THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 15, Number 2/2014, pp. 107–114

DOUBLY STOCHASTIC MODELS WITH ASYMMETRIC GARCH ERRORS

Muhammad SHERAZ

University of Bucharest, Faculty of Mathematics and Computer Science, Bucharest, Romania
E-mail: Muhammad_sheraz84@hotmail.com

The methods addressing volatility in computational finance and econometrics have been recently reported in financial literature. Recently Peiris et al. [8] have introduced doubly stochastic volatility models with GARCH innovations. Random coefficient autoregressive sequences are special case of doubly stochastic time series. In this paper, we consider doubly stochastic stationary time series with asymmetric GARCH errors. Some general properties of process, like variance and kurtosis are derived.

Key words: GARCH processes, AGARCH-(I)-(0,1), doubly stochastic time series, RCA(1), RCA-MA(1), Sign-RCA-MA(1).

1. INTRODUCTION

Recently, there has been growing development in the use of nonlinear volatility models in financial literature. Black-Scholes option pricing models has some contradictory assumptions, such as constant volatility and normally distributed log returns. In many financial time series, empirical studies reveal some facts such as stock returns and foreign exchange rates that exhibit leptokurtosis and stochastic volatility. Any distribution can be characterized by a number of features such as mean, variance, skewness and kurtosis. The measure of kurtosis is considered as whether the data are peaked or flat relative to Normal distribution.

The concept of stochastic volatility for financial time series, the autoregressive conditionally heteroscedastic (ARCH) models, was first studied by Engle [4], and its generalization, the GARCH models by Bollerslev [2]. The GARCH models assume symmetric effects on volatility, that is good and bad news have same effect on volatility, which is a shortfall of these models. The asymmetric GARCH (AGARCH) by Engle and Ng and various other nonlinear GARCH extensions have been proposed to capture asymmetric effects [3, 5]. Random coefficient autoregressive (RCA) time series were introduced by Nicholls and Quinn [7] and some of their properties were studied by Appadoo et al [1]. Thavaneswaran et al. [8] have also studied some RCA time series and volatility modeling. In this paper, we derive the kurtosis for various classes of doubly stochastic models with asymmetric GARCH innovations. Recently Peiris et al. [8] have introduced doubly stochastic volatility models with GARCH innovations. In Section 2 some random coefficient autoregressive time series have been discussed. In Section 3 some doubly stochastic models with asymmetric GARCH innovations are introduced and we derive a formula for kurtosis in terms of model parameters.

2. RANDOM COEFFICIENT AUTOREGRESSIVE TIME SERIES

Random coefficient autoregressive sequences are special case of doubly stochastic time series. Some random coefficient autoregressive time series are as follows:

$$y_t = \left(\phi + b_t\right) + \varepsilon_t, \quad t \in \mathbb{Z}, \tag{1}$$

$$y_t = (\phi + b_t) + \varepsilon_t + \theta\varepsilon_{t-1}, \quad t \in \mathbb{Z}, \tag{2}$$

$$y_t = (\phi + b_t + \Phi s_t) + \varepsilon_t + \theta\varepsilon_{t-1}, \quad t \in \mathbb{Z}, \tag{3}$$

where $s_t = \begin{cases} 1, & \text{if} \quad y_t > 0, \\ 0, & \text{if} \quad y_t = 0, \\ -1, & \text{if} \quad y_t < 0. \end{cases}$

In (1–3) we have Random coefficient autoregressive (RCA(1)), Random coefficient autoregressive-moving average (RCA-MA(1)), Sign-RCA-MA (1), respectively. The mean, variance and kurtosis for the above processes have been discussed in [10]. The two necessary and sufficient conditions for second order stationarity of $\{y_t\}$ are as follows:

$$\text{i.} \quad \begin{pmatrix} b_t \\ \varepsilon_t \end{pmatrix} \approx N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_b^2 & 0 \\ 0 & \sigma_\varepsilon^2 \end{pmatrix}\right),$$

$$\text{ii.} \quad \phi^2 + \sigma_b^2 < 1.$$

Here $\{b_t\}$ and $\{\varepsilon_t\}$ are errors in the model, while ϕ, Φ and s_t are parameters. Since $E(s_t^2) = 1$ and $E(s_t^4) = 1$, we can calculate the kurtosis. For more details see [10].

