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SOIL NONLINEARITY IMPACT ON SITE-STRUCTURE RESONANCE

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This paper presents an evaluation method for loading dependence of the site natural period using a nonlinear Kelvin-Voigt model and a combination of site-laboratory available information. Then, some consequences of this dependence on site-structure resonance are briefly presented.

Key words: soil dynamic degradation, nonlinear site natural period, nonlinear oscillating soil-structure system, soil-structure resonance.

1. INTRODUCTION

The significant dynamic strength degradation of the site materials (between 20–80%, in terms of loading magnitude) [2, 4, 8] lead to a substantial grows (in the same percent) of the natural period values of the site materials. Therefore the site natural period becomes strain, stress or loading function [5, 6] and from this reason all site-structure oscillating systems are nonlinear systems and the response of nonlinear system in resonant conditions can not be described by a linear model.

The site natural period variability is proved by seismic recordings during earthquakes with different magnitudes. The recorded dominant periods and maximum accelerations show a doubtless dependence of the natural site period on earthquake magnitude [12, 13].

In order to assess the natural period dependence on earthquake magnitude one must have all information about site natural periods from in situ seismic measurements. Unfortunately only for a few sites there are multiple seismic measurement values for different magnitudes beginning with low intensity events until the strong earthquakes. For most sites only seismic measurements for low and moderate events are available. For this reason, the usual method for site natural period determination is based on the "quarter length formula" $T_g = 4H / v_s$, where H is the site depth and v_s is the shear wave velocity. This formula treats the site as semi-infinite linear elastic space in contradiction with mechanical reality and gives a unique natural period value in contradiction with earthquake recordings [6, 12, 13].

In default of complete and reliable site information, this paper proposes a method for the nonlinear variability evaluation of the site natural period by a combination of available in situ and laboratory data. Such combination allows us to estimate a nonlinear dependence of the natural site period in terms of loading as $T_g = T_g\left(PGA\right)$ or $T_g = T_g\left(M_{GR}\right)$, where PGA (peak ground acceleration) and M_{GR} (magnitude Guttenberg-Richter) are usual seismic input characteristics.

Finally, we briefly present some effects on resonant avoidance strategy induced by the nonlinear form of the site natural period.

2. NATURAL SITE PERIODS FROM SEISMIC RECORDINGS

The seismic data recording during Vrancea earthquakes with different magnitudes shows a doubtless dependence of the natural periods and maximum accelerations on earthquake magnitude as is illustrated by the examples from Figs. 2.1, 2.2 and 2.3, where the data recorded at some Bucharest seismic station is presented and where the estimation of the maximum predicted event was added [12].

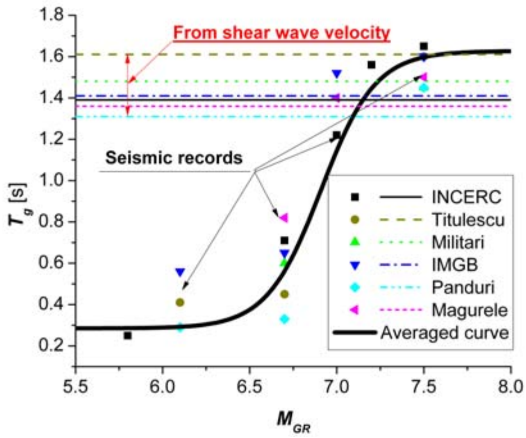

Fig. 2.1 – Nonlinear tendency of site natural periods.

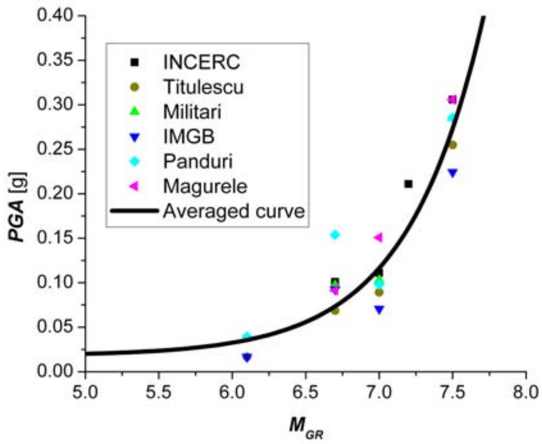

Fig. 2.2 – Nonlinear tendency of maximum accelerations.

