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DENSITY WAVES IN DIPOLAR BOSE-EINSTEIN CONDENSATES

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Density waves in cigar-shaped dipolar Bose-Einstein condensates are analysed by variational means and we show analytically how the dipole-dipole interaction between the atoms generates a roton-maxon excitation spectrum. A simple model is used to derive the effective equations which describe the emergence of the density waves.

Key words: dipolar Bose-Einstein condensates; density waves; variational treatment.

1. INTRODUCTION

The study of waveforms in nonlinear Schrödinger equations has a long-honored history which starts with the Ginzburg-Landau theory of superfluidity and includes research fields as diverse as Bose-Einstein condensates (BECs), nonlinear optics and plasmas. Investigations into Bose-Einstein condensates, in particular, have benefited greatly from the theoretical and numerical tools developed in nonlinear optics, where an equation similar to that used to describe the dynamics of condensates (namely the Gross-Pitaevskii equation (GPE)) was used to describe quasi-monochromatic wave trains propagating in weakly nonlinear dielectrics. These numerical tools (see Refs. [1, 2] for the main results) allow for rapid investigations into the dynamics of BECs without using the tedious quantum-mechanical methods developed to study the behavior of bosonic systems (see Refs. [3–7])

One particular characteristic of both nonlinear optical systems and BECs is that in addition to the wealth of nonlinear phenomena which stem from contact interactions (see Refs. [8–12] for a selection of such results) there is also the possibility to have *nonlocal nonlinear interactions*. In optics these nonlocal nonlinearities appear when the nonlinear mechanism involves transport, long-range forces, or many-body interactions, while in condensates consisting of atoms with dipolar interactions [13] these nonlocal interactions arise from either external electric fields or permanent magnetic moments. Interestingly enough, even though nonlocality involves some form of spatial averaging, nonlocal nonlinearities support waveforms similar to those observed in systems with local nonlinearities (albeit sometimes with fundamentally different characteristics), along with new families of waves which cannot exist in local, isotropic, nonlinear media, such as the "nematicons" observed experimentally in nematic liquid crystals [14].

Experimental investigations into dipolar condensates have been catalyzed by a series of experimental results which starts with the landmarking Bose-Einstein condensation of chromium in 2005 [15] and ends with the very recent condensation of dysprosium [16]. Theoretical investigations into dipolar condensates preceded the experiments by a couple of years and by the time the chromium condensate was achieved there were already noteworthy results such as the variational treatment of the dynamics of dipolar BECs reported in Ref. [17], the thorough investigations of ground state and elementary excitations of dipolar BECs [18] and the roton-maxon spectrum reported in Ref. [19]. Also noteworthy are the exact hydrodynamic equations of dipolar BECs [20] and the first prediction of stable two-dimensional bright solitons in dipolar BECs [21, 22]. In particular, it is shown in Refs. [21] and [22] that while stable bright solitons cannot exist in gases with short-range interactions (without additional confining potentials such as optical lattices), they can exist in dipolar condensates due to the anisotropy of the dipole-dipole interaction. This proof-of-concept was later refined in Ref. [23] by including three-body losses and noise in the scattering length to the extent of giving a detailed recipe for making two-dimensional matter-wave solitons. Similarly, in condensates with short-range interactions dark solitons are unstable against transverse excitations in two and three dimensions (without strong transversal confinement), while in dipolar condensates (loaded into optical lattices) they exist for arbitrarily large transversal sizes [24]. Related investigations into the properties of solitons in systems with nonlocal nonlinear interactions have been carried out in optics for solitons of even and odd parities [25], multipole vector solitons [26], fully three-dimensional spatiotemporal solitons [27], spinning bearing-shaped solitons [28] and crescent vortex solitons [29], to name only a few.

As the building blocks of our understanding have been only recently established, there are numerous open problems. For single component dipolar condensates, for instance, the collisional properties of solitons are subject to a new type of management through the joint tunability of the local and non-local interactions which should allow us to investigate the collisional spectrum from the inelastic to the elastic limit with improved control over the dynamics. Another fruitful research direction is that of studying two-component dipolar condensates where there are no analytical results (and few numerical results) concerning the level of miscibility of the stationary configurations and the taxonomy of symbiotic states is yet to be investigated. For nonpolar condensates the symbiotic states are those in which (even without the presence of a trap) one component effectively traps the other thereby forming a soliton molecule whose existence is very little touched upon in systems with nonlocal nonlinearities. Finally, let us mention that there are only incipient variational treatments of solitons in dipolar gases [30] and that there is a need for fit-to-measure variational models both for the study of stationary configurations and the collisions of solitons.