A process of the following form was considered in [9]:

$$b_{t+1} = ab_t + (1 + b_t)v_{t+1}, \quad t \in \mathbb{Z}. \tag{4}$$

Recently Peiris et al [8] have studied a doubly stochastic model of the form

$$\begin{aligned} y_t &= (\phi + b_t) + \varepsilon_t, \quad t \in \mathbb{Z}, \\ b_{t+1} &= ab_t + (1 + b_t)v_{t+1}, \quad t \in \mathbb{Z}. \end{aligned} \tag{5}$$

The moments for the model have been recently calculated [8]. Here ε_t is an identically distributed independent sequence of variables with mean zero and variance $\sigma_{\varepsilon_t}^2$.

$$\text{i.} \quad \begin{pmatrix} v_t \\ \varepsilon_t \end{pmatrix} \overset{N}{\sim} \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\upsilon^2 & 0 \\ 0 & \sigma_\varepsilon^2 \end{pmatrix}\right),$$

$$\text{ii.} \quad 1 - a^2 - \sigma_v^2 < 1,$$

$$\text{iii.} \quad 1 - a^2 - 2\sigma_v^2 - \phi^2 + \phi^2 a^2 + \phi^2\sigma_v^2 < 1.$$

In (5) the observed random variable is modeled in two steps. First, the distribution of the observed outcome is represented in a standard way and at a second step, b_t is treated as being itself a random variable.

THEOREM 2.1. *Consider* (5) *with conditions* i, ii *and* iii *then we have following results*:

$$1.\ E(b_t) = 0 \qquad 2.\ E(b_t^2) = \frac{\sigma_\upsilon^2}{1 - a^2 - \sigma_\upsilon^2};$$

$$3.\ E(b_t^3) = 6a\frac{\sigma_\upsilon^4}{(1 - a^3 - 3a\sigma_\upsilon^2)(1 - a^2 - \sigma_\upsilon^2)};$$

$$4.\ E(b_t^4) = \frac{3\sigma_\upsilon^4(-1 - a^2 - 5\sigma_\upsilon^2 + a^3 + a^5 - 16a^3\sigma_\upsilon^2 + 3a\sigma_\upsilon^2 - 9a\sigma_\upsilon^4)}{(-1 + a^3 + 3a\sigma_\upsilon^2)(-1 + a^2 + \sigma_\upsilon^2)(-1 + 6a^2\sigma_\upsilon^2 + a^4 + 3\sigma_\upsilon^4)};$$

$$5.\ K^{(b)} = 3\left(1 - a^2 - \sigma_\upsilon^2\right)\left(\frac{1 + a^2 + 5\sigma_\upsilon^2 - a^3 - a^5 + 16a^3\sigma_\upsilon^2 - 3a\sigma_\upsilon^2 + 9a\sigma_\upsilon^4}{\left(1 - a^3 - 3a\sigma_\upsilon^2\right)\left(1 - 6a^2\sigma_\upsilon^2 - a^4 - 3\sigma_\upsilon^4\right)}\right);$$

$$6.\ E\left(y_t^2\right) = \frac{\sigma_\varepsilon^2\left(1 - a^2 - \sigma_\upsilon^2\right)}{1 - a^2 - 2\sigma_\upsilon^2 - \phi^2 + \phi^2 a^2 + \phi^2\sigma_\upsilon^2};$$

$$7.\ E\left(y_t^4\right) = \frac{3\sigma_\varepsilon^4 + 6\sigma_\varepsilon^2\left(E\left(b_t^2\right) + \phi^2\right)E\left(y_{t-1}^2\right)}{\left(1 - \phi^4 - E\left(b_t^4\right) - 6\phi^2 E\left(b_t^2\right) - 4\phi E\left(b_t^3\right)\right)};$$

$$8.\ K^{(y)} = 3\left(\frac{1 - \phi^4 - 2\phi^2 E\left(b_t^2\right) - \left(E\left(b_t^2\right)\right)^2}{\left(1 - \phi^4 - E\left(b_t^4\right) - 6\phi^2 E\left(b_t^2\right) - 4\phi E\left(b_t^3\right)\right)}\right).$$

Proof. See details in [8].

3. DOUBLY STOCHASTIC MODELS WITH ASYMMETRIC GARCH ERRORS

In this section we discuss doubly stochastic volatility models with asymmetric GARCH errors. The following theorem provides the moments of doubly stochastic models with AGARCH- (I)-(0, 1).

THEOREM 3.1. *Consider the doubly stochastic volatility process* (5) *with* AGARCH (I) – (0,1) *errors of the following form then we have results* 1, 2 *and* 3 *as follows*:

$$y_t = \left(\phi + b_t\right)y_{t-1} + \varepsilon_t,\ \ t \in \mathbb{Z};$$

$$b_{t+1} = ab_t + \left(1 + b_t\right)v_{t+1},\ \ t \in \mathbb{Z};$$

$$\varepsilon_t = \sigma_t z_t;$$

$$\sigma_t^2 = \omega + \alpha\left(\varepsilon_{t-1} + r\right)^2$$

$$1.\ E\left(y_t^2\right) = \frac{\left(\omega + \alpha r^2\right)\left(1 - a^2 - \sigma_\upsilon^2\right)\sigma_z^2}{\left(1 - \alpha\sigma_z^2\right)\left(1 - a^2 - 2\sigma_\upsilon^2 - \phi^2 + \phi^2 a^2 + \phi^2\sigma_\upsilon^2\right)};$$

