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SOLITONS, KINKS AND SINGULAR SOLUTIONS OF COUPLED KORTEWEG-DE VRIES EQUATIONS

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This paper studies the coupled Korteweg-de Vries (KdV) equations that describe shallow two-layered water waves in ocean shores. The ansatz method is employed to retrieve the solitary wave solutions, topological and singular solitons of this coupled system of equations. The constraint conditions will fall out for the existence of these types of solitons. The study is also generalized to coupled KdV equations with power law nonlinearity. The numerical simulations supplement the analytical schemes.

Key words: solutions, integrability, ansatz method, numerics.

1. INTRODUCTION

Unique nonlinear phenomena exist in all research fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics, and so on. In the past work, the research of travelling wave solutions of several nonlinear evolution equations played an important role in the analysis of such nonlinear phenomena [1–15]. For the travelling wave solutions, many methods were attempted, such as the inverse scattering transform method [2], Hirota's bilinear transformation [9], the tanh method [6,11,12], sine-cosine method [15], homogeneous balance method [15], exp-function method [15] and so on. The above methods derived many types of solutions from most nonlinear evolution equations. It is now time to start focusing on such equations which appear in coupled vector format. They have practical applications too. Quite commonly, vector coupled evolutions are studied in two-layered or rather multiple-layered shallow water waves, or optical solitons in birefringent fibers and/or dense-wavelength division multiplexed systems [8]. The two-layered fluid waves near ocean shores can be modeled by coupled Korteweg-de Vries (KdV) equations or the Gear-Grimshaw models [3]. This paper will focus on coupled KdV equations; exact solitary waves, kinks (topological solitons) and singular solitons will be obtained.

2. GOVERNING EQUATIONS

The coupled KdV equations which describe the interaction process of two long waves are given by

Type I

$$q_t + a_1 qq_x + b_1 q_{xxx} + c_1 rr_x = 0 , \tag{1}$$

$$r_t + a_2 qr_x + b_2 r_{xxx} = 0 , \tag{2}$$

and the coupled KdV equations with power law nonlinearity are

Type II

$$q_t + a_1 q^n q_x + b_1 q_{xxx} + c_1 r^n r_x = 0 , \tag{3}$$

$$r_t + a_2 q^m r_x + b_2 r_{xxx} = 0. \tag{4}$$

These are the two models that represent the coupled KdV equations. The dependent variables are *q(x,t)* and *r(x,t)* which respectively represent the wave profile for the two layers of fluid. The spatially independent variable is *x* while the temporal independent variable is *t*. The constant coefficients are a_j , b_j, $j=1,2$ and c_1. In (1), the first term represents the evolution term, while the second term along with the fourth term are the nonlinear terms. The third term is the dispersion. Then for (2), the first term again represents evolution, the second term is the cross-nonlinear term while the third term is again the dispersion. For Type-II, the two additional parameters are *m* and *n* that represent the power law nonlinearity parameters, i.e., we get coupled KdV equations with power law nonlinearity. Thus, the power law nonlinearity is introduced on a generalized setting that could represent the regular KdV equation or the modified KdV (mKdV) depending on the values of *m* and *n*.

3. ANSATZ APPROACH

One of the most popular approaches to integration, that has lately surfaced is the *ansatz method* [3, 4, 7]. This method is similar to the trial solution method that is very commonly used in solving ordinary differential equations. Such a trial solution, for solitons, is assumed based on the known results of the single KdV equation. This trial solution is then substituted into the coupled system. The linearly independent functions are then identified, after the application of the balancing principle, and their respective coefficients are all set to zero. These lead to the computation of the soliton parameters.

3.1. Solitary waves

In this section the search is going to be for 1-soliton solution to the coupled KdV-type I equations given by (1), (2) and type II equations (3) and (4). To begin, let us assume the following solitary wave ansatz in two cases of the KdV equation.