Ripples on the density profile of condensates have been studied in a couple of distinct contexts, of which we mention the formation of Faraday patterns in pancake-shaped dipolar BECs and the periodic arrangement of density shells, disks and stripes which appear during the non-adiabatic collapse of dipolar condensates. The properties of Faraday patterns, in particular, are quite surprising because unlike those in condensates with short-range interactions (where the size of the patterns decreases monotonously with the frequency of the drive), patterns in dipolar gases present abrupt changes in the size of the patterns [31]. These abrupt changes in the size of the patterns have been shown numerically and there are so far no analytical results regarding the size of the patterns and the selection mechanism that sets their symmetry. Moreover, while in nonpolar BECs the emergence of Faraday waves can be understood qualitatively very well (and analytical results can be derived easily) using a Mathieu-type analysis on the homogeneous solution of a non-polynomial Schrödinger equation, these equations do not generally exist for dipolar condensates. The key feature of non-polynomial Schrödinger equations (see Refs. [32-35] for the main results) is that for cigar-shaped (pancake-shaped) nonpolar BECs they capture in an explicit form the transverse (longitudinal) forcing in an effectively longitudinal (transverse) equation which describes the dynamics of the condensate wave function. These equations are derived using a variational recipe which does not yield analytic results for dipolar condensates, so one is left with the hybrid equation used in Ref. [31] which contains both differential and integral terms. To the best of our knowledge, the only similarly looking equation is that introduced in Ref. [36], where the dipolar term is added to the usual terms which describe a nonpolar, cigar-shaped condensate with one component, but no derivation is given.

In this paper we introduce a fully variational model which is able to describe the emergence of density waves in dipolar cigar-shaped BECs with longitudinal homogeneity. The variational treatment resembles that in Refs. [9,10], with the difference that the Lagrangian of the condensate contains an extra integral term which accounts for the dipolar interaction. The main assumption behind our treatment is that for sufficiently strong transverse confinements we can neglect the transverse dynamics of the condensate, such that the ground state of the radial component of the trap can be used as the radial wave function. Investigations into the coupling of the transverse modes with the longitudinal density waves will be reported elsewhere.

2. VARIATIONAL TREATMENT

The starting point of our variational investigation is the Gross-Pitaevskii Lagrangian density of a dipolar cigar-shaped condensate with dipole-dipole interactions, namely

$$l=\frac{1}{2}\left(\psi(\vec{r},t)\frac{\partial\psi^*(\vec{r},t)}{\partial t}-\psi^*(\vec{r},t)\frac{\partial\psi(\vec{r},t)}{\partial t}\right)+\frac{1}{2}\left|\nabla\psi(\vec{r},t)\right|^2+\frac{g(t)}{2}\left|\psi(\vec{r},t)\right|^4+V(\vec{r},t)\left|\psi(\vec{r},t)\right|^2+$$
$$+g_d\left|\psi(\vec{r},t)\right|^2\int\mathrm{d}r'U(\vec{r}-\vec{r}')\left|\psi(\vec{r}',t)\right|^2, \tag{1}$$

where the dipole-dipole potential which appears in the final term is given by

$$U(\vec{r}-\vec{r}')=\frac{1-\dfrac{3\left(z-z'\right)^2}{\left(x-x'\right)^2+\left(y-y'\right)^2+\left(z-z'\right)^2}}{\left(\left(x-x'\right)^2+\left(y-y'\right)^2+\left(z-z'\right)^2\right)^{3/2}}. \tag{2}$$

In the previous equation we have considered implictly that all dipoles are polarized along the long axis of the condensate. Using the trial wave function

$$\psi(\vec{r},t)=\frac{1}{\pi\Omega}\sqrt{\frac{k}{2+u(t)^2+v(t)^2}}\exp\left(-\frac{x^2+y^2}{2\Omega^2}\right)\times(1+(u(t)+\mathrm{i}v(t))\cos kz), \tag{3}$$

we can integrate quasi-analitically the Lagrangian density considering that x and y extend over the entire real axis, while the integral over z is taken over one period of the potential,

$$L=\int\mathrm{d}\vec{r}l.$$

Following some straightforward computations, we find that the Lagrangian contains an interaction term of the form