$$2.\ E\left(y_t^4\right) = \frac{3\sigma_z^4 E\left(\sigma_t^4\right) + 6\sigma_z^2\left(E\left(b_t^2\right) + \phi^2\right)E\left(\sigma_t^2\right)E\left(y_{t-1}^2\right)}{\left(1 - \phi^4 - E\left(b_t^4\right) - 6\phi^2 E\left(b_t^2\right) - 4\phi E\left(b_t^3\right)\right)};$$

$$3.\ K^{(y)} = \frac{E\left(y_t^4\right)}{\left(E\left(y_t^2\right)\right)^2},\ K^{(y)} \textit{is kurtosis of the process.}$$

Proof. 1 $E\left(y_t^2\right) = E\left(\left(\phi + b_t\right)y_{t-1} + \varepsilon_t\right)^2$

$$E\left(y_t^2\right) = E\left(\phi^2 y_{t-1}^2 + b_t^2 y_{t-1}^2 + \varepsilon_t^2 + 2\phi b_t y_{t-1} + 2\phi\varepsilon_t y_{t-1}\right) = \phi^2 E\left(y_{t-1}^2\right) + E\left(b_t^2\right)E\left(y_{t-1}^2\right) + E\left(\sigma_t^2\right)\sigma_z^2;$$

$$= \frac{E\left(\sigma_t^2\right)\sigma_z^2}{\left(1 - \phi^2 - E\left(b_t^2\right)\right)} = \frac{\dfrac{\left(\omega + \alpha r^2\right)\sigma_z^2}{\left(1 - \alpha\sigma_z^2\right)}}{\left(1 - \phi^2 - E\left(b_t^2\right)\right)};$$

$$E\left(y_t^2\right)=\frac{\left(\omega+\alpha r^2\right)\left(1-a^2-\sigma_\upsilon^2\right)\sigma_z^2}{\left(1-\alpha\sigma_z^2\right)\left(1-a^2-2\sigma_\upsilon^2-\phi^2+\phi^2a^2+\phi^2\sigma_\upsilon^2\right)};$$

$$\begin{aligned}
2.\ E\left(y_t^4\right)&=E\left(\left(\phi+b_t\right)y_{t-1}+\varepsilon_t\right)^4=\\
&=\phi^4E\left(y_{t-1}^4\right)+E\left(b_t^4\right)E\left(y_{t-1}^4\right)+3\sigma_z^4E\left(\sigma_t^4\right)+6\phi^2E\left(b_t^2\right)E\left(y_{t-1}^4\right)+\\
&\quad+4\phi E\left(b_t^3\right)E\left(y_{t-1}^2\right)+6E\left(b_t^2\right)E\left(y_{t-1}^2\right)E\left(\varepsilon_t^2\right)+12E\left(b_t\right)E\left(y_{t-1}^2\right)E\left(\varepsilon_t\right)=\\
&=\frac{\sigma_z^4E\left(\sigma_t^4\right)+6\sigma_z^2E\left(b_t^2\right)E\left(y_{t-1}^2\right)E\left(\sigma_t^2\right)+6\sigma_z^2\phi^2E\left(y_{t-1}^2\right)E\left(\sigma_t^2\right)}{\left(1-\phi^4-E\left(b_t^4\right)-6\phi^2E\left(b_t^2\right)-4\phi E\left(b_t^3\right)\right)}=\\
&=E\left(y_t^4\right)=\frac{3\sigma_z^4E\left(\sigma_t^4\right)+6\sigma_z^2\left(E\left(b_t^2\right)+\phi^2\right)E\left(\sigma_t^2\right)E\left(y_{t-1}^2\right)}{\left(1-\phi^4-E\left(b_t^4\right)-6\phi^2E\left(b_t^2\right)-4\phi E\left(b_t^3\right)\right)};
\end{aligned}$$

$$E\left(y_t^2\right)=\frac{\left(\omega+\alpha r^2\right)\left(1-a^2-\sigma_\upsilon^2\right)\sigma_z^2}{\left(1-\alpha\sigma_z^2\right)\left(1-a^2-2\sigma_\upsilon^2-\phi^2+\phi^2a^2+\phi^2\sigma_\upsilon^2\right)};\ E\left(y_t^4\right)=\frac{f_1+f_2}{f_3};$$

$$f_1=3\sigma_z^4\frac{\left(\omega+\alpha r^2\right)}{\left(1-3\alpha^2\right)\left(1-\alpha\right)}\left\{\left(\omega+\alpha r^2\right)\left(1+\alpha\right)+4\alpha^2r^2\right\};$$