3.1.1. Coupled KdV equations

To obtain 1-soliton solution of (1) and (2) we assume that

$$q(x,t) = \frac{A_1}{\cosh^{p_1}\tau}, \tag{5}$$

$$r(x,t) = \frac{A_2}{\cosh^{p_2}\tau}, \tag{6}$$

where A_1 and A_2 represent the amplitudes of *q* and *r* solitons, respectively, and τ is given by

$$\tau = B(x - vt), \tag{7}$$

while B is another free parameter of the soliton and *v* is the velocity of the soliton.
Substituting (5) and (6) into (1) and (2) gives:

$$\frac{p_1 q A_1 B \tanh\tau}{\cosh^{p_1}\tau} - \frac{a_1 p_1 A_1^2 B \tanh\tau}{\cosh^{2p_1}\tau} - b_1\left\{\frac{p_1^3 A_1 B^3 \tanh\tau}{\cosh^{p_1}\tau} + \frac{p_1(p_1+1)(p_1+2)A_1 B^3 \tanh\tau}{\cosh^{(p_1+2)}\tau}\right\} - \frac{c_1 p_2 A_2^2 B \tanh\tau}{\cosh^{2p_2}\tau} = 0, \tag{8}$$

$$\frac{p_2 q A_2 B \tanh\tau}{\cosh^{p_2}\tau} - \frac{a_2 p_2 A_1 A_2 B \tanh\tau}{\cosh^{(p_1+p_2)}\tau} - b_2\left\{\frac{p_2^3 A_2 B^3 \tanh\tau}{\cosh^{p_2}\tau} - \frac{p_2(p_2+1)(p_2+2)A_2 B^3 \tanh\tau}{\cosh^{(p_2+2)}\tau}\right\} = 0. \tag{9}$$

Now, by equating the exponent p_1+2 and $2p_1$ and the exponent p_2+2 and p_1+2 in (8), (9) we get

$$p_1 = p_2 = 2, \tag{10}$$

By equating the coefficients of $\dfrac{\tanh\tau}{\cosh^{p+j}\tau}$, $j=0,2$ in (8) and (9) we have

$$v - 4b_1B^2 = 0, \tag{11}$$

$$-a_1A_1^2 + 24b_1A_1B^2 - 2c_1A_2^2 = 0, \tag{12}$$

$$v - 4b_2B^2 = 0, \tag{13}$$

$$-a_2A_1 + 12b_2B^2 = 0. \tag{14}$$

From the above equations we have

$$b_2 = b_1 = b, \tag{15}$$

$$B = \frac{1}{2}\sqrt{\frac{v}{b}}, \tag{16}$$

$$A_1 = \frac{3v}{a_2}, \tag{17}$$

$$A_2 = \frac{3v}{a_2}\sqrt{\frac{a_2-a_1}{c_1}}, \quad \frac{a_2-a_1}{c_1} > 0. \tag{18}$$

The following Fig. 1 displays 1-soliton solution *q(x,t)*, *r(x,t)* with the parameters $a_1=1,\ a_2=2,\ b=1,\ c=2,\ v=0.5,\ -20\le x\le 20,\ \ 0\le t\le 20$.

