# PROPOSED CHANGES TO DEFINITIONS OF TIME OF AVAILABILITY AND UNAVAILABILITY OF DIGITAL 64 KB/S CHANNEL DEFINED IN RECOMMENDATION ITU-T G.821 (ANNEX A)

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This paper explains the terms of system availability and availability of digital channels. Furthermore it indicates types of errors that can arise in the digital channel. We present specific recommendations which determine the availability of digital channels and two original ways of calculation of availability of digital 64 kb/s channels in the presence of random errors. At the end of the paper is presented a proposal to amend the definition of time availability and unavailability of digital 64 kb/s channel given in Recommendation ITU-T G.821, Annex A.

Key words: availability and unavailability of digital 64 kb/s channel, recommendation ITU-T G.821 (Annex A), bit error probability, Markov chains.

# 1. INTRODUCTION

System availability is defined as the probability that the system is able to perform its function in a random moment. The term availability A, is defined for systems and system components where we can identify the state of correct operation, the state of outage, and where we have the possibility of recovery, [1].

Availability of digital channels is the probability that in a random moment the digital channel is completely correct for the transmission of digital messages (section 2, the expression (2)). As will be seen, in some cases, the message transmission is performed via digital channels, but with such an error rate, that the digital channels are declared invalid. In this paper, the irregularity of the digital channels will be considered only in the case when the error rate is above some agreed level. Thus, the correct digital channel will be the one in which the message transfer is realized with error rate less than the agreed level. This will be discussed in more detail later (section 3).

Transmission quality in the digital channel is defined by the probability of bit errors, BER. If we define the average BER, this parameter is used in the recommendation Q.706 [2] to define a standard transmission quality, which is expected on the first level of the MTP (*Message Transfer Part*) protocol. Using only this parameter, we are not able to determine distribution of errors in the digital channel. To determine the transmission quality in the digital channel, the events that are based on time intervals of one second duration are used more and more. This was accepted and presented in Recommendation G.821 [3], for digital transmission systems with bit rate N·64 kb/s·

Definitions of time availability and unavailability of digital 64 kb/s channel are presented in section 3, (ITU - T Recommendation G.821, Annex A). These definitions allow the use of the finite Markov chain with state of absorption in the modeling of the digital 64 kb/s channels. Using these models and the results obtained in Section 3, we can calculate pretty accurately the mean duration time for the state of availability and unavailability of digital 64 kb/s channels, depending on the transmission bit error rate.

This paper suggests changes the definition of time availability and unavailability of digital 64 kb/s channel given in Recommendation ITU-T G.821 (Annex A). The proposed definition provides that the channel is considered available or unavailable at the time of determining the availability or unavailability, and also will not change the methods of calculation and the results of calculation.

## 2. AVAILABILITY

The most common way of representing the availability, i.e., the unavailability of the system, is by using the following variables:

- mean time to failure, MTTF,
- mean time to repair, MTTR, and
- mean time between failures, MTBF.

Mutual dependences between mean time to failure, MTTF, mean time of recovery, MTTR, and mean time between failures, MTBF, can be easily understood from Fig. 1. According to the designations in the Fig. 1 can be said that is:

- MTTF mean value of all time intervals to failure TTF,
- MTBF mean value of all time intervals between failures TBF and
- MTTR mean value of all time intervals of recovery TTR.

Using these values, availability, A, i.e., unavailability, UA, can be represented as:

$$A = (MTBF - MTTR) / MTBF = MTTF / (MTTF + MTTR) = 1 - UA.$$
 (1)

Availability, A, for systems and system components, is described as the ratio of mean time spent in functional state and a total running time:

$$A = (MTBF - MTTR) / MTBF,$$
 (2)

where: MTBF denotes the mean value of random variable that can be called the length of time interval between the errors of the system or part of the system, and MTTR denotes the mean value of random variable that can be called the length of the interval in which the system or part of the system recovers to a functional state.

Unavailability, UA, is the variable of opposite meaning and of complementary value to the availability and can be expressed as:

$$UA = = 1 - A = MTTR / (MTTF + MTTR).$$
(3)

In the function of digital channels, MTBF and MTTR are called or designated in a different way, but their meaning and impact on the availability of digital channels is the same as explained here.