$$E\left(\sigma_t^4\right)=\frac{\left(\omega+\alpha r^2\right)}{\left(1-3\alpha^2\right)\left(1-\alpha\right)}\left\{\left(\omega+\alpha r^2\right)\left(1+\alpha\right)+4\alpha^2r^2\right\};\ E\left(\sigma_t^2\right)=\frac{\left(\omega+\alpha r^2\right)}{\left(1-\alpha\sigma_z^2\right)},$$

$$f_2=6\sigma_z^4\left(\frac{\omega+\alpha r^2}{1-\alpha\sigma_z^2}\right)^2\left(\frac{\sigma_\upsilon^2+\phi^2\left(1-a^2-\sigma_\upsilon^2\right)}{1-a^2-2\sigma_\upsilon^2-\phi^2+\phi^2a^2+\phi^2\sigma_\upsilon^2}\right);$$

$$\begin{aligned}
f_3&=1-\phi^4-\frac{3\sigma_\upsilon^4\left(-1-a^2-5\sigma_\upsilon^2+a^3+a^5-16a^3\sigma_\upsilon^2+3a\sigma_\upsilon^2-9a\sigma_\upsilon^4\right)}{\left(-1+a^3+3a\sigma_\upsilon^2\right)\left(-1+a^2+\sigma_\upsilon^2\right)\left(-1+6a^2\sigma_\upsilon^2+a^4+3\sigma_\upsilon^4\right)}-6\phi^2\left(\frac{\sigma_\upsilon^2}{1-a^2-\sigma_\upsilon^2}\right)-\\
&\quad-4\phi\left(6a\frac{\sigma_\upsilon^4}{\left(1-a^3-3a\sigma_\upsilon^2\right)\left(1-a^2-\sigma_\upsilon^2\right)}\right);
\end{aligned}$$

$$3.\ K^{(y)}=\frac{E\left(y_t^4\right)}{\left(E\left(y_t^2\right)\right)^2}.$$

THEOREM 3.2. *Consider the following doubly stochastic time series satisfying conditions* i, ii, iii *as follows*:

$$y_t=\left(\phi+b_t\right)+\varepsilon_t+\theta\varepsilon_{t-1}\ ,\ t\in\mathbb{Z};$$

$$b_{t+1}=ab_t+\left(1+b_t\right)v_{t+1}\ ,\ t\in\mathbb{Z};$$

where ε_t *is an identically distributed independent sequence of variables with mean zero and variance* σ_ε^2 *and we have following conditions* i, ii *and* iii *then we have results* 1, 2 *and* 3 *as follows*:

$$\text{i.}\quad \begin{pmatrix} v_t \\ \varepsilon_t \end{pmatrix}\overset{N}{\sim}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix},\begin{pmatrix} \sigma_\upsilon^2 & 0 \\ 0 & \sigma_\varepsilon^2 \end{pmatrix}\right),$$

$$\text{ii.}\ \ 1-a^2-\sigma_v^2<1,$$

$$\text{iii.}\ \ 1-a^2-2\sigma_v^2-\phi^2+\phi^2a^2+\phi^2\sigma_v^2<1.$$

$$1.\ E\left(y_t^2\right)=\frac{\sigma_\varepsilon^2\left(1+\theta^2\right)\left(1-a^2-\sigma_\upsilon^2\right)}{\left(1-a^2-2\sigma_\upsilon^2-\phi^2+\phi^2a^2+\phi^2\sigma_\upsilon^2\right)};$$

$$2.\ E\left(y_t^4\right)=\frac{3\sigma_\varepsilon^4\left(1+6\theta^2+\theta^4\right)+6\sigma_\varepsilon^2\left(1+\theta^2\right)\left(E\left(b_t^2\right)+\phi^2\right)E\left(y_{t-1}^2\right)}{\left(1-\phi^4-E\left(b_t^4\right)-6\phi^2E\left(b_t^2\right)-4\phi E\left(b_t^3\right)\right)};$$

$$3.\ K^{(y)}=\frac{E\left(y_t^4\right)}{\left(E\left(y_t^2\right)\right)^2},\ K^{(y)}\textit{is kurtosis of the process.}$$