One can see from these examples the obvious nonlinear increase of the site natural period T_g and of maximum acceleration PGA *versus* the increasing earthquake magnitude. Certainly, the different local conditions from the seismic station sites lead to a large dispersion of the recorded natural period values. But, using only data recorded at the same seismic station this dispersion become acceptable (Fig. 2.3).

The direct evaluation of the nonlinear natural period functions in the form $T_g = T_g\left(M_{GR}\right)$ or $T_g = T_g\left(\text{PGA}\right)$ is an adequate method but is not always possible. The seismic station network is not so expanded and only in a few stations the recorded events are appropriate for determination of natural period functions with a reasonable precision. Only for INCERC station there are multiple seismic recorded values with different magnitudes beginning with low events until the strong March 4, 1977 earthquake.

For this reason, the usual method for site natural period determination is based on the "quarter length formula" $T_g = 4H / v_s$, where H is the site depth and v_s is the shear wave velocity. This formula treats the site as semi-infinite linear elastic space in contradiction with mechanical reality and gives a unique natural period value in contradiction with earthquake recordings (Figs. 2.1 and 2.3). Furthermore, as can see from Figs. 2.1 and 2.3 the natural period values obtained by using shear wave velocity for low excitation input are located in the strong earthquake range. This paradox was point out [6, 13] but until now remains without a reasonable explanation.

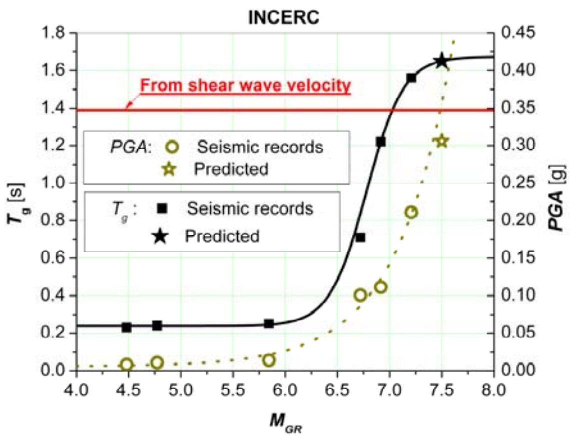

Fig. 2.3 – Seismic records at INCERC site.

3. NONLINEAR BEHAVIOR OF SITE MATERIALS

As known, the site materials, soils or rocks, are nonlinear materials with a dynamic behavior strongly dependent of loading level and this behavior affects the whole dynamic response including the system natural period values [2, 4, 6, 8].

Assuming that the geological site materials are nonlinear viscoelastic materials in the previous author's papers [2, 3, 4] this nonlinear behavior was modeled by using a nonlinear Kelvin-Voigt model which describes the variation of material mechanical parameters (shear modulus G and damping ratio ζ) in terms of shear strain invariant γ: $G = G\left(\gamma\right)$, $\zeta = \zeta\left(\gamma\right)$. Both these material function can be complete quantify by resonant column tests data [2, 3, 14]. As it can be seen from Figs. 3.1 and 3.2 the rigidity of the site materials may display during the strong events an important dynamic strength degradation associated with a substantial increase of the damping capacity.