$$L_1=\frac{kNg(t)\left(8+3u(t)^4+8v(t)^2+3v(t)^4+6u(t)^2\left(4+v(t)^2\right)\right)}{16\pi^2\left(2+u(t)^2+v(t)^2\right)^2\Omega^2}, \tag{4}$$

a potential term of the form

$$L_2=\frac{m\Omega^4}{2}, \tag{5}$$

a kinetic term of the form

$$L_3=\frac{\hbar^2\left(2+\left(u(t)^2+v(t)^2\right)\left(1+k^2\Omega^2\right)\right)}{2m\left(2+u(t)^2+v(t)^2\right)\Omega^2}, \tag{6}$$

while the time-dependent part is given by

$$L_4=\frac{\hbar\left(u(t)\dot{v}(t)-v(t)\dot{u}(t)\right)}{\left(2+u(t)^2+v(t)^2\right)}. \tag{7}$$

The term which takes into account the dipole-dipole interaction requires a separate discussion and we present below each step of the computation. First, let us say that we take n to be the density profile of the condensate which (in accordance with the variational ansatz in equation (3)) can be decomposed as

$$n(\vec{r})=n_x(x)n_y(y)n_z(z). \tag{8}$$

The dipolar term can then be written as

$$L_5=g_d\int\mathrm{d}\vec{r}n(r)\int\mathrm{d}\vec{r}'\frac{1-\dfrac{3\left(z-z'\right)^2}{\left(x-x'\right)^2+\left(y-y'\right)^2+\left(z-z'\right)^2}}{\left(\left(x-x'\right)^2+\left(y-y'\right)^2+\left(z-z'\right)^2\right)^{3/2}}n_x(x')n_y(y')n_z(z'), \tag{9}$$

$$L_5 = g_d \int \mathrm{d}\vec{r} n(r) \int \mathrm{d}z' n_z(z') F_{x,y}^{-1}\{F_{x,y}\left\{\frac{1-\frac{3(z-z')^2}{x^2+y^2+(z-z')^2}}{\left(x^2+y^2+(z-z')^2\right)^{3/2}}\right\} F_{x,y}\{n_x(x')n_y(y')\}\}, \tag{10}$$

$$\begin{aligned} L_5 = g_d \int \mathrm{d}z n_z(z) \int \mathrm{d}z' n_z(z') \int \mathrm{d}x\mathrm{d}y n_x(x) n_y(y) \times \\ \times F_{x,y}^{-1}\{F_{x,y}\left\{\frac{1-\frac{3(z-z')^2}{x^2+y^2+(z-z')^2}}{\left(x^2+y^2+(z-z')^2\right)^{3/2}}\right\} F_{x,y}\{n_x(x')n_y(y')\}\}, \end{aligned} \tag{11}$$

$$L_5 = g_d \int \mathrm{d}z n_z(z) \int \mathrm{d}z' n_z(z') \int \mathrm{d}k_x \mathrm{d}k_y F_{x,y}\{n_x(x')n_y(y')\}^2 \times F_{x,y}\left\{\frac{1-\frac{3(z-z')^2}{x^2+y^2+(z-z')^2}}{\left(x^2+y^2+(z-z')^2\right)^{3/2}}\right\}. \tag{12}$$

To go from equation (11) to equation (12) we have used Parseval's theorem which (in our case) states that the inner product of two real square-integrable functions is equal to the inner product of their Fourier transforms. The square Fourier transform which appears in equation (12) is, however, somewhat problematic, as it does not allow us to use Parseval's theorem directly, but we can recast the Fourier transform as

$$F_{x,y}\{n_x(x)n_y(y)\}^2 = F_{x,y}\{\tilde{n}_x(x)\tilde{n}_y(y)\} \tag{13}$$

which yields

$$L_5 = g_d \int \mathrm{d}z n_z(z) \int \mathrm{d}z' n_z(z') \int \mathrm{d}x\mathrm{d}y \tilde{n}_x(x) \tilde{n}_y(y) \frac{1-\frac{3(z-z')^2}{x^2+y^2+(z-z')^2}}{\left(x^2+y^2+(z-z')^2\right)^{3/2}}. \tag{14}$$