Proof. 1. $E\left(y_t^2\right)=E\left(\left(\phi+b_t\right)y_{t-1}+\varepsilon_t+\theta\varepsilon_{t-1}\right)^2$

$$E\left(y_t^2\right)=\phi^2E\left(y_{t-1}^2\right)+E\left(b_t^2\right)E\left(y_{t-1}^2\right)+E\left(\varepsilon_t^2\right)+\theta^2E\left(\varepsilon_{t-1}^2\right)=\frac{\sigma_\varepsilon^2\left(1+\theta^2\right)}{1-E\left(\phi+b_t\right)^2}$$

$$E\left(y_t^2\right)=\frac{\sigma_\varepsilon^2\left(1+\theta^2\right)\left(1-a^2-\sigma_\upsilon^2\right)}{\left(1-a^2-2\sigma_\upsilon^2-\phi^2+\phi^2a^2+\phi^2\sigma_\upsilon^2\right)};$$

$$2.\ E\left(y_t^4\right)=E\left(\left(\phi+b_t\right)y_{t-1}+\varepsilon_t+\theta\varepsilon_{t-1}\right)^4=$$

$$=E\left(\left(\phi+b_t\right)^2y_{t-1}^2+\varepsilon_t^2+\theta^2\varepsilon_{t-1}^2+2\left(\phi+b_t\right)y_{t-1}\varepsilon_t+2\varepsilon_t\theta\varepsilon_{t-1}+2\left(\phi+b_t\right)\theta\varepsilon_{t-1}y_{t-1}\right)^2=$$

$$=E\left(\phi+b_t\right)^4y_{t-1}^4+E\left(\varepsilon_t^4\right)+\theta^2E\left(\varepsilon_{t-1}^4\right)+6\theta^2E\left(\phi+b_t\right)^2E\left(\varepsilon_t^2\right)E\left(y_{t-1}^2\right)+6E\left(\phi+b_t\right)^2E\left(\varepsilon_t^2\right)E\left(y_{t-1}^2\right);$$

$$E\left(y_t^4\right)=\frac{3\sigma_\varepsilon^4\left(1+6\theta^2+\theta^4\right)+6\sigma_\varepsilon^2\left(1+\theta\right)^2\left(E\left(b_t^2\right)+\phi^2\right)E\left(y_{t-1}^2\right)}{\left(1-\phi^4-E\left(b_t^4\right)-6\phi^2E\left(b_t^2\right)-4\phi E\left(b_t^3\right)\right)};$$

$$3.\ K^{(y)}=\frac{E\left(y_t^4\right)}{\left(E\left(y_t^2\right)\right)^2}=\frac{\dfrac{3\sigma_\varepsilon^4\left(1+6\theta^2+\theta^4\right)+6\sigma_\varepsilon^2\left(1+\theta^2\right)\left(E\left(b_t^2\right)+\phi^2\right)E\left(y_{t-1}^2\right)}{\left(1-\phi^4-E\left(b_t^4\right)-6\phi^2E\left(b_t^2\right)-4\phi E\left(b_t^3\right)\right)}}{\left(\dfrac{\sigma_\varepsilon^2\left(1+\theta^2\right)\left(1-a^2-\sigma_\upsilon^2\right)}{\left(1-a^2-2\sigma_\upsilon^2-\phi^2+\phi^2a^2+\phi^2\sigma_\upsilon^2\right)}\right)^2}.$$

THEOREM 3.3. *Consider the doubly stochastic volatility process with* AGARCH (I) – (0,1) *errors of the following form then we have results* 1, 2 *and* 3 *as follows*:

$$y_t=\left(\phi+b_t\right)y_{t-1}+\varepsilon_t+\theta\varepsilon_{t-1},\ \ t\in\mathbb{Z};$$

$$b_{t+1}=ab_t+\left(1+b_t\right)v_{t+1},\ \ t\in\mathbb{Z};$$

$$\varepsilon_t=\sigma_tz_t;$$

$$\sigma_t^2=\omega+\alpha\left(\varepsilon_{t-1}+r\right)^2;$$

$$1.\ E\left(y_t^2\right)=\frac{\sigma_z^2\left(\omega+\alpha r^2\right)\left(1+\theta^2\right)\left(1-a^2-\sigma_\upsilon^2\right)}{\left(1-\alpha\sigma_z^2\right)\left(1-a^2-2\sigma_\upsilon^2-\phi^2+\phi^2a^2+\phi^2\sigma_\upsilon^2\right)};$$

$$2.\ E\left(y_t^4\right)=\frac{3\sigma_z^4E\left(\sigma_t^4\right)\left(1+6\theta^2+\theta^4\right)+6\sigma_z^2\left(1+\theta^2\right)\left(E\left(b_t^2\right)+\phi^2\right)E\left(\sigma_t^2\right)E\left(y_{t-1}^2\right)}{\left(1-\phi^4-E\left(b_t^4\right)-6\phi^2E\left(b_t^2\right)-4\phi E\left(b_t^3\right)\right)};$$

$$3.\ K^{(y)}=\frac{E\left(y_t^4\right)}{\left(E\left(y_t^2\right)\right)^2},\ K^{(y)}\textit{is kurtosis of the process.}$$