# 3. AVAILABILITY OF DIGITAL CHANNELS

In the use of digital channels there are time intervals when the digital channel functions correctly, and when it is faulty. Mean time between failures, MTBF is the mean length of time intervals between successive failures. It is obvious that during the time interval between two faults exists the time interval when the digital channel is faulty. Mean recovery time, MTTR, is the mean length of time intervals measured from the failure occurrence to digital channel recovery to the correct state. It can be concluded that the time of normal operation of the digital channel is the time interval between recovery and failure of the digital channel, and the recovery time of a digital channel is the time period measured from the failure origination to the recovery of digital channel in the correct state.

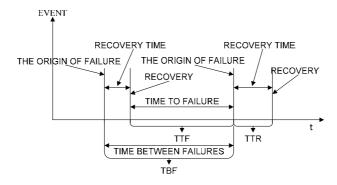


Fig. 1 – Graphical representation of the mutual dependence between time to failure TTF, recovery time TTR, and time between failures TBF.

## 4. ERRORS IN DIGITAL CHANNEL

An important characteristic of analog transmission systems is frequency spectrum, which is occupied by the transmitted signals, i.e. the frequency bandwidth of the system. In digital systems, a similar role has a value that, when considering digital signal, presents the rate at which the numbers, digits are generated and following one after the other, or when considering the transfer of such signals, presents the rate at which the digits flow, following one another in the transmition. The variable that expresses, in both cases, the number of transmitted digits in the unit of time, we shall call digital or numerical flow (*digit rate, franc. débit numérique*) and we shall express it by the number of digits in the second. Of course, if it is a binary digital transmission, we shall express this value by the number of binary digits or bits (*binary digit = bit*) per second.

The other variable, which is important in assessing the quality of signal transmission in analog systems, in addition to bandwidth, is the signal / noise ratio. Similar role in systems, where digital signals are transmitted, has intensity or probability of bit error (*Bit Error Rate*, BER).

In general, the term bit error rate is used to denote the ratio of incorrectly transmitted bits to the total number of bits, whether it is measured or expected ratio. The probability of bit error refers only to the expected value of this ratio for great intervals of time.

For comparison, we can define the average BER. Using this parameter in Recommendation Q.706 [2], quality transmission standards that are expected in the first level of the MTP protocol are presented:

- long-term bit error rate of less than  $1 \cdot 10^{-6}$ ,
- medium-term bit error rate of less than  $1.10^{-4}$ .

The criterion of availability or unavailability of digital channels is defined in recommendation G.821, Annex A, [3]. This definition determines the events that define the beginning of time of "errorless" and "faulty" channels. Based on these events we cannot conclude anything about the average intensity of bit errors in a longer time interval. The digital 64 kb/s channel becomes unavailable at the beginning of the first second of 10 consecutive one-second intervals in which the intensity of bit errors is greater than 10<sup>-3</sup>. Digital 64 kb/s channel becomes available again at the beginning of the first second of 10 consecutive one-second intervals in which the intensity of bit errors is less than 10<sup>-3</sup>. In the first case, the ten seconds are considered as the part of unavailability time, while in another case these ten seconds are part of the availability time. This, quite formal, definition will be discussed in section 6.

# 5. RECOMMENDATIONS THAT DETERMINE AVAILABILITY OF DIGITAL CHANNELS

In Recommendation G.826 [4], the parameter bit rate for flow rates greater than 64 kb/s is considered. Recommendation G.821 defines the following events:

- the second with an error (ES) is one-second interval in which one or more bits are transferred with error,
- the second with significant error (SES) is one-second interval in which the bit error rate is equal to or greater than  $10^{-3}$ .

As already noted (section 2), the criterion of availability or unavailability of digital 64 kb / s channel is defined in ITU-T Recommendation G.821 (Annex A), where errors are randomly distributed.

