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TWO-PHOTON QUANTUM DYNAMICS IN A NONLINEAR MICROMASER

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Nonlinear two-photon interactions of an atomic system mediated by a thermal quantized electromagnetic field mode are investigated. We analyze the properties of the electromagnetic field inside the resonator mode. In particular, for a random particle injection rate we obtain an exact analytical solution for the steady state of the cavity field. The mean number of photons into the resonator mode is quite sensitive to the temperature. For larger atomic injection rates we find a suppression of the mean photon number into the quantized mode.

Key words: two-photon dynamics, resonator modes, nonlinear micromaser.

1. INTRODUCTION

The interaction of a single fixed particle with a quantized field is perhaps the most fundamental problem of quantum electronics. An enormous amount of theoretical and experimental work has been carried out using such systems in the nonlinear or linear interaction regimes. Interesting results were also obtained when dilute atomic fluxes cross the quantized fields [1, 2, 3]. The quantum nature of the electromagnetic field was shown to be sensitive on the statistics of the atomic flux. The deflection pattern of atomic fluxes by quantized fields also depends on the statistics of the electromagnetic field. Thus, cavity quantum electrodynamics in the strong coupling regime has become an active field of research ranging from experimental tests of fundamental problems in quantum mechanics to the implementation of basic processes for quantum information [4]. In principle, the well-known Jaynes-Cummings model (JCM), where a single particle interacts with a single mode of the quantized electromagnetic field, or its generalizations, are suitable to describe those effects. There is a lot of research on the micromaser theory that focuses on the interplay between a flux of two-level atoms and a cavity field. Many non-classical features of the cavity field such as Fock states and trapping states [5], squeezed states [6] or Einstein-Podolsky-Rosen entangled radiation [7] have been observed in the one-photon two-level micromaser.

As we know, two-photon processes in atomic systems are very important in quantum optics and laser physics due to the high degree of correlation between the emitted photons. In particular, spatial entangled deflected particles can be created via twin-photon light beams [8]. Very recently, a scheme was proposed to obtain correlated photon pairs in different frequency ranges [9]. In this context, two-photon two-level systems are naturally another important subject of research. Accordingly, the two-photon JCM was developed [10, 11, 12, 13]. Since the realization of the first two-photon quantum oscillator in the laboratory by employing Rydberg atoms in a high-Q superconducting microwave cavity [12], the two-photon JCM has attracted a great deal of attention [13]. It is important to emphasize that the mere fact that the atomic system emits or absorbs photon pairs leads to field statistics which is very different from those of the ordinary maser fields. Many applications of two-photon maser have been suggested and developed. One example is the generation of sub-Poissonian light which can provide a signal with low quantum noise [14]. Another example is the squeezed-state generation. In the limit of a large number of photons, two-photon light is transformed to squeezed light, which is a nonclassical phenomenon [15].

The properties of various systems were shown recently to be similar to those of a micromaser, i.e. an atomic stream passing through a cavity quantized mode so that a steady state is created [16, 17]. For instance, the Josephson cascade micromaser or a micromaser for the molecular field obtained via photoassociation of fermionic atoms into bosonic molecules inside an optical lattice were investigated in [18] and [19], respectively. The dynamics of a resonator coupled to a superconducting single-electron transistor tuned to the Josephson quasiparticle resonance were shown to be similar in many ways to that found in a micromaser [20]. A tunable on-chip micromaser using a superconducting quantum circuit that can be used to generate single photons was analyzed in [21]. Additionally, the simultaneously cooling of an artificial atom and its neighboring quantum system via analogous processes were shown to occur in [22]. Therefore any novel systems describing the interaction of a particle with a quantized electromagnetic field mode are of great interest from these points of view.

Thus here our purpose is to investigate the nonlinear two-photon interaction of a flux of particles with a thermalized quantized cavity mode. We found that the quantum features of the system are quite sensitive to the temperatures. Moreover, higher order photon correlation functions characterize the nonlinear model. We use the properties of su(1,1) algebra to estimate theses correlations.

The paper is organized as follows. In Section 2, we describe the model as well as the analytical formalism. Section 3 characterizes the obtained results while the conclusions are summarized in the last section, i.e., in Section 4.