Proof. 1. $E\left(y_t^2\right)=E\left(\left(\phi+b_t\right)y_{t-1}+\varepsilon_t+\theta\varepsilon_{t-1}\right)^2$

$$E\left(y_t^2\right)=\phi^2E\left(y_{t-1}^2\right)+E\left(b_t^2\right)E\left(y_{t-1}^2\right)+E\left(\varepsilon_t^2\right)+\theta^2E\left(\varepsilon_{t-1}^2\right)=\frac{\sigma_z^2\left(1+\theta^2\right)E\left(\sigma_t^2\right)}{1-E\left(\phi+b_t\right)^2};$$

$$E\left(y_t^2\right)=\frac{\sigma_z^2\left(1+\theta^2\right)\left(\omega+\alpha r^2\right)\left(1-a^2-\sigma_\upsilon^2\right)}{\left(1-\alpha\sigma_z^2\right)\left(1-a^2-2\sigma_\upsilon^2-\phi^2+\phi^2a^2+\phi^2\sigma_\upsilon^2\right)};$$

2. $E\left(y_t^4\right)=E\left(\left(\phi+b_t\right)y_{t-1}+\varepsilon_t+\theta\varepsilon_{t-1}\right)^4=$

$$=E\left(\left(\phi+b_t\right)^2y_{t-1}^2+\varepsilon_t^2+\theta^2\varepsilon_{t-1}^2+2\left(\phi+b_t\right)y_{t-1}\varepsilon_t+2\varepsilon_t\theta\varepsilon_{t-1}+2\left(\phi+b_t\right)\theta\varepsilon_{t-1}y_{t-1}\right)^2$$

$$=\frac{3E\left(\sigma_t^4\right)\left(1+6\theta^2+\theta^4\right)+6\sigma_z^2\left(1+\theta^2\right)\left(E\left(b_t^2\right)+\phi^2\right)E\left(\sigma_t^2\right)E\left(y_{t-1}^2\right)}{\left(1-\phi^4-E\left(b_t^4\right)-6\phi^2E\left(b_t^2\right)-4\phi E\left(b_t^3\right)\right)};$$

$$E\left(y_t^4\right)=\frac{f_1+f_2}{f_3};$$

$$E\left(\sigma_t^4\right)=\frac{\left(\omega+\alpha r^2\right)}{\left(1-3\alpha^2\right)\left(1-\alpha\right)}\left\{\left(\omega+\alpha r^2\right)\left(1+\alpha\right)+4\alpha^2r^2\right\};$$

$$E\left(\sigma_t^2\right)=\frac{\left(\omega+\alpha r^2\right)}{\left(1-\alpha\sigma_z^2\right)},\ E\left(y_t^2\right)=\frac{\sigma_z^2\left(1+\theta^2\right)\left(\omega+\alpha r^2\right)\left(1-a^2-\sigma_\upsilon^2\right)}{\left(1-\alpha\sigma_z^2\right)\left(1-a^2-2\sigma_\upsilon^2-\phi^2+\phi^2a^2+\phi^2\sigma_\upsilon^2\right)};$$

$$f_1=3\sigma_z^4\left(1+6\theta^2+\theta^4\right)\frac{\left(\omega+\alpha r^2\right)}{\left(1-3\alpha^2\right)\left(1-\alpha\right)}\left\{\left(\omega+\alpha r^2\right)\left(1+\alpha\right)+4\alpha^2r^2\right\};$$

$$f_2=\frac{6\sigma_z^4\left(1+\theta^2\right)^2\left(\omega+\alpha r^2\right)^2\left(\phi^2-\phi^2a^2-\phi^2\sigma_\upsilon^2+\sigma_\upsilon^2\right)}{\left(1-\alpha\sigma_z^2\right)^2\left(1-a^2-2\sigma_\upsilon^2-\phi^2+\phi^2a^2+\phi^2\sigma_\upsilon^2\right)};$$

$$f_3=1-\phi^4-\frac{3\sigma_\upsilon^4\left(-1-a^2-5\sigma_\upsilon^2+a^3+a^5-16a^3\sigma_\upsilon^2+3a\sigma_\upsilon^2-9a\sigma_\upsilon^4\right)}{\left(-1+a^3+3a\sigma_\upsilon^2\right)\left(-1+a^2+\sigma_\upsilon^2\right)\left(-1+6a^2\sigma_\upsilon^2+a^4+3\sigma_\upsilon^4\right)}-6\phi^2\left(\frac{\sigma_\upsilon^2}{1-a^2-\sigma_\upsilon^2}\right)-$$

$$-4\phi 6a\frac{\sigma_\upsilon^4}{\left(1-a^3-3a\sigma_\upsilon^2\right)\left(1-a^2-\sigma_\upsilon^2\right)};$$

$$3.\ K^{(y)}=\frac{E\left(y_t^4\right)}{\left(E\left(y_t^2\right)\right)^2},\ K^{(y)}\textit{is kurtosis of the process follows the definition.}$$