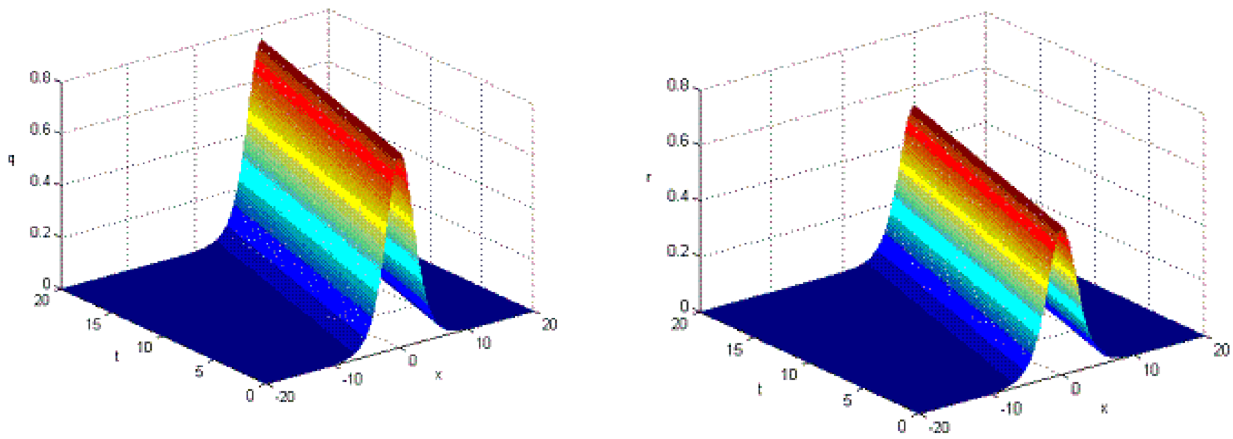

Fig. 1. – Soliton solution: – left: *q(x,t)*, right: *r(x,t)*.

3.1.2. Coupled KdV equation with power-law nonlinearity

Starting with the same ansatz method as (5) and (6), equations (3) and (4) give

$$\frac{p_1 q A_1 B \tanh\tau}{\cosh^{p_1}\tau} - \frac{a_1 p_1 A_1^{n+1} B \tanh\tau}{\cosh^{np_1+p_1}\tau} - b_1\left\{\frac{p_1^3 A_1 B^3 \tanh\tau}{\cosh^{p_1}\tau} + \frac{p_1(p_1+1)(p_1+2)A_1 B^3 \tanh\tau}{\cosh^{(p_1+2)}\tau}\right\} - \\ -\frac{c_1 p_2 A_2^{n+1} B \tanh\tau}{\cosh^{np_2+p_2}\tau} = 0 \tag{19}$$

$$\frac{p_2 q A_2 B \tanh\tau}{\cosh^{p_2}\tau} - \frac{a_2 p_2 A_1^m A_2 B \tanh\tau}{\cosh^{(p_1+p_2)}\tau} - \\ -b_2\left\{\frac{p_2^3 A_2 B^3 \tanh\tau}{\cosh^{p_2}\tau} - \frac{p_2(p_2+1)(p_2+2)A_2 B^3 \tanh\tau}{\cosh^{(p_2+2)}\tau}\right\} = 0 \tag{20}$$

Equating the exponent of $np_1 + p_1$, $p_1 + 2$ and exponent of $np_2 + p_1$, $p_1 + 2$ in (19) and (20) we get

$$p_1 = p_2 = \frac{2}{n}. \tag{21}$$

From (19) and (20), by equating the coefficients of $\cosh^{p_1+j}\tau$, $j = 0, 2$ we get

$$v - \frac{4}{n^2} b_1 B^2 = 0, \tag{22}$$

$$a_1 A_1^{n+1} - \frac{(n+2)(2n+2)}{n^2} b_1 A_1 B^2 - c A_2^{n+1} = 0, \tag{23}$$

$$v - \frac{4}{m^2} b_2 B^2 = 0, \tag{24}$$

$$a_2 A_1^m - \frac{(m+2)(2m+2)}{m^2} b_2 B^2 = 0. \tag{25}$$

From the above equations we get

$$n = m, \tag{26}$$

$$b_2 = b_1 = b, \tag{27}$$

$$B = \frac{m}{2}\sqrt{\frac{v}{b_1}}, \tag{28}$$

$$A_1 = \left(\frac{v(m+2)(2m+2)}{4a_2}\right)^{\frac{1}{m}}, \tag{29}$$

$$A_2 = \left(\frac{v(m+2)(2m+2)b_1 A_1 B^2}{m^2} - a_1 A_1^{m+1}\right)^{\frac{1}{m+1}}. \tag{30}$$

In Fig. 2 we display 1-soliton solution *q(x,t)*, *r(x,t)* with the parameters :

$$m = 2,\ a_1 = 1,\ a_2 = 2,\ b_1 = b_2 = 1,\ c_1 = -2,\ v = 0.5,\ -20 \le x \le 20,\quad 0 \le t \le 20$$

3.2. Topological solitons

Topological solitons are also known as *kinks* or *shock waves* [13, 14]. A topological soliton is a nonlinear wave where we have a transition from one stable state to another. Thus, these are typically modeled by the *tanh* functions. Hence, the following ansatz will be utilized to get a solution of the coupled KdV equations.

$$q(x,t) = A_1 \tanh^{p_1} \tau\ , \tag{31}$$

$$r(x,t) = A_2 \tanh^{p_2} \tau\ , \tag{32}$$

where A_1 and A_2 are free parameters of *q* and *r* soliton respectively and τ is given by (7).

