I\right],\end{aligned} \tag{10}$$

with j = 1 and 2.

The parameter which describes the cloning is the average fidelity defined as follows

$$\overline{F} = p_0F_0 + p_1F_1, \tag{11}$$

where F_j is the fidelity computed with the help of ρ_j. In Fig. 1 and Fig. 2 we plot the average fidelity versus the parameters a, which defines the initial Alice's qubit, and n that characterizes the quantum channel shared between Alice, Bob, Charlie, and Daniel.

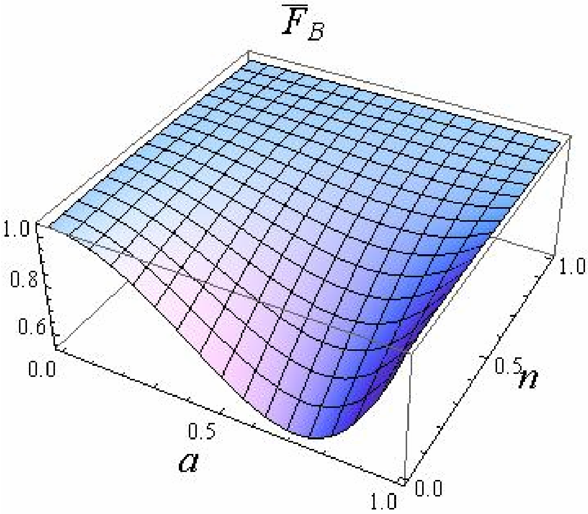

Fig. 1 The average fidelity of the clone transmitted to Bob versus *a* and *n*.

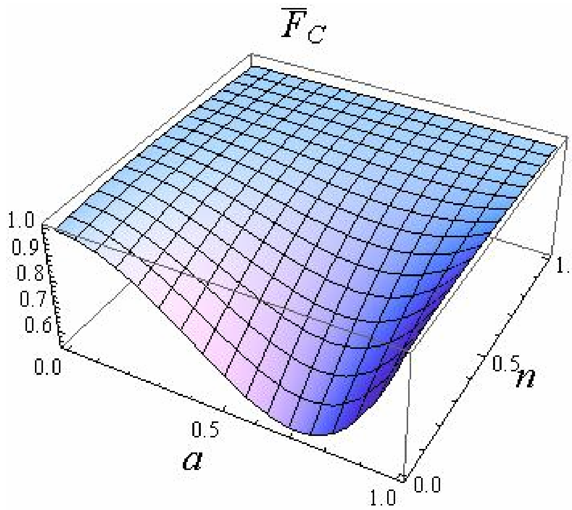

Fig. 2 The average fidelity of the clone transmitted to Charlie versus *a* and *n*.

3. CONCLUSIONS

In this work we considered the quantum telecloning of qubits based on a non-maximally entangled channel shared between the sender and receiver. The two final transmitted clones are non-identical and non-optimal. This happens because the quantum channel is built with the help of the states $\left|\phi_0\right\rangle$ and $\left|\phi_1\right\rangle$ given by Eq. (3), which are not obtained by the optimal symmetric cloning machine.

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