THEOREM 3.4. *Consider the doubly stochastic volatility process with* AGARCH(I) – (0,1) *errors of the following form then we have results* 1, 2 *and* 3 *as follows*:

$$y_t = \left(\phi + b_t + \Phi s_t\right) y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1},\ t \in \mathbb{Z}$$

$$b_{t+1} = ab_t + \left(1 + b_t\right) v_{t+1},\ t \in \mathbb{Z}$$

$$\varepsilon_t = \sigma_t z_t$$

$$\sigma_t^2 = \omega + \alpha\left(\varepsilon_{t-1} + r\right)^2$$

$$1.\ E\left(y_t^2\right) = \frac{\sigma_z^2\left(\omega + \alpha r^2\right)\left(1 + \theta^2\right)\left(1 - a^2 - \sigma_\upsilon^2\right)}{\left(1 - \alpha\sigma_z^2\right)\left(1 - a^2 - 2\sigma_\upsilon^2 - \Phi^2 + \Phi a^2 + \Phi^2\sigma_\upsilon^2 - \phi^2 + \phi^2 a^2 + \phi^2\sigma_\upsilon^2\right)};$$

$$2.\ E\left(y_t^4\right) = \frac{3\sigma_z^4 E\left(\sigma_t^4\right)\left(1 + 6\theta^2 + \theta^4\right) + 6\sigma_z^2\left(1 + \theta^2\right)\left(E\left(b_t^2\right) + \phi^2 + \Phi^2\right)E\left(\sigma_t^2\right)E\left(y_{t-1}^2\right)}{\left(1 - \phi^4 - E\left(b_t^4\right) - 6\phi^2 E\left(b_t^2\right) - \Phi^4 - 6\Phi^2\left(\phi^2 + E\left(b_t^2\right)\right)\right)};$$

$$3.\ K^{(y)} = \frac{E\left(y_t^4\right)}{\left(E\left(y_t^2\right)\right)^2},\ K^{(y)} \textit{is kurtosis of the process.}$$

Proof. 1. $E\left(y_t^2\right) = E\left(\left(\phi + b_t + \Phi s_t\right) y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}\right)^2$

$$E\left(y_t^2\right) = \phi^2 E\left(y_{t-1}^2\right) + \Phi^2 E\left(y_{t-1}^2\right) + E\left(b_t^2\right)E\left(y_{t-1}^2\right) + E\left(\varepsilon_t^2\right) + \theta^2 E\left(\varepsilon_{t-1}^2\right) = \frac{\sigma_z^2\left(1 + \theta^2\right)E\left(\sigma_t^2\right)}{1 - \Phi^2 - E\left(\phi + b_t\right)^2};$$

$$E\left(y_t^2\right) = \frac{\sigma_z^2\left(1 + \theta^2\right)\left(\omega + \alpha r^2\right)\left(1 - a^2 - \sigma_\upsilon^2\right)}{\left(1 - \alpha\sigma_z^2\right)\left(1 - a^2 - 2\sigma_\upsilon^2 - \Phi^2 + \Phi a^2 + \Phi^2\sigma_\upsilon^2 - \phi^2 + \phi^2 a^2 + \phi^2\sigma_\upsilon^2\right)};$$

$$2.\ E\left(y_t^4\right) = E\left(\left(\phi + b_t + \Phi s_t\right) y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}\right)^4 =$$

$$= \frac{E\left(\varepsilon_t^4\right)\left(1 + 6\theta^2 + \theta^4\right) + 6\left(1 + \theta^2\right)\left(E\left(b_t^2\right) + \phi^2 + \Phi^2\right)E\left(\varepsilon_t^2\right)E\left(y_{t-1}^2\right)}{\left(1 - \phi^4 - E\left(b_t^4\right) - 6\phi^2 E\left(b_t^2\right) - \Phi^4 - 6\Phi^2\left(\phi^2 + E\left(b_t^2\right)\right)\right)} =$$

$$= \frac{3\sigma_z^4 E\left(\sigma_t^4\right)\left(1 + 6\theta^2 + \theta^4\right) + 6\sigma_z^2\left(1 + \theta^2\right)\left(E\left(b_t^2\right) + \phi^2 + \Phi^2\right)E\left(\sigma_t^2\right)E\left(y_{t-1}^2\right)}{\left(1 - \phi^4 - E\left(b_t^4\right) - 6\phi^2 E\left(b_t^2\right) - \Phi^4 - 6\Phi^2\left(\phi^2 + E\left(b_t^2\right)\right)\right)};$$

$$E\left(y_t^4\right) = \frac{f_1 + f_2}{f_3};$$

$$E\left(\sigma_t^4\right) = \frac{\left(\omega + \alpha r^2\right)}{\left(1 - 3\alpha^2\right)\left(1 - \alpha\right)}\left\{\left(\omega + \alpha r^2\right)\left(1 + \alpha\right) + 4\alpha^2 r^2\right\};$$

