THE PUBLISHING HOUSE OF THE ROMANIAN ACADEMY
PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 14, Number 1/2013, pp. 3–8

A CLASSIFICATION OF CUBIC EDGE-TRANSITIVE GRAPHS OF ORDER $4p^3$

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A graph is called edge-transitive if its automorphism group acts transitively on its edge set. In this paper, we classify all connected cubic edge-transitive graphs of order $4p^3$ for each prime p.

Key words: Regular coverings, Edge-transitive graphs, Semisymmetric graphs, Symmetric graphs.

1. INRODUCTION

Throughout this paper, graphs are assumed to be finite, simple, undirected and connected. For the group-theoretic concepts and notations not defined here we refer to [14].

For a graph X, we denote by $V(X)$, $E(X)$, $A(X)$ and $Aut(X)$ the vertex set, the edge set, the arc set and the full automorphism group of X, respectively. If a subgroup G of $Aut(X)$ acts transitively on $V(X)$, $E(X)$ and $A(X)$ we say that X is G-vertex-transitive, G-edge-transitive and G-arc-transitive, respectively. In the special case, when $G = Aut(X)$ we say that X is vertex-transitive, edge-transitive and arc-transitive (or symmetric), respectively. A regular G-edge-transitive but not G-vertex-transitive graph will be referred to as a G-semisymmetric graph. In particular, if $G = Aut(X)$, the graph is said to be semisymmetric.

An s-arc in a graph X is an ordered $(s+1)$-tuple $(v_0, v_1, \ldots, v_{s-1}, v_s)$ of vertices of X such that v_{i-1} is adjacent to v_i for $1 \le i \le s$ and $v_{i-1} \ne v_{i+1}$ for $1 \le i \le s$. A graph X is said to be s-arc-transitive if $Aut(X)$ is transitive on the set of s-arcs in X. A graph X is said to be s-regular if $Aut(X)$ acts regularly on the set of s-arcs in X. Tutte [23] showed that every finite connected cubic symmetric graph is s-regular for some s, $1 \le s \le 5$. A subgroup of $Aut(X)$ is said to be s-regular if it acts regularly on the set of s-arcs in X. The classification of cubic symmetric or semisymmetric graphs of different orders is given in many papers. Note that a cubic edge-transitive graph is either symmetric or semisymmetric and then, for classifying cubic edge-transitive graphs of certain order, we must investigate both symmetric and semisymmetric ones. So far, cubic edge-transitive graphs of orders $2p$ [13, 11], $2p^2$ [13,11], $4p$ [4,12], $4p^2$ [3,12], $6p$ [7,12], $6p^2$ [17, 12], $8p$ [2,8], $8p^2$ [1, 8], $10p$ [7,10], $10p^2$ [24, 10], $14p$ [7, 20] and $2p^3$ [19, 9] have been classified. In this paper, we want to classify all connected cubic edge-transitive graphs of order $4p^3$, where p is a prime. It is sufficient to classify cubic symmetric graphs of order $4p^3$ for each prime p, because in [4, Theorem 1.1] we proved that there is no cubic semisymmetric graph of order $4p^3$, where p is a prime.

Now, we need to introduce a new graph as titled EC_{p^3} in [12]. Let K_4 be the complete graph of order 4. We identify the vertex (Fig. 1) set of K_4 with $\{a,b,c,d\}$. Let p be a prime and Z_p^3 be the 3-dimensional row vector space over the field Z_p. Take the standard basis vectors $e_1 = (1,0,0)$, $e_2 = (0,1,0)$ and $e_3 = (0,0,1)$. The graph EC_{p^3} is defined with the vertex set $V(EC_{p^3}) = V(K_4) \times Z_p^3$ and the edge set $E(EC_{p^3})$ as following:

$$E(EC_{p^3}) = \{(a,x)(b,x),(a,x)(c,x),(a,x)(d,x),(b,x)(c,x+e_1),(c,x)(d,x+e_2),(d,x)(b,x+e_3) \mid x \in Z_p^3\}.$$