2. APPROACH

In the following, we present shortly the theory of the nonlinear two-photon micromaser. In a typical cavity QED experiment, an atom in the selected state enters the cavity at such a rate that at most one atom is allowed to interact with the field for a fixed time duration. For the operation of micromasers, the atomic injection dynamics have to be considered. The resonant nonlinear interaction of an atomic flux crossing the quantized cavity electromagnetic field is characterized by the relation: $\rho(t_k+\tau)=M(\tau)\rho(t_k)$, where t_k is the injection time of the kth atom, and τ gives the time-duration of this interaction. The gain operator $M(\tau)$ is defined as:

$$M(\tau)\rho = \mathrm{Tr}_a[U(\tau)\rho\otimes|1\rangle\langle 1|U(\tau)^{-1}], \tag{1}$$

with $U(\tau)=\exp(\mathrm{i}H\tau/\hbar)$ being the time evolution operator, and $\mathrm{Tr}_a[\cdots]$ denotes the trace over the atomic variables. With the help of Eq. (1), the electromagnetic field distribution after interacting with the kth atom can be obtained.

As mentioned above, two-photon effects are considered in this paper. As a matter of fact, a two-photon process concerns transitions from one level to another via an intermediate level involving a single photon in each transition. When the intermediate level is far from one-photon resonance, this three-level system can be reduced to an effective two-level system coupled by a single mode field via a two-photon process. The validity of this model has been discussed in detail in [23]. Such processes in a high-Q cavity, the two-photon micromaser, have been experimentally demonstrated in [12]. The atomic system we are interested in is shown in Fig. 1. A resonant transition from the ground state $|0\rangle$ to the excited state $|1\rangle$ is driven by a microwave field via a two-photon process. The effective Hamiltonian describing the nonlinear matter-field resonant interaction shown in Fig. 1a, while involving the transition $|0\rangle\leftrightarrow|1\rangle$, can be written in the rotating wave approximation as follows:

$$H=\hbar\lambda(a_+^2S^- + S^+a^2). \tag{2}$$

a_+ and a are here the electromagnetic field creation and annihilation operators, while S^+ and S^- the corresponding ones for the atomic subsystem, respectively. λ gives the nonlinear matter-field coupling strength. This is the Hamiltonian which is broadly used to describe the degenerate two-photon processes and has received extensive attention during the last decades.

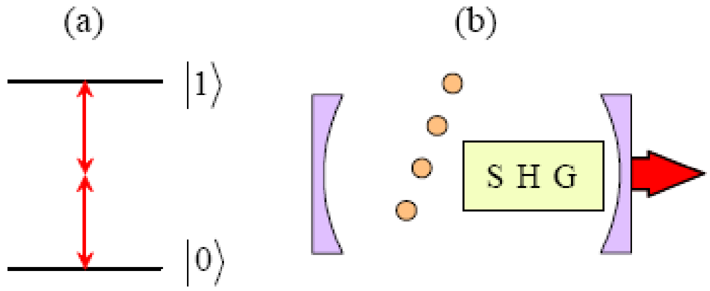

Fig. 1 – a) The atomic level configuration used in this study; b) Micromaser scheme proposal. The dipole-forbidden atoms are sent into the cavity and interact with cavity mode via a two-photon process. The second harmonic generation (SHG) crystal converts two cavity photons into a single one.

Generally, we assume the intra-cavity system can exchange information with thermal environment due to the cavity dissipation. Then, the entire quantum dynamics of the density matrix ρ, corresponding to degrees of freedom of electromagnetic field, is given as:

$$\dot{\rho} = r[M(\tau) - 1]\rho + L\rho. \tag{3}$$

r is here the atomic pumping rate, i.e. the number of particles entering the interaction region per unit time. Thus, the time evolution of the electromagnetic field in the micromaser is determined by Eq. (3). The superoperator L describes the interaction of the cavity mode containing a two-photon absorber with the thermal environment via cavity losses. At finite temperatures, this superoperator L for the cavity damping is given as follows:

$$L\rho = -\frac{\chi}{2}(1+\bar{n})\{[a_+^2, a^2\rho] + [\rho a_+^2, a^2]\} - \frac{\chi}{2}\bar{n}\{[a^2, a_+^2\rho] + [\rho a^2, a_+^2]\}, \tag{4}$$

where the nonlinearity of the process is taken into account. χ is here the damping rate of the quantized electromagnetic field mode. The mean thermal photon number at the dipole-forbidden atomic frequency is given by $\bar{n}$. The nonlinear damping described by Eq. (4) can be realized in practice by utilizing a second harmonic generation (SHG) crystal [24, 25, 26, 27]). The SHG couples with the cavity mode via a two-photon process which is shown in Fig. 1b and is characterized by the Hamiltonian, $H_b = \hbar\kappa_b(b_+ a^2 + b a_+^2)$. We assume further that the mode $\{b, b_+\}$ is damped faster than the mode $\{a, a_+\}$ and one can adiabatically eliminates it (the details have been shown in Appendix) to arrive at Eq. (4).

For further convenience we introduce the field operators

$$I^+ = a_+^2/2,\; I^- = a^2/2,\; I_z = (a_+ a + 1/2)/2,$$

that obey the commutation relations for su(1,1) algebra, i.e. $[I^+, I^-] = -2I_z$ and $[I_z, I^\pm] = \pm I^\pm$. These operators act on the corresponding bases states of the su(1,1) algebra in the following way

$$\begin{aligned} I^+|j,m\rangle &= \sqrt{(m+1)(m+2j)}\,|j,m+1\rangle, \\ I^-|j,m\rangle &= \sqrt{m(m+2j-1)}\,|j,m-1\rangle, \\ I_z|j,m\rangle &= (m+j)|j,m\rangle. \end{aligned} \tag{5}$$

Here $m \in \{0,1,2,\cdots,\infty\}$, while for a single mode field, as considered in our approach, the allowed value of the Bargmann index (i.e., j) is 1/4 (3/4) for an even (odd) photon number. The correspondence between the number state of the single mode field $|n\rangle$ and the su(1,1) basis states $|j,m\rangle$ is $|n\rangle \leftrightarrow |j,m\rangle$ for $n = 2(m+j) - 1/2$ [28].

In order to obtain the field properties of the nonlinear two-photon quantum micromaser, the diagonal elements of the master equation (3), i.e. $P_m = \langle j,m|\rho|j,m\rangle$, are obtained using Eqs. (1-5) and represented as follows

$$\dot{P}_m = -r\sigma_{m+1}P_m + r\sigma_m P_{m-1} - 4\chi(1+\overline{n})\{m(m+2j-1)P_m - (m+1)(m+2j)P_{m+1}\} - \\ - 4\chi\overline{n}\{(m+1)(m+2j)P_m - m(m+2j-1)P_{m-1}\}, \tag{6}$$

where $\sigma_m = \sin^2[2\lambda\tau\sqrt{m(m+2j-1)}]$. The expression for σ_m shows that the interaction time τ plays an important role on the photon-number dynamics of the nonlinear two-photon quantum micromaser.

3. RESULTS AND DISCUSSIONS

The resulting field after the interaction with atoms possesses nonclassical features and statistical properties significantly different from the original thermal state. In the following, we shall calculate the quantities which are relevant to characterize the generated state. In this section, we derive the steady-state density matrix of the electromagnetic field and compare the photon statistics, the mean photon numbers, the second-order as well as the fourth-order photon correlation functions in different situations of interest.

With the help of Eq. (6), the steady-state solution for diagonal elements of the density matrix can be obtained from the following recursion relation:

$$P_{m+1} = [\frac{\overline{n}}{1+\overline{n}} + \frac{\overline{r}\sigma_{m+1}}{(1+\overline{n})(m+1)(m+2j)}]P_m. \tag{7}$$

Eq. (7) contains all information about the statistical properties of the steady-state field reached by the nonlinear two-photon micromaser and may lead to an explicit form for P_m. One then obtains the following steady-state solution of Eq. (6):

$$P_m = P_0 \prod_{k=1}^{m} \frac{\overline{n} + \overline{r}\sigma_k [k(k-1+2j)]^{-1}}{1+\overline{n}}, \tag{8}$$

where the probability P_0 has to be determined from the normalization relation $\sum_{m=0}^{\infty} P_m = 1$, and $\overline{r} = r/4\chi$. Eq. (7) and Eq. (8) shows that the system can place itself in more than one steady state depending on the interaction time, which is the usual situation in micromasers. As an illustration, Fig. 2a shows the steady state photon-number distributions in different cases. Inspecting this figure, we can see that the photon distributions are changed by varying injection rate $\overline{r}$ and mode temperature $\overline{n}$. With the photon-number probabilities P_m at hand, one can immediately calculate the mean photon number and the corresponding correlation effects.