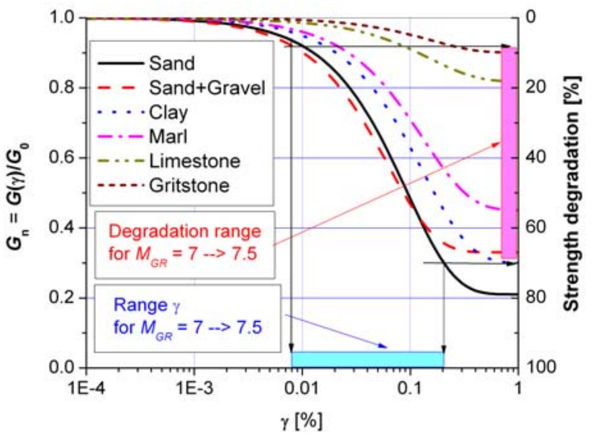

Fig. 3.1 – Strength degradation.

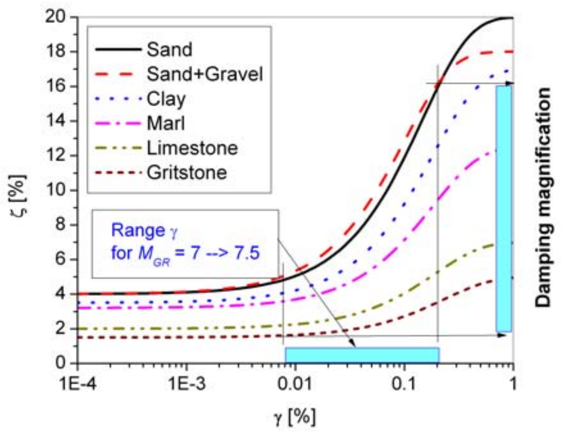

Fig. 3.2 – Damping magnification.

Due to these nonlinear characteristics of the site materials, every site oscillating system become a nonlinear system and for every site instead of a unique linear natural period value T_g a function $T_g = T_g(\bullet)$ in terms of strain, stress or loading input must be evaluated [5, 6]. For full quantification of such function we will prove in the next that the resonant column data can have an important contribution.

4. NATURAL PERIOD OF RESONANT COLUMN SPECIMEN

From resonant column test under harmonic torsional inputs with different amplitudes $M^i = M_0^i \sin\omega t$ we can obtain the corresponding strain level γ_i, the modulus-function value G^i and damping value ζ^i [4], [14]. The shear-modulus value G^i is obtained using the relationship:

$$G^i = \rho\left(v_s^i\right)^2 = \rho\left(\frac{\omega_0^i h}{\Psi}\right)^2 \quad \text{with} \quad \Psi = \sqrt{R - \frac{1}{3}R^2 + \frac{4}{45}R^3}\,, \tag{4.1}$$

where ρ is the mass density of specimen, v_s^i and ω_0^i are the shear wave velocity and the specimen natural pulsation at level *i*, *h* is the specimen height and Ψ is the root of torsional frequency equation with analytical form in terms of the ratio *R* between torsional inertia of the specimen and the torsional inertia of the top cap system: $R = J / J_{top}$.

After several tests with different strain level $\gamma_i\,(i = 1, 2 \ldots n)$ the shear-modulus function $G = G(\gamma)$ and the damping function $\zeta = \zeta(\gamma)$ can be obtained in the normalized forms:

$$G(\gamma) = G_0 \cdot G_n(\gamma), \quad \text{with}: G_0 = G(\gamma)\big|_{\gamma=0} \text{ and } G_n(\gamma) = G(\gamma)/G_0, \tag{4.2}$$

$$\zeta = \zeta(\gamma) = \zeta_0 \cdot \zeta_n(\gamma), \quad \text{with}: \zeta_0 = \zeta(\gamma)\big|_{\gamma=0} \text{ and } \zeta_n(\gamma) = \zeta(\gamma)/\zeta_0, \tag{4.3}$$

where G_0 is the initial value of the shear modulus-function, $G_n(\gamma)$ is the normalized shear-modulus function ζ_0 is the initial damping value and $\zeta_n(\gamma)$ is the normalized damping function.