THEOREM 1.1. *Let* p *be a prime and* X *be a edge-transitive cubic graph of order* $4p^3$. *Then,* X *is isomorphic to one of* EC_{p^3} *for a prime* p. *Moreover,* X *is a 2-regular symmetric graph.*

2. PRELIMINARIES

Let X be a graph and N be a subgroup of $Aut(X)$. For $u,v \in V(X)$, denote by $\{u,v\}$ the edge incident to u and v in X, and by $N_X(u)$ we denote the set of vertices adjacent to u in X. The quotient graph X_N induced by N is defined as the graph such that the set Σ of N-orbits in $V(X)$ is the vertex set of X_N and $B,C \in \Sigma$ are adjacent if and only if there exist $u \in B$ and $v \in C$ such that $\{u,v\} \in E(X)$.

A graph $\tilde{X}$ is called a covering of a graph X with projection $\wp : \tilde{X} \to X$ if there is a surjection $\wp : V(\tilde{X}) \to V(X)$ such that $\wp|_{N_{\tilde{X}}(\tilde{v})} : N_{\tilde{X}}(\tilde{v}) \to N_X(v)$ is a bijection for any vertex $v \in V(X)$ and $\tilde{v} \in \wp^{-1}(v)$. A covering $\tilde{X}$ of X with a projection $\wp$ is said to be regular (or K-covering) if there is a semiregular subgroup K of the automorphism group $Aut(\tilde{X})$ such that graph X is isomorphic to the quotient graph $\tilde{X}_K$, say by h, and the quotient map $\tilde{X} \to \tilde{X}_K$ is the composition $\wp h$ of $\wp$ and h;to emphasize this we sometimes write $\wp_K$ instead of just $\wp$. The fibre of an edge or a vertex is its preimage under $\wp$. An automorphism of $\tilde{X}$ is said to be fibre-preserving if it maps a fibre to a fibre, while every covering transformation maps a fibre on to itself. All of fibre-preserving automorphisms form a group called the fibre-preserving group.

Let K be a finite group. A voltage assignment (or, K-voltage assignment) of X is a function $\xi : A(X) \to K$ with the property that $\xi(a^{-1}) = (\xi(a))^{-1}$ for each arc $a \in A(X)$. The values of ξ are called voltages, and K is the voltage group. The graph $Cov(X,\xi) = X \times_{\xi} K$ derived from a voltage assignment $\xi : A(X) \to K$ has vertex set $V(X) \times K$ and edge set $E(X) \times K$, so that an edge (e,g) of $E(X) \times K$ joins a vertex (u,g) to $(v, g\xi(a))$ for $a = (u,v) \in A(X)$ and $g \in K$, where $e = \{u,v\}$. Giving a spanning tree T of the graph X, a voltage assignment ξ is said to be T-reduced if the voltages on the tree arcs are the identity.

Gross and Tucker [15] showed that every regular covering $\tilde{X}$ of a graph X can be derived from a T-reduced voltages assignment ξ with respect to an arbitrary fixed spanning tree T of X. It is clear that if ξ is reduced, the derived graph $X \times_{\xi} K$ is connected if and only if the voltages on the cotree arcs generate the voltages group K.

Let $\tilde{X}$ be a K-covering of X with a projection $\wp$. If $\alpha \in Aut(X)$ and $\tilde{\alpha} \in Aut(\tilde{X})$ satisfy $\tilde{\alpha}\wp = \wp\alpha$, we call $\tilde{\alpha}$ a lift of α, and α the projection of $\tilde{\alpha}$. The lifts and the projections of such subgroups are of course subgroups in $Aut(\tilde{X})$ and $Aut(X)$, respectively. A regular covering projection $\wp$ is called arc-transitive if a some subgroup $G \le Aut(\tilde{X})$ lifts along $\wp$, which G is an arc-transitive subgroup.