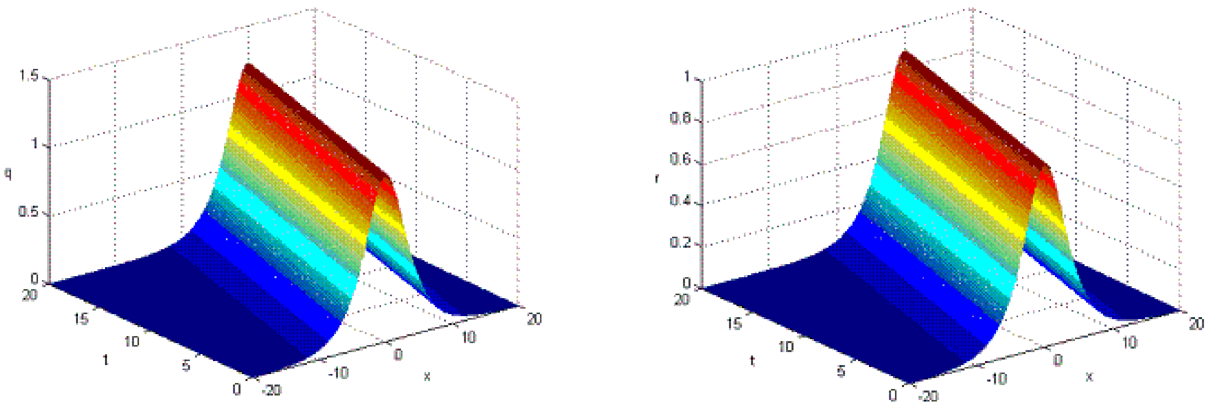

Fig. 2 – Soliton solution for *m* = 2: – left: *q(x,t)*, right: *r(x,t)*.

3.2.1. Coupled KdV equations

Substituting (31) and (32) into (1) and (2) respectively yields

$$\begin{aligned} &p_1 v A_1 B(\tanh^{p_1+1}\tau - \tanh^{p_1-1}\tau) + a_1 p_1 A_1^2 B(\tanh^{2p_1-1}\tau - \tanh^{2p_1+1}\tau) + p_1 A_1 B^3 \\ &\{(p_1-1)(p_1-2)\tanh^{p_1-3}\tau - (p_1+1)(p_1+2)\}\tanh^{p_1+3}\tau - \{2p_1^2 + (p_1-1)(p_1-2)\} \\ &\tanh^{p_1-1}\tau + \{2p_1^2 + (p_1+1)(p_1+2)\}\tanh^{p_1+1}\tau\} + c_1 p_2 A_2^2(\tanh^{2p_2-1}\tau - \tanh^{2p_2+1}\tau) = 0, \end{aligned} \tag{33}$$

$$\begin{aligned} &p_2 v A_2 B(\tanh^{p_2+1}\tau - \tanh^{p_2-1}\tau) + a_2 p_2 A_1 A_2 B(\tanh^{p_1+p_2-1}\tau - \tanh^{p_1+p_2+1}\tau) \\ &+ b_2\, p_2 A_2 B^3\{(p_2-1)(p_2-2)\tanh^{p_2-3}\tau - (p_2+1)(p_2+2)\tanh^{p_2+3}\tau \\ &-\{2p_2^2 + (p_2-1)(p_2-2)\}\tanh^{p_2-1}\tau + \{2p_2^2 + (p_2+1)(p_2+2)\}\tanh^{p_2+1}\tau\} = 0. \end{aligned} \tag{34}$$