The expectation values of the operators needed for evaluating the properties of the nonlinear micromaser are obtained from Eq. (5) and Eq. (8). The mean photon number in the cavity mode is given by the following expression

$$\langle n \rangle \equiv \langle a_+ a \rangle = 2\sum_{m=0}^{\infty}(m+j)P_m - 1/2. \tag{9}$$

The generation of nonclassical light is of interest, primarily, from the fundamental point of view. The applications of nonclassical states of electromagnetic fields are discussed in connection with the problem of quantum information and the concept of quantum lithography [29]. Quantum states of light may be classified according to their optical coherence properties. For example, it is considered to be a quantum regime if the one-mode second-order coherence function is less than one. Therefore, the second-order correlation function of the nonlinear two-photon micromaser can be obtained from the following relation:

$$\langle I^+ I^- \rangle = \sum_{m=0}^{\infty} m(m+2j-1)P_m. \tag{10}$$

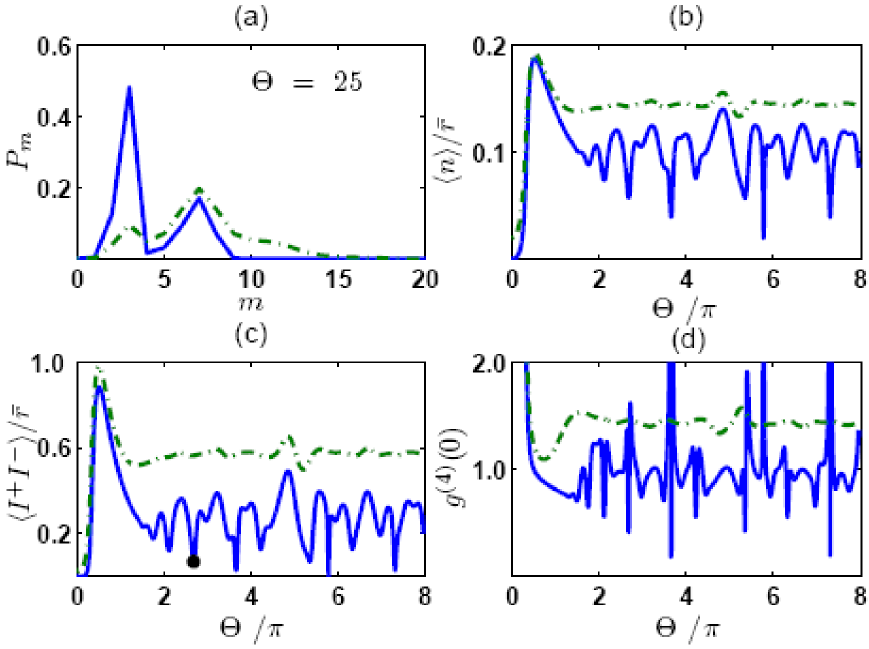

Fig. 2 – a) The steady-state photon distribution P_m for $\Theta = 25$; b), c) and d) are the normalized mean photon number $\langle n\rangle/\bar{r}$, the normalized second-order correlation function $\langle I^+I^-\rangle/\bar{r}$, and the normalized fourth-order correlation $g^{(4)}(0)$ as a function of generalized coupling $\Theta = 2\lambda\tau\sqrt{\bar{r}}$, respectively. Here $\bar{r} \equiv r/4\chi = 100$, $j = 1/4$ while $\bar{n} = 0$ and 1 for the solid and the dot-dashed curves, respectively. The symbol ‘ • ’ in (c) characterizes a particular trapping state.