Let $X \times_{\phi} K \to X$ be a connected K-covering. Given $\alpha \in Aut(X)$, we define a function $\bar{\alpha}$ from the set of voltages on fundamental closed walks based at a fixed vertex $v \in V(X)$ to the voltage group K by

$$(\phi(C))^{\bar{\alpha}} = \phi(C^{\alpha}),$$

where C ranges over all fundamental closed walks at v, and $\phi(C)$ and $\phi(C^{\alpha})$ are the voltages on C and C^{α}, respectively.

The next proposition is a special case of [18, Theorem 4.2].

PROPOSITION 2.1. *Let* $X \times_{\alpha} K \to X$ *be a connected* K*-covering. Then, an automorphism* α *of* X *lifts if and only if* $\phi(u,v)^{\sigma} = \psi(u,v)$ *extends to an automorphism of* K.

Two coverings $\tilde{X}_1$ and $\tilde{X}_2$ of X with projections $\wp_1$ and $\wp_2$ respectively, are said to be isomorphic if there exists a graph isomorphism $\tilde{\alpha} : \tilde{X}_1 \to \tilde{X}_2$ such that $\tilde{\alpha}\wp_2 = \wp_1$.

We quote the following propositions.

PROPOSITION 2.2 [22]. *Two connected regular coverings* $X \times_{\phi} K$ *and* $X \times_{\psi} K$, *where* ϕ *and* ψ *are* T*-reduced are isomorphic if and only if there exists an automorphism* $\sigma \in Aut(K)$ *such that* $\phi(u,v)^{\sigma} = \psi(u,v)$ *for any cotree arc* (u,v) *of* X.

PROPOSITION 2.3 [16, Theorem 9]. *Let* X *be a connected symmetric graph of prime valency and* G *an* s*-regular subgroup of* $Aut(X)$ *for some* $s \geq 1$. *If a normal subgroup* N *of* G *has more than two orbits, then it is semiregular and* G/N *is an* s*-regular subgroup of* $Aut(X_N)$, *where* X_N *is the quotient graph of* X *corresponding to the orbits of* N. *Furthermore,* X *is a* N*-regular covering of* X_N.

PROPOSITION 2.4 [6, Propositions 2-5]. *Let* X *be a connected cubic symmetric graph and* G *be an* s*-regular subgroup of* $Aut(X)$. *Then the stabilizer* G_v *of* $v \in V(X)$ *is isomorphic to* $Z_3, S_3, S_3 \times Z_2, S_4$, *or* $S_4 \times Z_2$ *for* $s = 1, 2, 3, 4$ *or 5, respectively.*

PROPOSITION 2.5 [12, Theorem 6.2]. *Let* X *be a connected cubic symmetric graph of order* $4p$ *or* $4p^2$ *for a prime* p. *Then* X *is isomorphic to the 2-regular hypercube* Q_3 *of order 8, the 2-regular Petersen generalized graphs P (8, 3) or P (10, 7) of order 16 or 20 respectively, the 3-regular Desargues graph of order 20 or the 3-regular Coxeter graph* C_{28} *of order 28.*

3. MAIN RESUALTS

For a positive integer n, we denote by Z_n the cyclic group of order n. Note that up to isomorphism there are exactly five groups of order p^3 for each odd prime p. These five groups are given by the following presentations:

$$Z_{p^3}, Z_p^3, Z_{p^2} \times Z_p,$$

$$N(p^2, p) := \langle x, y \mid x^{p^2} = y^p = 1, [x, y] = x^p \rangle,$$

$$N(p, p, p) := \langle x, y, z \mid x^p = y^p = z^p = 1, [x, y] = z, [z, x] = [z, y] = 1 \rangle.$$