Now, by equating the exponent $2p_1 + 1$ and $p_1 + 3$ in (33) equating the exponent $2p_2 + 1$ and $p_1 + 3$ in (34)give

$$p_1 = p_2 = 2\,. \tag{35}$$

By equating the coefficients of $\tanh^{p_1+j}\tau$, $j=\pm 1,\pm 3$ in (33), by equating the coefficients of $\tanh^{p_2+j}\tau$, $j=\pm 1,\pm 3$ in (34) and put $p_1=p_2=2$ we have

$$v+8b_1B^2=0\,, \tag{36}$$

$$vA_1+a_1A_1^2+20b_1A_1B^2+c_1A_2^2=0\,, \tag{37}$$

$$a_1A_1^2+12b_1A_1B^2+c_1A_2^2=0\,, \tag{38}$$

$$v+8b_1B^2=0\,, \tag{39}$$

$$v+a_2A_1+20b_2B^2=0\,, \tag{40}$$

$$a_2A_1+12b_2B^2=0\,. \tag{41}$$

Using Maple program to solve the above equations we obtain the solutions of the form

$$b_1=b_2=b\,, \tag{42}$$

$$B=\sqrt{\frac{-v}{8b}}\,, \tag{43}$$

$$A_1=\frac{3v}{2a_2}\,, \tag{44}$$

$$A_2=\frac{3v}{2a_2}\sqrt{\frac{a_2-a_1}{c_1}},\quad \frac{a_2-a_1}{c_1}>0\,. \tag{45}$$

Figure 3 displays topological 1-soliton solution *q(x,t)*, *r(x,t)*, respectively, with the parameters $a_1=1,\ a_2=2,\ b=-2, c=2, v=0.5, -20\le x\le 20,\ \ 0\le t\le 20$

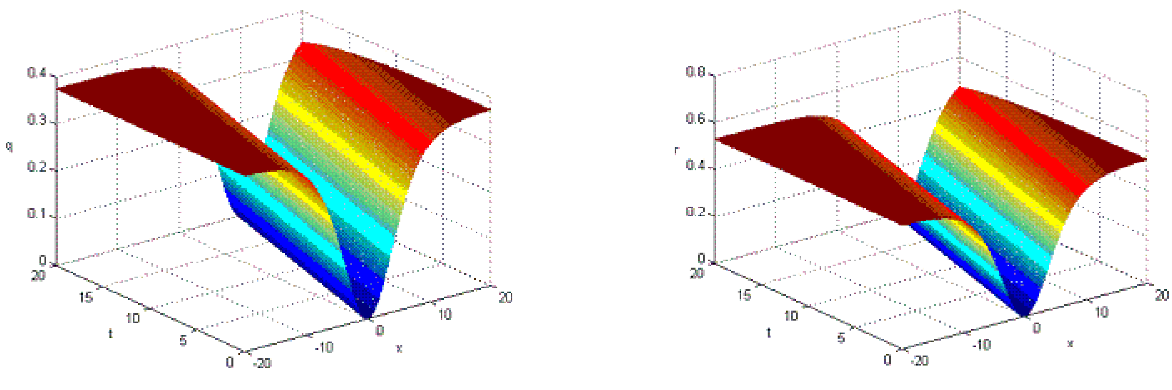

Fig. 3. – Topological 1-soliton solution: – left: *q(x,t)*, right: *r(x,t)*.

3.2.2. Coupled KdV equation with power-law nonlinearity

For the coupled KdV equation with power law nonlinearity, the starting hypothesis stays the same as in (33), (34). Thus, (3) and (4) give

$$p_1 v A_1 B(\tanh^{p_1+1}\tau - \tanh^{p_1-1}\tau) + a_1 p_1 A_1^{n+1} B(\tanh^{np_1+p_1-1}\tau - \tanh^{np_1+p_1+1}\tau)$$
$$+ p_1 A_1 B^3\{(p_1-1)(p_1-2)\tanh^{p_1-3}\tau - (p_1+1)(p_1+2)\tanh^{p_1+3}\tau - \{2p_1^2 + (p_1-1)(p_1-2)\}\tanh^{p_1-1}\tau \tag{46}$$
$$+\{2p_1^2 + (p_1+1)(p_1+2)\}\tanh^{p_1+1}\tau\} + c_1 p_2 A_2^2(\tanh^{np_2+p_2-1}\tau - \tanh^{np_2+p_2+1}\tau) = 0,$$