The specimen natural period for a level γ_i is:

$$T^i = \frac{2\pi}{\omega_0^i} = \frac{2\pi h\sqrt{\rho}}{\Psi} \cdot \frac{1}{\sqrt{G^i}} \tag{4.4}$$

and using eq.(4.2), *the nonlinear natural period function* of the soil specimen results:

$$T_g(\gamma) = T_0 \cdot T_n(\gamma) \quad \text{with:} \quad T_0 = T_g(\gamma)\big|_{\gamma=0} = \frac{2\pi h\sqrt{\rho}}{\Psi} \cdot \frac{1}{\sqrt{G_0}} \quad \text{and} \quad T_n(\gamma) = \frac{T_g(\gamma)}{T_0} = \frac{1}{\sqrt{G_n(\gamma)}}. \tag{4.5}$$

We mention that the natural periods obtained by resonant column test in the form (4.5) is the natural periods of the single degree of freedom oscillating system composed by a single mass (the vibration device) supported by a spring and a damper represented by the specimen. This system is much different in comparison with site-structure system. But, as can see from eq. (4.5) the physical and geometrical sample properties (h, ρ, J, J_{top}) are included only in the initial value T_0. Thus, the resonant column test can offer accurate data for obtaining only the nonlinear dependence of the normalized natural period $T_n = T_n(\gamma)$.

5. NORMALIZED NATURAL PERIODS IN TERMS OF LOADING

For practical applications it is necessary to determine the normalized natural period in terms of loading amplitude usually described by *peak ground acceleration* (*PGA*). For this conversion – $T_n = T_n(\gamma)$ into $T_n = T_n(PGA)$ – one can use the numerical simulation of the resonant column specimen behavior, modeled as nonlinear Kelvin-Voigt model subjected to abutment motion $\ddot{x}_g(t) = \ddot{x}_g^0 \sin\omega t$ with different acceleration amplitude $\ddot{x}_g^0$. In this loading case, the motion equation reads as [4]:

$$\ddot{x} + 2\omega_0\zeta(x)\cdot\dot{x} + \omega_0^2 G_n(x)\cdot x = -\ddot{x}_g^0 \sin\omega t. \tag{5.1}$$

Using the change of variable $\tau = \omega_0 t$ and introducing the new time function $\varphi(\tau) = x(t) = x(\tau/\omega_0)$ yields we can obtain the dimensionless form eq. (5.1) [3]:

$$\varphi'' + C(\varphi)\cdot\varphi' + K(\varphi)\cdot\varphi = \mu\sin\upsilon\tau, \tag{5.2}$$

where the superscript accent denotes the time derivative with respect to τ, and:

$$C(\varphi) = \frac{c(x)}{m\omega_0} = 2\zeta(x) \quad ; \quad K(\varphi) = \frac{k(x)}{m\omega_0^2} = G_n(x) \quad ; \quad \mu = \frac{\ddot{x}_g^0}{\omega_0^2} = x_{static} \quad ; \quad \upsilon = \frac{\omega}{\omega_0}. \tag{5.3}$$

The steady-state solution of the equation (5.2) can be numerically obtained using a computer program based on Newmark algorithm [3, 7, 9]. The solution can be written in the form: $\varphi(\tau,\upsilon,\mu) = \mu\Phi(\upsilon,\mu)\sin(\upsilon\tau-\psi)$, where $\Phi(\upsilon,\mu)$ is *the nonlinear magnification function:*

$$\Phi(\upsilon,\mu) = \frac{\max\limits_{\tau}\left[\varphi(\tau,\upsilon,\mu)\right]}{\mu} = \frac{x_{dynamic}}{x_{static}} \tag{5.4}$$

a ratio of maximum dynamic amplitude $\varphi_{\max} \equiv x_{dynamic}$ to static displacement $\mu = x_{static}$.

By numerical simulations with different values of normalized loading amplitudes μ we can obtain a set of nonlinear magnification functions $\Phi(\upsilon) = \Phi(\upsilon;\mu)\big|_{\mu=ct.}$. Figure 5.1 presents some magnification functions obtained by using the material functions $G_n(x)$ and $\zeta(x)$ of a clay specimen tested in the resonant column [2].