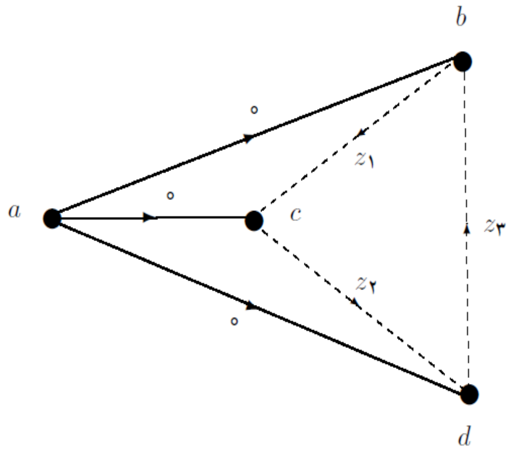

Fig. 1 – A spanning tree and a voltage assignment on K_4.

At the first, we shall classify the cubic symmetric graphs of order $4p^3$ for each prime p. For each prime $p \le 7$ by [5], there exists one unique cubic symmetric graph of order $4p^3$. Moreover, these graphs are 2-regular. So, we can assume that $p \ge 11$.

LEMMA 3.1. *Suppose that* X *is a cubic symmetric graph of order* $4p^3$, *where* $p \ge 11$ *is an odd prime. Set* $A := Aut(X)$. *Moreover suppose that* $Q := O_p(A)$ *is the maximal normal* p*-subgroup of* A. *Then* $|Q| = p^3$.

Proof. Let X be a cubic symmetric graph of order $4p^3$, where $p \ge 11$ is an odd prime. Then by [23], X is at most 5-regular. By Proposition 2.4, the stabilizer A_v of $v \in V(X)$ is a $\{2,3\}$-group. Moreover, $|A_v| = 2^{s-1}3$ and hence $|A| = 2^{s+1}3p^3$, for some $1 \le s \le 5$. Now, we intend to prove that $|Q| = p^3$.

We first suppose that $|Q| = 1$. Let N be a minimal normal subgroup of A. It is obvious that N must be solvable because otherwise N is isomorphic to A_5 or $PSL(2,7)$, a contradiction to $p \ge 11$. So, N is an elementary abelian 2-group, 3-group or p-group. Since $|Q| = 1$, N can not be an elementary abelian p-group. Also, N can not be an elementary abelian 3-group because otherwise $N \le A_v$, where $v \in V(X)$ and N is not semiregular, which contradicts Proposition 2.3. Thus, N is an elementary abelian 2-group. It is easy to check that N has more than two orbits and then by Proposition 2.3, it is semiregular. Therefore, $|N| = 2$ or 4. Now suppose that $|N| = 2$. Let M/N be a minimal normal subgroup of A/N. By Proposition 2.3, A/N is an s-regular subgroup of $Aut(X_N)$. Clearly, M/N is solvable and then elementary abelian. If M/N is an elementary abelian 2-group, it is semiregular by Proposition 2.3, so that $|M/N| = 2$. It follows that the quotient graph X_M has odd number of vertices and valency 3, which is impossible. Also, similarly as above M/N can not be an elementary abelian 3-group. Thus, M/N is an elementary abelian p-group. So, $|M| = 2p, 2p^2$ or $2p^3$. Let $P \in Syl_p(M)$. Then we can easily see that P is normal and also characteristic in M. Then, A has a normal subgroup of order p, p^2 or p^3, a contradiction to $|Q| = 1$. It leads to $|N| \ne 2$.
Now, if $|N| = 4$, then the quotient graph X_N must have order p^3, a contradiction. Therefore, $|Q| \ne 1$.
Finally, if $|Q| = p$ or p^2, then Q has more than two orbits and then by Proposition 2.3, A/Q is an s-regular subgroup of $Aut(X_Q)$, where X_Q is of order $4p^2$ or $4p$, respectively. But by Proposition 2.5, there is no symmetric cubic graph X_Q of these orders for prime $p \ge 11$, a contradiction. Therefore, $|Q| = p^3$. Similarly as previous, Q has more than two orbits and then by Proposition 2.3, X_Q is a symmetric cubic graph of order 4. Then X_Q must isomorphic to the complete graph K_4. Indeed, X is a Q-regular covering of the complete graph K_4, where $|Q| = p^3$. ∎