A parameter providing a good qualitative comparison of different one-atom maser regimes is called generalized coupling, given by $\Theta = 2\lambda\tau\sqrt{\bar{r}}$. It is useful for locating features of the maser operating under a wide variety of conditions. For example, through the mean photon number $\langle n\rangle$, we can get the effect of generalized coupling Θ on the normalized mean photon number $\langle n\rangle/\bar{r}$ into the cavity mode in different cases of interest, which is shown in Fig. 2b. Though the injection rate is relatively large, i.e. $\bar{r} = 100$, we observe that $\langle \bar{n}\rangle/\bar{r} \ll 1$. Thus, for larger atomic injection rates, we found the suppression of the mean photon number into the quantized mode. The reason is that due to the two-photon nature of the interaction the photons are emitted in pairs and, thus, the role of the mean number of photon pairs is played by $\langle I^+I^-\rangle$. Therefore, in Fig. 2c, we depict the behavior of the generated mean number of photon pairs $\langle I^+I^-\rangle$ as function of the generalized coupling Θ. Figure 2 (a,b,c) shows that the mean number of photons into the resonator mode is quite sensitive to the temperature.

In order to further understand the characteristics of the two-photon quantum micromaser, it is necessary to know the fourth-order photon correlation (that is the correlations among photon pairs) for the electromagnetic fields in the cavity. The fourth-order correlation function is represented as follows:

$$\langle I^{+2}I^{-2}\rangle = \sum_{m=0}^{\infty} m(m-1)(m+2j-1)(m+2j-2)P_m,$$
$$g^{(4)}(0) = \frac{\langle I^{+2}I^{-2}\rangle}{\langle I^+I^-\rangle^2}. \quad (11)$$

One can see in Fig. 2d that the normalized fourth-order correlation function $g^{(4)}(0)$ is varying with generalized coupling Θ in different cases. We can also find that the fourth-order correlation function of the cavity field is quite sensitive to the temperatures.

To interpret the dip structure in Fig. 2c we shall now briefly analyze the properties of the trapping states. It is well-known that trapping states can separate the Fock space into disconnected blocks, which plays an essential role in state evolution [4]. The trapping states can be formed in our model as well. With the help of the time evolution operator $U(\tau)$ defined at the beginning, the time evolution of an arbitrary state of the atom-field coupling, i.e. $\sum_m S_m \,|\, j,m\rangle(\alpha\,|\,1\rangle + \beta\,|\,0\rangle)$, is given by

$$\sum_m S_m U(\tau)\,|\, j,m\rangle(\alpha\,|\,1\rangle + \beta\,|\,0\rangle) = |1\rangle \sum_m S_m \{\alpha\cos[2\tau\lambda\sqrt{(m+1)(m+2j)}]\,|\, j,m\rangle -$$
$$-\mathrm{i}\beta\sin[2\tau\lambda\sqrt{m(m+2j-1)}]\,|\, j,m-1\rangle\} + |0\rangle\sum_m S_m\{-\mathrm{i}\alpha\sin[2\tau\lambda\sqrt{(m+1)(m+2j)}]\,|\, j,m+1\rangle +$$
$$+\beta\cos[2\tau\lambda\sqrt{m(m+2j-1)}]\,|\, j,m\rangle\}.$$

Here $\alpha\,|\,1\rangle + \beta\,|\,0\rangle$ presents an arbitrary superposition state; S_m expresses the initial distribution of the cavity mode. Now, one can immediately find the upward and the downward trapping conditions [30] for some particular $m = M$, i.e., $2\tau\lambda\sqrt{M(M+2j-1)} = q\pi$ and $2\tau\lambda\sqrt{(M+1)(M+2j)} = p\pi$ (p and q are integer numbers). When these conditions are satisfied, the cavity photon number is unchanged after atom-field interaction and hence the photon number is "trapped" [5]. The upward and downward trapping condition can be expressed in terms of the parameters Θ and $\bar{r}$:

$$\Theta\sqrt{M(M+2j-1)} = \sqrt{\bar{r}}\,q\pi \quad ; \quad \Theta\sqrt{(M+1)(M+2j)} = \sqrt{\bar{r}}\,p\pi. \tag{12}$$

As an example, we analyze the dip at $\Theta = 2.672$ which is denoted by the symbol '•' in Fig. 2(c). This dip corresponds to the trapping state $|\, j,m\rangle$ with $q = 1$ and $M = 4$ (or $p = 1$ and $M = 3$). Similarly, we can find other dips corresponding to different trapping states. As the mean number of thermal photons $\bar{n}$ increases, the thermal fluctuations allow the system to pass through partial trapping states (see the dot-dashed curves in Fig. 2c, which is similar to the case of single-photon JCM [16]. Therefore, we have studied the impact of each parameter on the properties of generated cavity field.