Because $\upsilon = \omega/\omega_0 = T_0/T = 1/T_n$ one can obtain the magnification functions Φ in terms of normalized period T_n (Fig. 5.2) and because $\mu = \ddot{x}_g^0/\omega_0^2 = (g \cdot \mathrm{PGA})/\omega_0^2$ a relationship $T_n = T_n(\mathrm{PGA})$ results (Fig. 5.3). For instance, in Fig. 5.4 some functions $T_n = T_n(\mathrm{PGA})$ for different site materials are given.

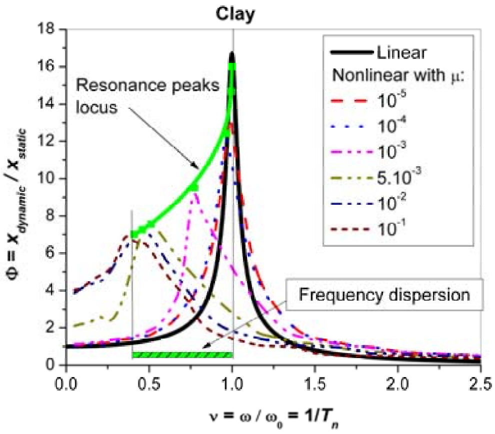

Fig. 5.1 – Nonlinear magnification functions in terms of normalized frequency (for a clay specimen).

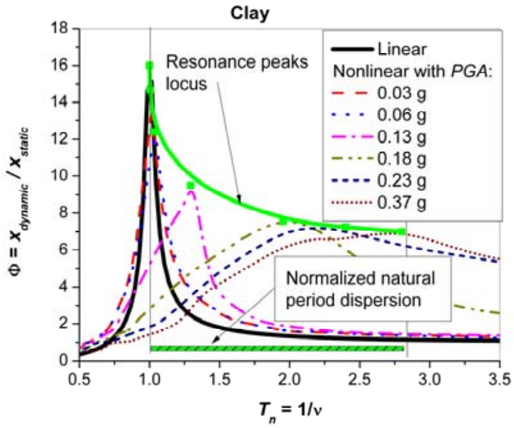

Fig. 5.2 – Nonlinear magnification functions in terms of normalized periods (for a clay specimen).

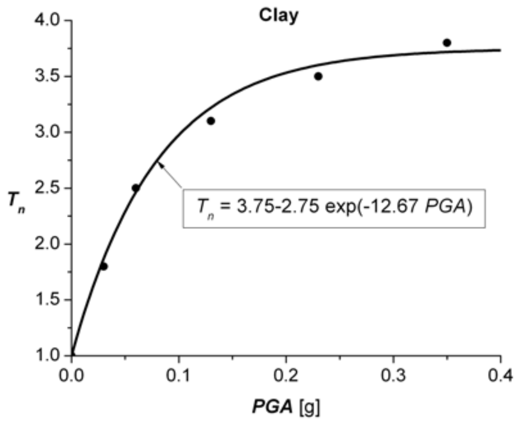

Fig. 5.3 – Relationship $T_n = T_n(\mathrm{PGA})$.

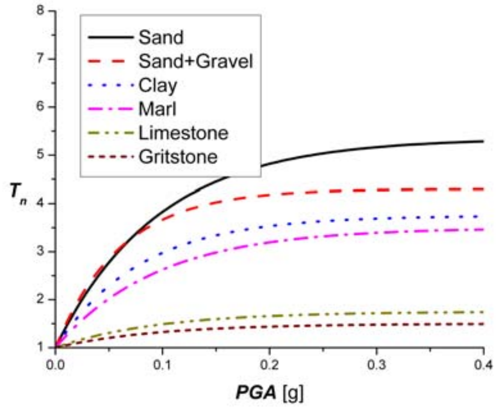

Fig. 5.4 – Some functions $T_n = T_n(\mathrm{PGA})$.