$$p_2 v A_2 B(\tanh^{p_2+1}\tau - \tanh^{p_2-1}\tau) + a_2 p_2 A_1 A_2 B(\tanh^{mp_1+p_2-1}\tau - \tanh^{mp_1+p_2+1}\tau)$$
$$+b_2 p_2 A_2 B^3\{(p_2-1)(p_2-2)\tanh^{p_2-3}\tau - (p_2+1)(p_2+2)\tanh^{p_2+3}\tau \tag{47}$$
$$-\{2p_2^2 + (p_2-1)(p_2-2)\}\tanh^{p_2-1}\tau + \{2p_2^2 + (p_2+1)(p_2+2)\}\tanh^{p_2+1}\tau\} = 0.$$

Now, by the aid of balancing principle, equating the exponent $np_1 + p_1 + 1$ and $p_1 + 3$ and equating the exponent $np_2 + p_2 + 1$ and $p_1 + 3$ in (46) gives

$$p_2 = 2/n. \tag{48}$$

By setting the coefficients of $\tanh^{p_1+j}\tau, \tanh^{p_2+j}\tau,\ j = \pm 1, \pm 3$ in (46) and (47) to zero, since these are linearly independent functions, we have

$$v + (2p_1^2 + (p_1-1)(p_1-2))b_1 B^2 = 0, \tag{49}$$

$$vA_1 + a_1 A_1^{n+1} + (2p_1^2 + (p_1+1)(p_1+2))b_1 A_1 B^2 + c_1 A_2^{n+1} = 0, \tag{50}$$

$$a_1 A_1^{n+1} + (p_1+1)(p_1+2)b_1 A_1 B^2 + c_1 A_2^{n+1} = 0, \tag{51}$$

$$p_1(p_1-1)(p_1-2)b_1 A_1 B^3 = 0, \tag{52}$$

$$v + (2p_2^2 + (p_2-1)(p_2-2))b_2 B^2 = 0, \tag{53}$$

$$v + a_2 A_1^m + (2p_2^2 + (p_2+1)(p_2+2))b_2 B^2 = 0, \tag{54}$$

$$a_2 A_1^n + (p_2+1)(p_2+2)b_2 B^2 = 0. \tag{55}$$

It follows from (52) that $p_1 = 1$ or $p_1 = 2$, i.e., the case 3.2.1. Now, substituting $p_1 = 1$ in (49), (53) we find

$$n = m = 2. \tag{56}$$

Now, we put $p_1 = p_2 = 1$ and $n = m = 2$ in (49–55) and we get

$$B = \sqrt{-v/2b}, \tag{57}$$

$$A_1 = \sqrt{v/a_2}, \tag{58}$$

$$A_2 = \left(A_1(a_2 - a_1)/a_2 c_1\right)^{1/3}. \tag{59}$$

Figure 4 displays the kink soliton solution with the following parameters:

$$a_1 = 1,\ a_2 = 2,\ b = -2,\ c = 2,\ v = 0.5,\ -20 \le x \le 20,\ \ 0 \le t \le 20.$$

3.3. Singular solitons

There are several nonlinear evolution equations that support singular solutions and they are applicable in various areas of mathematics and physics. Let us assume the following singular solutions:

$$q(x,t) = \frac{A_1}{\sinh^{p_1} \tau}, \tag{60}$$

$$r(x,t) = \frac{A_2}{\sinh^{p_2} \tau}, \tag{61}$$

where A_1 and A_2 are constants of the q and r soliton, respectively, and τ is given by (7).