LEMMA 3.2. *Let* $p \ge 11$ *be a prime and* X *be an arc-transitive* Q*-regular covering of the complete graph* K_4, *where* $|Q| = p^3$. *Then,* X *is a* Z_p^3*-covering of* K_4 *and moreover,* X *is 2-regular.*

Proof. Let $X = K_4 \times_\phi Q$ be a connected Q-covering of K_4 satisfying the hypotheses, where $\phi = 0$ on the spanning tree T as illustrated by plain lines in Fig. 1. We assign voltages z_1, z_2 and z_3 in Q to the cotree arcs $(b,c), (c,d)$ and (d,b), respectively. The connectivity of X implies that $Q = \langle z_1, z_2, z_3 \rangle$. Set $\alpha = (ab)(cd)$ and $\beta = (bcd)$.The arc-transitivity of the regular projection ϕ implies that α and β lift. Let C be a fundamental cycle in K_4. Then, C is abc, acd or adb and their images with corresponding voltages on K_4 are given in Table 1.

Table 1

Fundamental cycles and their images with corresponding voltages on K_4

C	$\phi(C)$	C^{α}	$\phi(C^{\alpha})$	C^{β}	$\phi(C^{\beta})$
abc	z_1	*bad*	z_3	*acd*	z_2
acd	z_2	*bdc*	$-z_1-z_2-z_3$	*adb*	z_3
adb	z_3	*bca*	z_1	*abc*	z_1

The mapping $\bar{\alpha}$ from the set of voltages on the three fundamental cycles of K_4 to the voltage group Q is defined by $\phi(C)^{\bar{\alpha}} = \phi(C^{\alpha})$, where C ranges over these three cycles. Similarly, one can define $\bar{\beta}$. Since α and β lift, by Proposition 2.1, $\bar{\alpha}$ and $\bar{\beta}$ can be extended to automorphisms of Q, say α^* and β^*, respectively. Then, $z_1^{\beta^*} = z_2$ and $z_2^{\beta^*} = z_3$ imply that z_1, z_2 and z_3 have the same order. As $|Q| = p^3$, we have five possible cases: $Q = Z_{p^3}, Z_p^3, Z_{p^2}\times Z_p, N(p^2,p)$ or $N(p,p,p)$.

Case I: $Q = Z_{p^3}$. In this case, because z_1, z_2 and z_3 have the same order, $Q = \langle z_1\rangle = \langle z_2\rangle = \langle z_3\rangle$. Thus, one may assume $z_1 = 1$. Let $1^{\beta^*} = k$. By considering the images of z_1, z_2 and z_3 under β^*, we have $z_2 = k, z_3 = k^2$ and $k^3 = 1$ in Z_{p^3}. Let $1^{\alpha^*} = l$. Similarly, by considering the images of z_1, z_2 and z_3 under α^*, we have $l = k^2$ and $lk^2 = 1$. Thus, $k = l$ and so $k = 1$. It follows that $z_1 = z_2 = z_3 = 1$. Since $z_2^{\alpha*} = -z_1 - z_2 - z_3$, we can conclude that $4 = 0 \pmod{p^3}$ that is impossible.

Case II: $Q = Z_p^3$. In proof of [12, Theorem 6.1], this case has been investigated and it has been proved that X is isomorphic to one of graphs EC_{p^3} for a prime $p > 7$. Moreover, X is 2-regular.