6. SITE NORMALIZED NATURAL PERIOD

For the evaluation of the entire site normalized natural periods first one must determine from resonant column tests the nonlinear variation T_n^i for each site stratum, and then one can obtain the average natural period variation for the entire site layers T_n^{av} as the average of the strata normalized natural period T_n^i weighted with its thickness h_i [6, 13]:

$$T_n^{av} = \left(\sum T_n^i \times h_i\right) / \sum h_i \,. \tag{6.1}$$

This method was validated using the site emplacement of the seismic station INCERC with known stratification [1]. First, for each constituent layer the material functions $G = G(\gamma)$ and $\zeta = \zeta(\gamma)$ were estimated and by numerical simulation, some functions $T_n^i = T_n^i(PGA)$ one for each stratum i was obtained.

Then, for some PGA values (0.05, 0.10, 0.15, 0.20, 0.25 and 0.30 g) using eq. (6.1) the site natural period values $T_n^{av}\big|_{\mathrm{PGA}=ct.}$ was obtained and by statistical fit from these period values a normalized averaged natural period function $T_n^{av} = T_n^{av}(\mathrm{PGA})$ results.

This method was validated using the necessary data (material functions and seismic records) from seismic station INCERC site.

In Fig. 6.1 the validation result is given, by comparison between nonlinear function $T_n^{av} = T_n^{av}(\mathrm{PGA})$ obtained with the aid of resonant column data and the same function directly obtained from seismic measurements $T_n^{rec} = T_n^{rec}(\mathrm{PGA})$ [22]. As can see from Fig. 6.1 the differences between seismic records and resonant column simulations are acceptable.

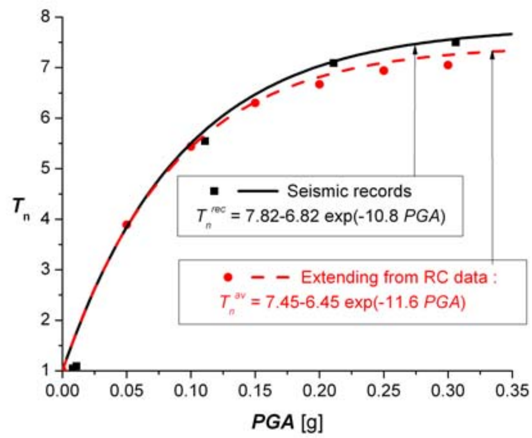

Fig. 6.1 – Dependence T_n – *PGA* provided both resonant column data and seismic records.

7. T_0 ESTIMATION FROM SEISMIC RECORDS

We remember that only resonant column data is not enough for complete determination of the natural period function $T_g(\mathrm{PGA}) = T_0 \cdot T_n(\mathrm{PGA})$ and besides of the normalized function $T_n^{av} = T_n^{av}(\mathrm{PGA})$ given by the resonant column data, the initial value T_0 obtained from seismic measurements it is necessary.

When T_0 value is not available or is to difficult to obtain from processing of seismic records one can use any known site pair of values $\left(T_g^{kn}, \mathrm{PGA}^{kn}\right)$. In this case, the T_0 may be obtained by translation the normalized resonant column curve $T_n^{av} = T_n^{av}(\mathrm{PGA})$ in any known "point" $\left(T_g^{kn}, \mathrm{PGA}^{kn}\right)$ of the $\left(T_g, \mathrm{PGA}\right)$ space:

$$T_0 = \frac{T_g^{av}\big|_{\mathrm{PGA}=\mathrm{PGA}^{kn}}}{T_n\big|_{\mathrm{PGA}=\mathrm{PGA}^{kn}}}. \tag{7.1}$$