$$E\left(\sigma_t^2\right) = \frac{\left(\omega + \alpha r^2\right)}{\left(1 - \alpha\sigma_z^2\right)};$$

$$E\left(y_t^2\right) = \frac{\sigma_z^2\left(\omega + \alpha r^2\right)\left(1 + \theta^2\right)\left(1 - a^2 - \sigma_\upsilon^2\right)}{\left(1 - \alpha\sigma_z^2\right)\left(1 - a^2 - 2\sigma_\upsilon^2 - \Phi^2 + \Phi a^2 + \Phi^2\sigma_\upsilon^2 - \phi^2 + \phi^2 a^2 + \phi^2\sigma_\upsilon^2\right)};$$

$$f_1 = 3\sigma_z^4\left(1+6\theta^2+\theta^4\right)\frac{\left(\omega+\alpha r^2\right)}{\left(1-3\alpha^2\right)\left(1-\alpha\right)}\left\{\left(\omega+\alpha r^2\right)\left(1+\alpha\right)+4\alpha^2 r^2\right\};$$

$$f_2 = \frac{6\sigma_z^4\left(1+\theta^2\right)^2\left(\omega+\alpha r^2\right)^2\left(\left(\phi^2+\Phi^2\right)\left(1-a^2-\sigma_\upsilon^2\right)+\sigma_\upsilon^2\right)}{\left(1-\alpha\sigma_z^2\right)^2\left(1-a^2-2\sigma_\upsilon^2-\Phi^2+\Phi a^2+\Phi^2\sigma_\upsilon^2-\phi^2+\phi^2 a^2+\phi^2\sigma_\upsilon^2\right)};$$

$$f_3 = 1-\phi^4-\frac{3\sigma_\upsilon^4\left(-1-a^2-5\sigma_\upsilon^2+a^3+a^5-16a^3\sigma_\upsilon^2+3a\sigma_\upsilon^2-9a\sigma_\upsilon^4\right)}{\left(-1+a^3+3a\sigma_\upsilon^2\right)\left(-1+a^2+\sigma_\upsilon^2\right)\left(-1+6a^2\sigma_\upsilon^2+a^4+3\sigma_\upsilon^4\right)}-$$

$$-6\left(\phi^2+\Phi^2\right)\left(\frac{\sigma_\upsilon^2}{1-a^2-\sigma_\upsilon^2}\right)-\Phi^4-6\phi^2\Phi^2;$$

3. $K^{(y)} = \dfrac{E\left(y_t^4\right)}{\left(E\left(y_t^2\right)\right)^2}$, $K^{(y)}$ *is kurtosis of the process follows the definition.*

4. CONCLUSIONS

In this paper, the kurtosis of doubly stochastic models with AGARCH-(I)-(0,1) innovations are derived. Statistical inferences for these doubly stochastic models with asymmetric GARCH errors and state space modeling can be viewed as a special case for nonlinear time series.

REFERENCES

1. APPADOO, S.S., GHAHRAMANI, M., THAVANESWARAN, A., *Moment properties of sometime series models*, The Mathematical Scientist, **30**, pp. 50–63, 2005.
2. BOLLERSLEV, T., *Generalized autoregressive conditional heteroscedasticity,* Journal of Econometrics, **31**, pp. 307-327, 1986.
3. CHRISTIAN, F., JEAN, Z.M., *GARCH models,* Jhon Wiley and Sons, 2010.
4. ENGLE, R.F., *Autoregressive conditional heteroscedasticity with estimates of the variance of U.K.inflation,* Econonometrica, **50**, pp. 987–1008, 1982.
5. GEORGE, L., *Computational finance: Numerical methods for pricing financial instruments*, Butterworth-Heinemann, Elsevie, 2004.
6. JURGEN, F., WOLFGANG, H.K., CHRISTIAN, H.M., *Statistics of Financial Markets: An Introduction,* Springer, 2011.
7. NICHOLLS, D., QUINN, B.G., *Random Coefficient Autoregressive Models, An Introduction: Lector Notes in Statistics,* Springer, New York, 1982.
8. PEIRIS, S., THAVANESWARAN,A., APPADOO,S.S., *Doubly stochastic models with GARCH models,* Applied Mathematics Letters, **24**, pp. 1768–1773, 2011.
9. SHIRYAEV, A.N., *Probability*, 2nd ed.,Springer-Verlag, New York Inc, 2005.
10. THAVANESWARAN, A., APPADOO, S.S., BECTOR,C.R, *Recent developments in volatility modeling and applications*, Journal of Applied Mathematics and decision sciences, Article ID86320, pp.1–23.

Received October 31, 2012