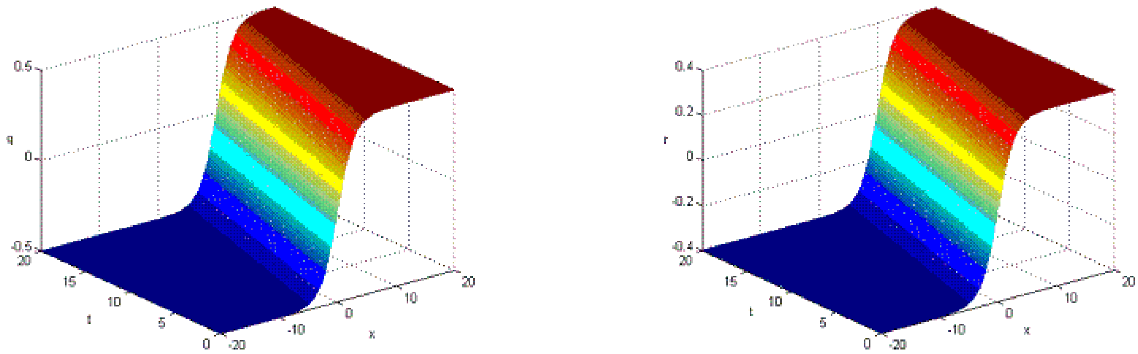

Fig. 4. – Kink solution: – left: *q(x,t)*, right: *r(x,t)*.

3.3.1. Coupled KdV equation

From (60) and (61) we have

$$\frac{p_1 v A_1 B \coth\tau}{\sinh^{p_1}\tau} - \frac{a_1 p_1 A_1^2 B \coth\tau}{\sinh^{2p_1}\tau} - b_1\left\{\frac{p_1^3 A_1 B^3 \coth\tau}{\sinh^{p_1}\tau} + \frac{p_1(p_1+1)(p_1+2)A_1 B^3 \coth\tau}{\cosh^{(p_1+2)}\tau}\right\} - \\ -\frac{c_1 p_2 A_2^2 B \coth\tau}{\sinh^{2p_2}\tau} = 0 \tag{62}$$

$$\frac{p_2 v A_2 B \coth\tau}{\sinh^{p_2}\tau} - \frac{a_2 p_2 A_1 A_2 B \coth\tau}{\cosh^{(p_1+p_2)}\tau} - b_2\left\{\frac{p_2^3 A_2 B^3 \coth\tau}{\sinh^{p_2}\tau} + \frac{p_2(p_2+1)(p_2+2)A_2 B^3 \coth\tau}{\sinh^{(p_2+2)}\tau}\right\} = 0. \tag{63}$$

Finally we get

$$B = \frac{1}{2}\sqrt{\frac{v}{b}}, \tag{64}$$

$$A_1 = \frac{-3v}{a_2}, \tag{65}$$

$$A_2 = \frac{3v}{a_2}\sqrt{\frac{-(a_1+a_2)}{c_1}}, \quad \frac{a_1+a_2}{c_1} > 0. \tag{66}$$

3.3.2. Coupled KdV equations with power-law nonlinearity

In this case, (3) and (4) give

$$\frac{p_1 v A_1 B\coth\tau}{\sinh^{p_1}\tau}-\frac{a_1 p_1 A_1^{n+1} B\coth\tau}{\sinh^{np_1+p_1}\tau}-b_1\left\{\frac{p_1^3 A_1 B^3\coth\tau}{\sinh^{p_1}\tau}+\frac{p_1(p_1+1)(p_1+2)A_1 B^3\coth\tau}{\sinh^{p_1+2}\tau}\right\}-$$
$$-\frac{c_1 p_2 A_2^{n+1} B\coth\tau}{\sinh^{np_2+p_2}\tau}=0 \tag{67}$$

$$\frac{p_2 v A_2 B\coth\tau}{\sinh^{p_2}\tau}-\frac{a_2 p_2 A_1^{m} A_2 B\coth\tau}{\sinh^{mp_1+p_2}\tau}-b_2\left\{\frac{p_2^3 A_2 B^3 \cot \mathrm{h}\,\tau}{\sinh^{p_2}\tau}+\frac{p_2(p_2+1)(p_2+2)A_2 B^3\coth\tau}{\sinh^{p_2+2}\tau}\right\}=0. \tag{68}$$