4. SUMMARY

In this paper we investigate the nonlinear two-photon interactions of an atomic system mediated by a thermal quantized electromagnetic field mode. The effect of the mode temperatures on the properties of the electromagnetic field inside the resonator mode is given here. We obtain an exact analytical solution for the steady state of the cavity field. Our results show that the two-photon micromaser properties is quite sensitive to the temperature. We estimate the second-order and fourth-order correlation functions of the generated electromagnetic field to further characterize the nonlinear two-photon quantum micromaser.

APPENDIX: DERRIVATION OF EQ. (4)

Here we will show a brief derivation of Eq. (4). Consider a system with the following Hamiltonian

$$\tilde{H} = \hbar\omega_a a_+ a + \hbar\omega_b b_+ b + \hbar\kappa_b[b_+ a^2 + b a_+^2]. \tag{13}$$

Here a_+ and a are the creation and annihilation operators of the fundamental mode corresponding to the cavity mode while the second harmonic field operators b_+ and b denote, respectively, the creation and annihilation operators of bath mode. The bath couples to the cavity mode via a SHG process (Fig. 1b). κ_b characterizes the coupling constant between the bath and the cavity mode. By introducing the unitary operator $U_0 = \exp[\mathrm{i}(\omega_a a_+ a + \omega_b b_+ b)t]$, the Hamiltonian in the interaction picture is derived as ($\tilde{H}_i = U_0 \tilde{H} U_0^\dagger$)

$$\tilde{H}_i = \hbar\kappa_b[\exp(\mathrm{i}\Delta t) b_+ a^2 + \exp(-\mathrm{i}\Delta t) b a_+^2].$$

Here $\Delta = \omega_b - 2\omega_a$. In the Born-Markov approximation, the density operator ρ of the system is determined by the following equation [31]

$$\dot{\rho}(t) = -\frac{1}{\hbar^2}\int_0^t \mathrm{d}\tau \mathrm{Tr}_{\mathrm{R}}[\tilde{H}_i(t),[\tilde{H}_i(t-\tau), R_0\rho(t)]], \tag{14}$$

where R_0 is the initial reservoir density operator. After a standard calculation, the master equation is then obtained as:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\rho = L\rho = (a_+^2\rho a^2 - \rho a^2 a_+^2)W^{(1)} + (a^2\rho a_+^2 - \rho a_+^2 a^2)W^{(2)} + (a_+^2\rho a_+^2 - \rho a_+^2 a_+^2)W^{(3)} + \\ + (a^2\rho a^2 - \rho a^2 a^2)W^{(4)} + hc,\end{aligned} \tag{15}$$

where $W^{(i)}$ are related with the two-time averages of the reservoir operators by

$$\begin{aligned} W^{(1)} &= \kappa_b^2\int_0^t \mathrm{d}\tau\, \mathrm{Tr}_{\mathrm{R}}[\exp(\mathrm{i}\Delta\tau)R_0 b_+(t-\tau)b(t)], \\ W^{(2)} &= \kappa_b^2\int_0^t \mathrm{d}\tau\, \mathrm{Tr}_{\mathrm{R}}[\exp(-\mathrm{i}\Delta\tau)R_0 b(t-\tau)b_+(t)], \\ W^{(3)} &= \kappa_b^2\int_0^t \mathrm{d}\tau\, \mathrm{Tr}_{\mathrm{R}}[\exp(-\mathrm{i}\Delta\tau)R_0 b(t-\tau)b(t)], \\ W^{(4)} &= \kappa_b^2\int_0^t \mathrm{d}\tau\, \mathrm{Tr}_{\mathrm{R}}[\exp(\mathrm{i}\Delta\tau)R_0 b_+(t-\tau)b_+(t)]. \end{aligned} \tag{16}$$