Relationship between the availability time and the number of already defined one-second intervals allows the following definitions for the transmission quality:

- the ratio of significant error seconds (SES-ratio, SESR) is the ratio of SES to total time available for the specified measurement time;
- the ratio of error seconds (ES-ratio, ESR) is the ratio of seconds with errors to total time in seconds for which the system was available at the time the measurement was performed.

Requirements for quality of transmission of one, 27 500 km long, hypothetical ISDN connection (I.325) include the following values of these ratios: ESR < 0.002 or SESR < 0.8.

The characteristics of this recommendation are:

- 1. It defines the start and end times of unavailability of digital 64 kb/s channels;
- 2. It defines the start and end times of availability of digital 64 kb/s channels;
- 3. Beginnings and ends of these intervals do not coincide with their discovery.

As can be seen, comparing this section to section 2, an important event for determination of the availability of digital channels is the appearance of seconds with significant errors (SES), and it is considered that the seconds with an error belong to the time of available functionality.

# 6. CALCULATION OF THE AVAILABILITY OF DIGITAL CHANNELS WITH RANDOM ERRORS

In previous sections we described a digital channel, errors in the digital channel and recommendations which define the time of availability and unavailability of digital 64 kb/s channels. This section will present two original methods for the calculation of the availability of digital 64 kb/s channels in the presence of random errors.

In the first method, it is possible to present the calculation of the availability of digital 64 kb/s channels in the presence of random errors using the Markov chain (Fig. 2). Digital 64 kb/s channel is considered to be in a state of availability. One-second interval during which is BER  $\geq 10^{-3}$  will be called the probability (*S*), and one-second interval during which is BER  $< 10^{-3}$  will be called the probability (*F*), if each second is considered as the outcome of Bernoulli trial. It is valid that F = 1 - S.

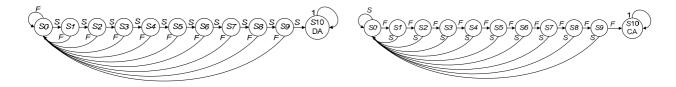


Fig. 2 – Markov chain process alarm detection (digital channels in a state availability).

Fig. 3 – Markov chain for the process of exiting from a state of alarm (digital channels in a state of unavailability).

Let us suppose, therefore, that S is the probability of the appearance of the second with a significant error, i.e. that at intervals of one second duration is satisfied condition BER  $\geq 10^{-3}$ . Condition  $s_i$  ( $s_i = 0,1,2,3,...,9$ ) is reached after i consecutive one-second intervals, where the BER  $\geq 10^{-3}$ . After each one-second interval when the BER  $< 10^{-3}$ , Markov chain returns to the initial state,  $s_0$ . The moment of entry into a state of unavailability, s10, is designated as detection of an alarm, DA, [7]. Exit from the alarm state, clearance of alarm CA, is the moment of entering the state of availability of digital channels. State of alarm detection can be considered as an absorption state, because the process terminates when it enters this state. Digital channel at that moment enters the state of unavailability.

Let the matrix  $P_1$  be transition matrix, corresponding to the model from Fig. 2. It's dimension is  $(11\times11)$  and has the following form:

$$\boldsymbol{P}_{1} = \begin{pmatrix} 1 - \boldsymbol{p} & \boldsymbol{p} & 0 & 0 & \cdots & 0 & 0 \\ 1 - \boldsymbol{p} & 0 & \boldsymbol{p} & 0 & \cdots & 0 & 0 \\ 1 - \boldsymbol{p} & 0 & 0 & \boldsymbol{p} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 - \boldsymbol{p} & 0 & 0 & 0 & \cdots & 0 & \boldsymbol{p} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix} = \begin{pmatrix} \boldsymbol{Q}_{1} & \boldsymbol{d}_{1} \\ 0 & 1 \end{pmatrix},$$

where the matrix  $Q_1$  is sub-stochastic matrix of the size  $(10\times10)$ , while  $d_1$  is the vector of dimension  $(10\times1)$  (not so important for the further calculation), with the transposed form  $d_1^T = (0,0,0,...,0,p)$ . The fundamental matrix  $M_1 = [m1_{ij}] = (I - Q_1)^{-1}$  can be calculated from sub-stochastic matrix  $Q_1$ , where I is the unit matrix of dimension  $(10\times10)$  and p is the probability of the appearance of a second with an error [5]. The main property of the implemented fundamental matrix is that  $(i, j)^{th}$  element of the matrix,  $m_{1ij}$ , represents the average number of occurrences of state j until entering the state of absorption if the initial state is i. A detailed description of this matrix can be found in [5] and [6].