Doing the same calculation of subsection 3.1.2 we get

$$B=\frac{m}{2}\sqrt{\frac{v}{b}}, \tag{69}$$

$$A_1=\left(\frac{-(m+1)(m+2)v}{2a_2}\right)^{\frac{1}{m}}, \tag{70}$$

$$A_2=\left(\left(\frac{-a_1(m+1)(m+2)v}{2a_2}\right)^{1+\frac{1}{m}}-\frac{(m+2)^2 v}{4}\right)^{1/(1+m)}. \tag{71}$$

4. CONCLUSIONS

This paper studied the coupled KdV equations that appear in shallow two-layered water waves in lakes, rivers and ocean shores. Solitary waves, kinks, and singular soliton solutions were obtained. The ansatz method was applied in order to obtain these special solutions. The soliton solutions also introduced several constraint conditions that must remain valid in order for these solutions to exist. The numerical simulations clearly illustrate the solutions that were analytically obtained. Other approaches that will be used to study these coupled KdV equations which will reveal further nonlinear wave solutions will be reported elsewhere.

REFERENCES

1. M.A. ABDOU, *The extended tanh-method and its applications for solving nonlinear physical models*, Appl. Math. Comput., **190**, pp. 988–996, 2007.
2. M.J. ABLOWITZ, P.A. CLARKSON, *Solitons, Non-linear Evolution Equations and Inverse Scattering Transform*, Cambridge University Press, Cambridge, 1991.
3. A. BISWAS, M.S. ISMAIL, *1-soliton solution of the coupled KdV equation and Gear-Grimshaw model*, Appl. Math. Comp., **216**, pp. 3662–3670, 2010.
4. A. BISWAS, A. YILDIRIM, T. HAYAT, O. M. ALDOSSARY, R. SASSAMAN, *Soliton perturbation theory for the generalized Klein-Gordon equation with full nonlinearity*, Proc. Romanian Acad. A, **13**, pp. 32–41, 2012.
5. G. EBADI, A. YILDIRIM, A. BISWAS, *Chiral solitons with Bohm potential using G'/G method and exponential function method*, Rom. Rep. Phys., **64**, pp. 357–366, 2012.
6. E.G. FAN, *Extended tanh-function method and its applications to nonlinear equations*, Phys. Lett. A, **277**, pp. 212–218, 2000.

7. P. GREEN, D. MILOVIC, A.K. SARMA, D.A. LOTT, A. BISWAS, *Dynamics of super-sech solitons in optical fibers*, J. Nonl. Opt. Phys. and Materials, **19**, pp. 339–370, 2010.
8. J.H. HE, X.H. WU, *Exp-function method for nonlinear wave equations*, Chaos. Soliton. Fract., **30**, pp. 700–708, 2006.
9. R. HIROTA, *Exact envelope soliton solutions of a nonlinear wave equation*, J. Math. Phys., **14**, pp. 805–810, 1973.
10. A.G. JOHNPILLAI, A. YILDIRIM, A. BISWAS, *Chiral solitons with Bohm potential by Lie group analysis and traveling wave hypothesis*, Rom. J. Phys., **57**, pp. 545–554, 2012.
11. W. MALFLIET, *Solitary wave solutions of nonlinear wave equations*, Am. J. Phys., **60**, pp. 650–654, 1992.
12. E.J. PARKES, B.R. DUFFY, *An automated tanh-function method for finding solitary wave solutions to nonlinear evolution equations*, Comput. Phys. Commun. **98**, pp. 288–300, 1996.
13. H. TRIKI, A. YILDIRIM, T. HAYAT, O.M. ALDOSSARY, A. BISWAS, *Topological and non-topological soliton solutions of the Bretherton equation*, Proc. Romanian Acad. A, **13**, pp. 103–108, 2012.
14. H. TRIKI, S. CRUTCHER, A. YILDIRIM, T. HAYAT, O.M. ALDOSSARY, A. BISWAS, *Bright and dark solitons of the complex Ginzburg-Landau equation with parabolic and dual-power law nonlinearity*, Rom. Rep. Phys., **64**, pp. 367–380, 2012.
15. M.L. WANG, *Exact solutions of a compound KdV-Burgers equation*, Phys. Lett. A, **213**, pp. 279–287, 1996.

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