Assuming that the electromagnetic field reservoir satisfies the thermal distribution, one obtains that $\mathrm{Tr}_{\mathrm{R}}[R_0 b_+(t-\tau)b(t)] = e^{-\chi_b\tau}\langle b_+ b\rangle_R = e^{-\chi_b\tau}\bar{n}$, $\mathrm{Tr}_{\mathrm{R}}[R_0 b(t-\tau)b_+(t)] = e^{-\chi_b\tau}\langle bb_+\rangle_R = e^{-\chi_b\tau}(\bar{n}+1)$ and $\mathrm{Tr}_{\mathrm{R}}[R_0 b(t-\tau)b(t)] = \mathrm{Tr}_{\mathrm{R}}[R_0 b_+(t-\tau)b_+(t)] = 0$. χ_b and $\bar{n}$ are the damping rate and the mean thermal photon number for the mode $\{b, b_+\}$. Therefore, the formulas for the coefficients $W^{(i)}$ can be written finally as:

$$\begin{aligned} W^{(1)} &= \frac{\chi}{2}\bar{n}, \\ W^{(2)} &= \frac{\chi}{2}(\bar{n}+1), \\ W^{(3)} &= W^{(4)} = 0, \end{aligned} \tag{17}$$

where $\chi = \dfrac{2\kappa_b^2}{\chi_b}$ for $\Delta = 0$. One then arrive at Eq. (4).

REFERENCES

1. M. O. SCULLY, M. S. ZUBAIRY, *Quantum Optics*, Cambridge University Press, 1997.
2. M. ORSZAG, *Quantum Optics. Including Ions, Quantum Trajectories, and Decoherences*, Springer Verlag Press, 1999.
3. W. P. SCHLEICH, *Quantum Optics in Phase Space*, Wiley-VCH Press, 2001.
4. H. WALTHER, B. T. H. VARCOE, B.-G. ENGLERT, T. BECKER, *Cavity quantum electrodynamics*, Rep. Prog. Phys., **69**, pp. 1325–1382, 2006.
5. M. WEIDINGER, B. T. H. VARCOE, R. HEERLEIN, H. WALTHER, *Trapping States in the Micromaser*, Phys. Rev. Lett., **82**, pp. 3795–3798, 1999.
6. S. QAMAR, M. S. ZUBAIRY, *Effect of finite Q on squeezed states in a micromaser*, Phys. Rev. A, **48**, pp. 1559–1563, 1993.
7. S. PIELAWA, G. MORIGI, D. VITALI, L. DAVIDOVICH, *Generation of Einstein-Podolsky-Rosen-Entangled Radiation through an Atomic Reservoir*, Phys. Rev. Lett., **98**, 240401, 2007.
8. G. Yu. KRYUCHKYAN, M. MACOVEI, C. H. KEITEL, *Spatially entangled atomic deflections in twin-photon light beams*, Phys. Rev. A, **77**, 035603, 2008.
9. F. OSTER, C. H. KEITEL, M. MACOVEI, *Generation of correlated photon pairs in different frequency ranges*, Phys. Rev. A, **85**, 063814, 2012.
10. C. V. SUKUMAR, B. BUCK, *Multi-phonon generalisation of the Jaynes-Cummings model*, Phys. Lett. A, **83**, pp. 211–213, 1981.
11. L. DAVIDOVICH, J. M. RAIMOND, M. BRUNE, S. HAROCHE, *Quantum theory of a two-photon micromaser*, Phys. Rev. A, **36**, pp. 3771–3787, 1987.
12. M. BRUNE, J. M. RAIMOND, P. GOY, L. DAVIDOVICH, and S. HAROCHE, *Realization of a two-photon maser oscillator*, Phys. Rev. Lett. **59**, pp. 1899–1902, 1987.