If  $S_{av}$  indicates the time of the digital channel operation in a state of availability, to the alarm detection, then, on the basis of the already mentioned features of transition matrix  $M_1$ , we can calculate the mean duration of the availability of channel state as follows  $[7-8]^*$ .

$$E(S_{av}) = \sum_{j=1}^{10} m_{11j}$$
 (4)

A similar model can be defined for the operation of digital channels in a state of unavailability (Fig 3). Markov chain, which presents the process of digital channel operation in the state of its unavailability is obtained by replacing the transition probabilities *S* with 1–*S*, or *p* with 1–*p*. Exit from the alarm state, the state of CA, s10 (clearance of alarm), in this case is the absorption condition which presents the return to the state of availability of digital channel.

On the basis of the chain obtained in a manner identical to the previously presented manner, we can calculate the mean duration of unavailability of digital channels. The matrix of transition,  $P_2$ , in this case is of the form:

$$\mathbf{P}_{2} = \begin{pmatrix} \mathbf{p} & 1 - \mathbf{p} & 0 & 0 & \cdots & 0 & 0 \\ \mathbf{p} & 0 & 1 - \mathbf{p} & 0 & \cdots & 0 & 0 \\ \mathbf{p} & 0 & 0 & 1 - \mathbf{p} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{p} & 0 & 0 & 0 & \cdots & 0 & 1 - \mathbf{p} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix} = \begin{pmatrix} \mathbf{Q}_{2} & \mathbf{d}_{2} \\ 0 & 1 \end{pmatrix}.$$

In the same way we get the fundamental matrix  $M_2 = [m_{2ij}] = (I - Q_2)^{-1}$ . If  $S_{uav}$  presents the length of unavailability time of digital channel, then the average time duration of the state of channel unavailability [7] and [8], can be calculated as:

$$E(S_{uav}) = \sum_{j=1}^{10} m_{2_{1j}}.$$
 (5)

If we present now the probability S with the relation (Eq. 4.3.4, Section 4.3, [7]):

$$S = 1 - F = 1 - \sum_{k=0}^{63} p_n(k) = 1 - e^{-\lambda} \cdot \sum_{k=0}^{63} \frac{\lambda^k}{k!}, \dots \text{ where } ... \lambda = n \cdot P_b,$$
 (6)

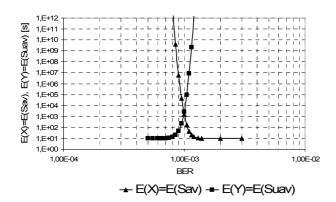
then we get the mean duration of states of digital channel ( $E(S_{av})$  and  $E(S_{uav})$ ), depending on the intensity of bit errors in transmission over the digital channel.

The calculated values for  $E(S_{av})$  and  $E(S_{uav})$  in function of BER are presented in Fig. 4. The mean values,  $E(S_{av})$  and  $E(S_{uav})$ , of random variables  $S_{av}$  and  $S_{uav}$ , are essential components to calculate the probability that the channel is found, at some random moment, in a state of availability or unavailability. On this basis, the availability, A, or unavailability, UA, of digital channel, depending on the intensity of bit error, is represented by the following equation:

$$A = \frac{E(S_{av})}{E(S_{av}) + E(S_{uav})} = 1 - UA.$$
(7)

Fig. 5 presents the availability, A, and unavailability, UA, of digital 64 kb/s channels, depending on the intensity of bit errors in transmission.