Finally, the calculated form of the site natural period function becomes:

$$T_g^{calc}(\mathrm{PGA}) = T_0 \cdot T_n^{av}(\mathrm{PGA}). \tag{7.2}$$

The validation of this method can be done by comparison between T_g^{calc} and T_g^{rec} curves both obtained from the same site. Thus, in Fig. 7.1 such comparison is given using the laboratory and in situ data for INCERC site. The calculated curve $T_g^{calc} = T_g^{calc}(\mathrm{PGA})$ was obtained by translation of the normalized curve $T_n^{av} = T_n^{av}(\mathrm{PGA})$ into the 1977 earthquake "point" $\left(\mathrm{PGA} = 0.21\mathrm{g}\ ;\ T_g = 1.56\mathrm{s}\right)$ and the recorded curve $T_g^{rec} = T_g^{rec}(PGA)$ was obtained directly from seismic measurements processing.

Also, the translation can be done and in another measurement points. Thus, in Fig. 7.2 the normalized curve $T_n^{av} = T_n^{av}(\mathrm{PGA})$ was translated in three known points of the strong events: 1977 point $\left(\mathrm{PGA} = 0.21\mathrm{g}\ ;\ T_g = 1.56\mathrm{s}\right)$, 1986 point $\left(\mathrm{PGA} = 0.11\mathrm{g}\ ;\ T_g = 1.22\mathrm{s}\right)$ and in the point $\left(PGA = 0.306\mathrm{g}\ ;\ T_g = 1.65\mathrm{s}\right)$ corresponding to maximum predicted event. In all these cases the differences between calculated and measured curves was reasonable: $\Delta T_g = \left|T_g^{rec} - T_g^{calc}\right| \le 0.1$ s .

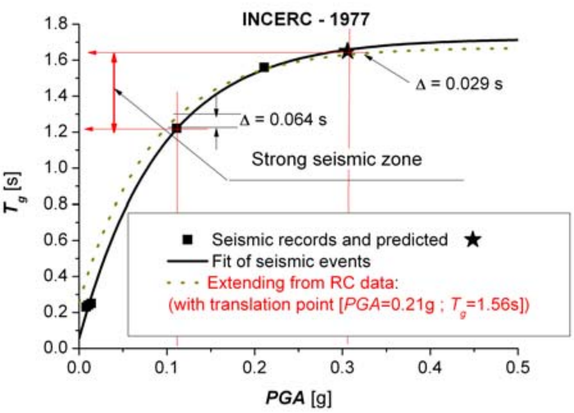

Fig. 7.1 – Translation in the 1977 point.

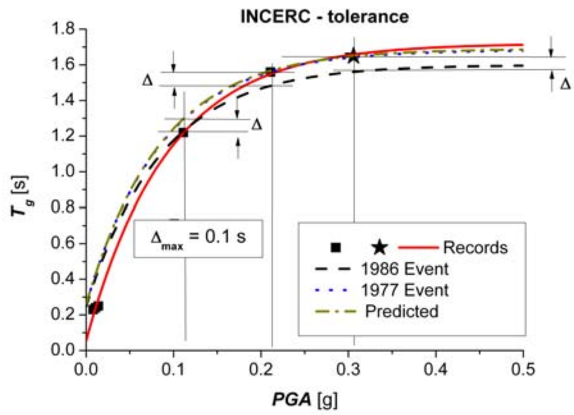

Fig. 7.2 – Translation in some strong seismic points.

8. RESONANCE IN NONLINEAR CONDITIONS

8.1. Overestimation trap

Treating the site as linear oscillator means a unique natural period as provided the low loading methods (the wave velocity, H/V method, or else [12]). Even if this value is overestimated the site-structure resonance avoidance can not be assured. In the usual strength design an overvaluation of the external loadings assures a safe structural response to inferior loadings. But, in the resonance case, the overestimations of the natural site period values do not assure the resonance avoidance.

Thus, for example, if for INCERC site it is considered only a unique site natural period with the maximum predicted values $T_g = 1.56$ s, it seems that for this site the resonance danger arises only for buildings with the same natural period. Therefore, for a building designed with an inferior natural period $T_s = 1.4$ s, placed on this site the occurrence of resonance is unlikely. But, if we take into account the loading dependence, the natural site period of $T_g = 1.4$ s can be reach under inferior earthquake loading as $M_{GR} = 7.1$ and the resonant magnification becomes quite possible (Fig. 8.1).