13. M. MAJEED, M. S. ZUBAIRY, *Effect of phase fluctuations on noise-free amplification and phase squeezing in a two-photon laser*, Phys. Rev. A, **44**, pp. 4688–4692, 1999.
14. L. DAVIDOVICH, *Sub-Poissonian processes in quantum optics*, Rev. Mod. Phys., **68**, pp. 127–173, 1996.
15. H. P. YUEN, *Two-photon coherent states of the radiation field*, Phys. Rev. A, **13**, pp. 2226–2243, 1976.
16. P. FILIPOWICZ, J. JAVANAINEN, P. MEYSTRE, *Theory of a microscopic maser*, Phys. Rev. A, **34**, pp. 3077–3087, 1986; P. MEYSTRE, G. REMPE, H. WALTHER, *Very-low-temperature behavior of a micromaser*, Opt. Lett., **13**, pp. 1078–1080, 1988; E. M. WRIGHT, P. MEYSTRE, *Collapse and revival in the micromaser*, Opt. Lett., **14**, pp. 177–179, 1989.
17. I. ASHRAF, J. GEA-BANACLOCHE, M. S. ZUBAIRY, *Theory of the two-photon micromaser: Photon statistics* , Phys. Rev. A, **42**, pp. 6704–6712, 1990.
18. N. HATAKENAKA, S. KURIHARA, *Josephson cascade micromaser*, Phys. Rev. A, **54**, pp. 1729–1732, 1996.
19. Ch. P. SEARCH, W. ZHANG, P. MEYSTRE, *Molecular Micromaser*, Phys. Rev. Lett., **91**, 190401, 2003;
Ch. SEARCH, T. MIYAKAWA, P. MEYSTRE, *Lattice array of molecular micromasers*, Opt. Express, **12**, pp. 30–41, 2004.
20. D. A. RODRIGUES, J. IMBERS, A. D. ARMOUR, *Quantum Dynamics of a Resonator Driven by a Superconducting Single-Electron Transistor: A Solid-State Analogue of the Micromaser*, Phys. Rev. Lett., **98**, 067204, 2007.
21. J. Q. YOU, Y.-x. LIU, C. P. SUN, F. NORI, *Persistent single-photon production by tunable on-chip micromaser with a superconducting quantum circuit*, Phys. Rev. B, **75**, 104516, 2007,
22. J. Q. YOU, Y.-x. LIU, F. NORI, *Simultaneous Cooling of an Artificial Atom and Its Neighboring Quantum System*, Phys. Rev. Lett., **100**, 047001, 2008.
23. A. H. TOOR, M. S. ZUBAIRY, *Validity of the effective Hamiltonian in the two-photon atom-field interaction*, Phys. Rev. A, **45**, pp. 4951–4959, 1992.
24. M. J. COLLETT, R. B. LEVIEN, *Two-photon-loss model of intracavity second-harmonic generation*, Phys. Rev. A, **43**, pp. 5068–5072, 1991.
25. E. S. GUERRA, B. M. GARRAWAY, P. L. KNIGHT, *Two-photon parametric pumping versus two-photon absorption: A quantum jump approach*, Phys. Rev. A, **55**, pp. 3842–3857, 1997.
26. L. GILLES, P. L. KNIGHT, *Two-photon absorption and nonclassical states of light*, Phys. Rev. A, **48**, pp. 1582–1593, 1993.
27. M. S. ZUBAIRY, J. J. YEH, *Photon statistics in multiphoton absorption and emission processes*, Phys. Rev. A. **21**, pp. 1624–1631, 1980.
28. C. C. GERRY, *Remarks on the use of group theory in quantum optics*, Opt. Express, **8**, pp. 76–85, 2001.
29. A. N. BOTO, P. KOK, D. S. ABRAMS, S. L. BRAUNSTEIN, C. P. WILLIAMS, J. P. DOWLING, *Quantum Interferometric Optical Lithography: Exploiting Entanglement to Beat the Diffraction Limit* , Phys. Rev. Lett., **85**, pp. 2733–2736, 2000.
30. J. J. SLOSSER, P. MEYSTRE, S. L. BRAUNSTEIN, *Harmonic oscillator driven by a quantum current*, Phys. Rev. Lett., **63**, pp. 934–937, 1989.
31. H. CARMICHAEL, *An Open System Approach to Quantum Optics*, Springer Verlag Press, 1993.

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