<sup>\*</sup>  $m_1(2)_{ij}$ , is  $(i,j)^{\text{th}}$  element of the transition matrix  $\mathbf{M}_1(2) = [m_1(2)_{ij}] = (\mathbf{I} - \mathbf{Q}_{1(2)})^{-1}$ . Detailed description of this matrix can be found in [5] and [6].  $\mathbf{Q}_{1(2)}$  is sub-stochastic matrix derived from the transition matrix, which corresponds to the model of Fig. 2 and Fig 3.



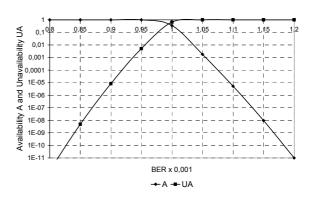


Fig. 4 – The average time of availability and unavailability, depending on the intensity of bit errors in transmission.

Fig. 5 – Availability and unavailability of digital 64 kb/s channel in function of BER, calculated on the basis of the recommendation ITU-T G.821

Later in this section, we shall present another way of calculating the availability or unavailability of digital 64 kb/s channel in the case of uniformly distributed errors, as described in [3], depending on the intensity of bit errors. First we shall consider digital 64 kb/s channel in a state of availability. With  $P_{bit}$  we shall designate the probability that one bit is wrong. The probability that in one one-second interval BER is equal to or worse than the  $10^{-3}$ , according to Annex A of ITU-T recommendation G.821, is equal to the probability that k bits (where  $k \ge 64$ ) are erroneous of total 64 000 bits. This can be represented by the following expression:

$$S = P(BER \ge 1 \cdot 10^{-3}) = 1 - F =$$

$$= \sum_{i=64}^{64000} {64000 \choose i} \cdot P_b^i \cdot (1 - P_b)^{64000 - i} \approx 1 - e^{-\lambda} \cdot \sum_{i=0}^{63} \frac{\lambda^i}{i!},$$
(8)

where is  $\lambda = 64~000 \cdot P_{bit}$ .

We shall call success (S) one-second interval during which the BER  $\geq 10^{-3}$ , and failure (F) one-second interval during which the BER  $< 10^{-3}$ , if each second represents the outcome of Bernoulli trials. As the event B (Better), will mark the ten consecutive one-second interval F, a similar event W (Worse), will mark the ten consecutive one-second intervals S. The time between events B and W is called the time available (X) with its mean value E(X), and the time between events W and W is called the unavailability time (Y) with an average value E(Y) (Fig. 6). The mean time to occurrence of these events can be calculated using the theory of recurrent events [4]. According to this theory, the mean number of attempts for ten consecutive successes, i.e. average time for ten consecutive one-second intervals where the BER  $\geq 10^{-3}$ , is the mean recurrent time (expression 7.7, Section XIII in [9]):

$$E(X) = \frac{1 - S^{10}}{F \cdot S^{10}}. (9)$$

Let's now digital 64 kb/s channel be in a state of unavailability. In the same manner as for state of availability, mean time for the event F or the average time of unavailability is expressed by the following equation:

$$E(Y) = \frac{1 - F^{10}}{S \cdot F^{10}} \,. \tag{10}$$

On the basis of the mean values of availability time (E(X)) and unavailability time (E(Y)) we can easily calculate availability, A, and unavailability, UA = 1 - A, of digital 64 kb/s channels, according to equations (9) and (10) as the function of BER. Calculated values for A and UA as the function of BER are presented in Fig. 5.

$$A = E(X) / (E(X) + E(Y)), \tag{11}$$

$$UA = 1 - A = E(Y) / (E(X) + E(Y)).$$
(12)

Both presented methods give identical results –  $(E(X) = E(S_{av}))$  and  $E(Y) = E(S_{uav})$  – Figs. 4 and 5. If we now replace the probability of S in the expressions (9) and (10) by the relation from equation (8), we get a mean duration time of states of availability  $(E(X) = E(S_{av}))$ , expression (13)), and unavailability  $(E(Y) = E(S_{uav}))$ , expression (14)), of a digital channel, depending on the intensity of bit error in transmission over the digital channel.