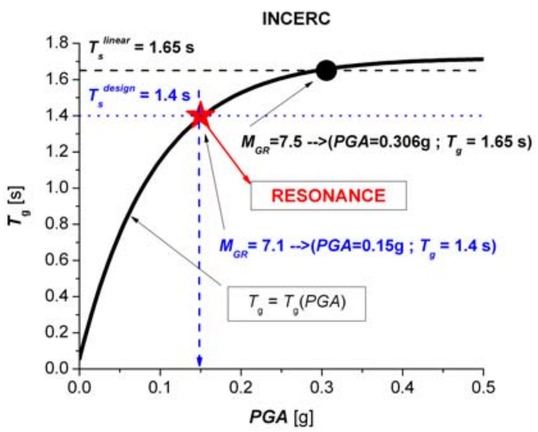

Fig. 8.1 – Overestimation trap (site INCERC).

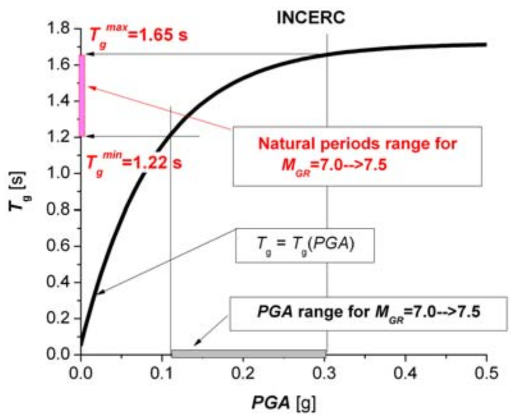

Fig. 8.2 – Dangerous resonant zone (site INCERC).

8.2. Dangerous periods range

The post-earthquake observations show that seismic loading level (magnitude, PGA) play an important role in the resonance consequences because only strong events (over $M_{GR} \geq 7$) may lead to an important structural damages which can growth until structural collapse [12]. Thus, for safe avoidance of resonance is necessary to define foe each site a natural period value range corresponding to strong earthquakes.

As example, for seismic site INCERC the seismic recordings show that for magnitudes between $M_{GR} = 7$ and maximum expected magnitude $M_{GR} = 7.5$ correspond an acceleration range $\mathrm{PGA} \simeq 0.1 \to 0.3$ g and a dangerous natural period range $T_g \simeq 1.22 \to 1.65$ s (Fig. 8.2). So, it is not recommended to use this site for buildings with natural period T_s in the same range $T_s \simeq 1.22 \to 1.65$ s .

9. CONCLUDING REMARKS

- The seismic data recording during Vrancea earthquakes with different magnitudes shows a doubtless dependence of the site natural periods and maximum accelerations on earthquake magnitude.
- The "quarter length formula" ($T_g = 4H / v_s$) treats the site as linear elastic space in contradiction with mechanical reality and gives a unique natural period value in contradiction with earthquake recordings.
- The nonlinear natural site period – $T_g\left(PGA\right) = T_0 \cdot T_n\left(\mathrm{PGA}\right)$ – can be obtained from recorded seismic data if these data cover the entire expected *PGA* value range.
- In default of complete and reliable site information, the site nonlinear natural period can be done by a combination of available in situ – T_0 – and resonant column data - $T_n\left(\mathrm{PGA}\right)$.
- Using only a linear natural period value T_g^{linear} by neglecting nonlinear variability, the resonance avoidance strategy may be compromise because the resonance may come for different building natural periods even if $T_s \neq T_g^{linear}$.
- Due to nonlinear variability of the site natural periods it is advisable to avoid a building emplacement if the structural natural period T_s is situated in the same range as site natural period T_g given by strong earthquakes (usual with $M_{GR} \geq 7$).

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