Case III: $Q = Z_{p^2}\times Z_p$. Let $Q = Z_{p^2}\times Z_p = \langle x, y\rangle$, where x has order p^2 and y has order p. Since z_1, z_2 and z_3 have the same order and $Z_{p^2}\times Z_p$ can not be generated by elements of order p, each z_i (i = 1,2,3) hase order p^2. By Proposition 2.2, we can assume that $z_1 = x, z_2 = x^{i_1}y^{j_1}$ and $z_3 = x^{i_2}y^{j_2}$ such that $j_1, j_2 \neq 0 \pmod{p}$. By Table 1, we have the following relations:

$$x^{\alpha^*} = x^{i_2}y^{j_2}, (y^{j_1})^{\alpha^*} = x^{-1-i_1-i_2-i_1i_2}y^{-j_1-j_2-i_1j_2}, (y^{j_2})^{\alpha^*} = x^{1-i_1i_2^2}y^{-i_2j_2},$$

$$x^{\beta^*} = x^{i_1}y^{j_1}, (y^{j_1})^{\beta^*} = x^{i_2-i_1^2}y^{j_2-i_1j_1}, (y^{j_2})^{\beta^*} = x^{1-i_1i_2}y^{-i_2j_1}.$$

Since $(y^{j_1})^{\alpha^*}, (y^{j_2})^{\alpha^*}, (y^{j_1})^{\beta^*}$ and $(y^{j_2})^{\beta^*}$ have order p, we have the following equations:

$$(1)-1-i_1-i_2-i_1i_2 = 0, (2) 1-i_2^2 = 0,$$

$$(3) i_2 - i_1^2 = 0, (4) 1 - i_1i_2 = 0.$$

where all equations containing the scalars in Z_p are to be taken modulo p and the symbol mod p is omitted. By Eq. (2), we have $i_2 = 1$ or $i_2 = -1$. Suppose $i_2 = 1$. Then, by Eq. (4), $i_1 = 1$ and so by Eq. (1), 4 = 0, but it is impossible.

Case IV: $Q = N(p^2, p)$. We have $(yx)^i = z^{\frac{1}{2}i(i-1)}y^ix^i$ where $z = [x,y]$. By using this relation, we can get the equations similar to Case III. Thus, the proof of it is omitted.

Case V: $Q = N(p,p,p)$. Let $N(p,p,p) := \langle x,y,z \mid x^p = y^p = z^p = 1, [x,y] = z, [z,x] = [z,y] = 1\rangle$. Since $N(p,p,p) = \langle z_1, z_2, z_3\rangle$, we assume that $z_1 = x, z_2 = y$ and $z_3 = z$ by Proposition 2.2. In this case, one can easily check that β^* can not be an automorphism of Q. Thus, β does not lift, a contradiction. ■

We remark that the graph EC_{p^3} is defined for each prime p. On the other hand, for prime $p \leq 7$, there is one unique cubic symmetric graph of order $4p^3$, so we can identify these graphs with EC_{p^3}. Furthermore, these graphs are 2-regular.

Now, let X be a cubic symmetric graph of order $4p^3$, where p is a prime. By above for prime $p \leq 7$, X is isomorphic to EC_{p^3}. By Lemma 3.1, for prime $p > 7$, it is proved that X is a Q-regular covering of K_4. The normality of Q implies that the fibre-preserving group is arc-transitive and then, by Lemma 3.2, X is isomorphic to EC_{p^3}. So,

Corollary 3.2. *Let p be a prime and X be a cubic symmetric graph of order $4p^3$. Then, X is isomorphic to one of graphs EC_{p^3}. Moreover, X is 2-regular.*

Notice that there is no cubic semisymmetric graph of order $4p^3$, where p is a prime. So, by [4, Theorem 1.1] and Corollary 3.2, Theorem 1.1 is easily proved. Then, we omit extra explanations.

ACKNOWLEDGMENTS

This research work has been supported by Islamic Azad University, South-Tehran Branch.

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Received November 9, 2011