$$E(X) = E(S_{av}) = \frac{1 - \left(1 - e^{-\lambda} \sum_{k=0}^{63} \frac{\lambda^{k}}{k!}\right)^{10}}{\left(e^{-\lambda} \sum_{k=0}^{63} \frac{\lambda^{k}}{k!}\right) \left(1 - e^{-\lambda} \sum_{k=0}^{63} \frac{\lambda^{k}}{k!}\right)^{10}},$$
(13)

$$E(Y) = E(S_{uav}) = \frac{1 - \left(e^{-\lambda} \sum_{k=0}^{63} \frac{\lambda^{k}}{k!}\right)^{10}}{\left(1 - e^{-\lambda} \sum_{k=0}^{63} \frac{\lambda^{k}}{k!}\right) \left(e^{-\lambda} \sum_{k=0}^{63} \frac{\lambda^{k}}{k!}\right)^{10}},$$
(14)

where is  $\lambda = 64~000 \cdot P_{bit}$ .

Proposed new definition of time availability and unavailability is motivated by the following reasoning, [10].

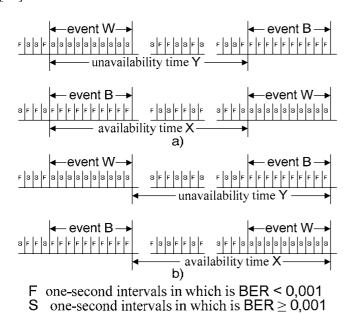


Fig. 6 – Definition of time availability and unavailability of digital 64 kb/s channel:
a) the recommendation G.821, Annex A;
b) the suggestion in this paper.

Let us suppose that the channel is in a state of availability and that in nine consecutive one-second intervals intensity of bit errors is greater than 10<sup>-3</sup>. What can be said about the availability of channel during the tenth one-second interval? According to the current definition of time availability, this channel is available in tenth seconds. If intensity of bit error in the tenth one-second interval is also greater than  $1.10^{-3}$ , then, retroactively, we can say that it was unavailable also in the tenth second. From this, simple example we can conclude that a valid definition does not uniquely determine the availability and that it is abstract. The same happens if the channel is in the state of unavailability.

We propose that the start time of unavailability is at the end of the tenth second of 10 consecutive one-second intervals in which the intensity of bit error is greater than 10<sup>-3</sup>. Thus, unavailability of channel begins after counting 10 one-second intervals in

which the intensity of bit error is greater than  $10^{-3}$ , not counting the one-second intervals back, and this period (event W) is not included in the time of unavailability of channels. Starting moment of the channel availability occurs at the end of the tenth second of 10 consecutive one-second intervals in which the intensity of bit errors is less than  $10^{-3}$ . Availability of channel begins after counting 10 one-second intervals in which the intensity of bit error is less than  $10^{-3}$ , also without including one-second intervals back, and this period (event B) does not contribute to the time availability of the channel, Fig. 6b). In short, the proposal is

that the beginning of the period of availability and unavailability be at the end of the events B and W, respectively, instead that the beginning of time of availability and unavailability, be at the beginning of these events, B and W (Fig. 6).

#### 7. CONCLUSION

Availability, A, for systems and system components, is defined as the ratio of mean time spent in functional state and a total running time.

Availability of digital channels is the probability that in some instant of time, the digital channel is completely correct for the transmission of digital messages (Section 2, expression (2)).

In Section 6 we propose two original calculation methods of the availability of digital 64 kb/s channels in the presence of random errors. The results of calculating a mean time duration of the channel function to alarm detection and a mean time duration of the channel to the exit from alarm condition, are quite reliable and may be used in future work, [10]. The average time of availability and unavailability availability and unavailability of digital 64 kb/s channel (with uniformly distributed errors) can be precisely calculated taking into account the explanation from the recommendation ITU-T G.821 (Annex A).

The proposed definition also excludes retroactive calculation of the start of availability and unavailability, i.e. allows the channel is considered available or unavailable at the time of determining the availability or unavailability, and also will not change the ways of calculation and the results of calculation. This means that it is closer to practical use because, for example, it allows us to include safeguards against errors and alarm signaling in the moment of unavailability determination, which is not possible according to the existing definition.

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