As the radial densit profile of the condensate is of Gaussian form, equation (13) can be written as

$$F_{x,y}\left\{\exp\left(-\frac{x^2+y^2}{\Omega^2}\right)\right\}^2 = F_{x,y}\left\{\left(\frac{\Omega}{2}\right)^2 \exp\left(-\frac{x^2+y^2}{2\Omega^2}\right)\right\} \tag{15}$$

and we have that

$$L_5 = g_d \int \mathrm{d}z n_z(z) \int \mathrm{d}z' n_z(z') K(z-z'), \tag{16}$$

where

$$K(z-z') = \frac{\pi\left(2\sqrt{(z-z')^2}\Omega - \exp\left(\frac{(z-z')^2}{2\Omega^2}\right)\mathrm{Erfc}\left(\frac{\sqrt{(z-z')^2}}{\sqrt{2}\Omega}\right)\right)}{4\Omega} \tag{17}$$

$$K\left(z-z'\right)\approx-\frac{\pi}{4}\sqrt{2\pi}\Omega\mathrm{Erfc}\left(\frac{\sqrt{(z-z')^{2}}}{\sqrt{2}\Omega}\right). \tag{18}$$

Please note that the key ingredient in our computations was the Gaussian radial ansatz which allowed us to find easily two density functions such that equation (13) is fullfiled.

With this last approximation for the kernel in equation (16) one can easily compute the remaining two integrals using most computer algebra systems, but the result is long enough that we do not write here explicitly. The Euler-Lagrane equations for $u(t)$ and $v(t)$, however, can be cast in a very simple form

$$\dot{v}(t)=u(t)\left(\frac{g_{d}\exp\left(-\frac{1}{2}k^{2}\Omega^{2}\right)\mathrm{Erfi}\left(\frac{k\Omega}{\sqrt{2}}\right)}{\sqrt{2}\pi^{3/2}\Omega^{3}\hbar}-\frac{k^{2}\hbar}{2m}-\frac{2\pi\rho g(t)}{\pi\Omega^{2}\hbar}\right) \tag{19}$$

and

$$\dot{u}(t)=\frac{k^{2}\hbar v(t)}{2m}, \tag{20}$$

which allows us to write them in the form of a Mathieu equation for $u(t)$. In equation (19) ρ represents the linear density of the condensate and is obtained from the condition that there are N bosons over one period of the density wave.

3. THE MAXON-ROTON SPECTRUM

From equations (19) and (20) we have that the energy of the density waves is given explicitly by

$$\varepsilon(k)=\frac{k^{2}\hbar}{2m}\left(\frac{k^{2}\hbar}{2m}-\frac{g_{d}\exp\left(-\frac{1}{2}k^{2}\Omega^{2}\right)\mathrm{Erfi}\left(\frac{k\Omega}{\sqrt{2}}\right)}{\sqrt{2}\pi^{3/2}\Omega^{3}\hbar}+\frac{2\pi\rho g(t)}{\pi\Omega^{2}\hbar}\right). \tag{21}$$

This is the main result of our paper and to the best of our knowledge it is the first time the dispersion relation of the density waves is obtained analytically from an energy minimizing treatment. The most important feature of the above dispersion relation is that it exhibits the maxon-roton (maximum-minimum) structure in the excitation spectrum just like ^{4}He. To illustrate the maxon-roton structure we show in Fig. 1 a typical energy spectrum.

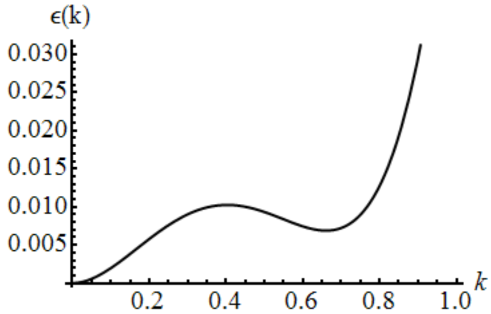

Fig. 1 – Typical energy spectrum of a Bose-Einstein condensate with dipole-dipole interactions. Here we have used $m = 1$, $\Omega = 1$, $\rho = 1.3$, $g = 1$, $g_d = 1$, and $\hbar = 1$.

4. CONCLUSIONS

Density waves in cigar-shaped dipolar Bose-Einstein condensates are analysed by variational means and we show analytically how the dipole-dipole interaction between atoms generates a roton-maxon excitation spectrum similar to that observed in ^{4}He. We show in detail how to treat analytically the term which accounts for the dipole-dipole interaction and derive an explicit analytic formula for the energy spectrum.

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