THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 15, Number 3/2014, pp. 254–261

GROUND FAULT LOCATION INFLUENCE ON AC POWER LINES IMPEDANCES

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In this paper it will be presented an analytical method in order to determine the ground fault current distribution in effectively grounded power networks. In our previous works it has been assumed that the transmission line has only one ground wire. In this paper, additionally it is considered the case where there are two ground wires. There are also presented some useful quantitative results obtained through a dedicated developed MATLAB 7.0 program.

Key words: power system faults, transmission lines, ground fault current.

1. INTRODUCTION

When a ground fault occurs on an overhead transmission line in a three-phase power network with grounded neutral, the fault current returns to the grounded neutral through the towers, ground return path and ground wires. The estimation of the ground current distribution has many major applications in power systems design, especially in grounding systems. Extensive work has been undertaken in order to model transmission network for ground fault current analysis [2–13]. The ground fault divides the line into two sections, each extending from the fault towards one end of the line. Depending of the number of towers between the faulted tower and the stations, respectively of the distance between the towers, these two sections of the line may be considered infinite, in which case the ground fault current distribution is independent on the termination of the network; otherwise, they must be regarded as finite, in which case the ground fault current distribution may depend greatly on the termination of the network [3, 4, 8, 12, 13]. In our previous works it was presented an analytical method in order to determine the ground fault current distribution in effectively grounded power network. It was assumed that the transmission line has only one ground wire [14, 15, 16]. In this paper it will be presented the case when there are two ground wires, too. It will be considered the case that those two sections of the line are both infinite, respectively both finite. The calculation method is based on the following assumptions: the network is considered linear in the sinusoidal steady-state; only the fundamental frequency is considered and impedances are considered as lumped parameters in each span of the transmission.

2. GROUND FAULT CURRENT DISTRIBUTION ON AN INFINITE TRANSMISSION LINE WITH ONE GROUND WIRE

A long line is one which has a middle section where the line is divided by each tower into infinite half-lines. In Fig. 1 the fault occurs at large distance from both terminals, at the tower number *0*. It is assumed that the fault is fed from both directions. The impedance of the ground wire connected between two grounded towers, called the self impedance per span, it is noted with Z_{cp_d}. The mutual impedance between the ground wire and the faulted phase conductor, per span, was noted with Z_m. Those two section of the line being both infinite, the situation in the left part from the faulted tower is identically with the situation in the right part.

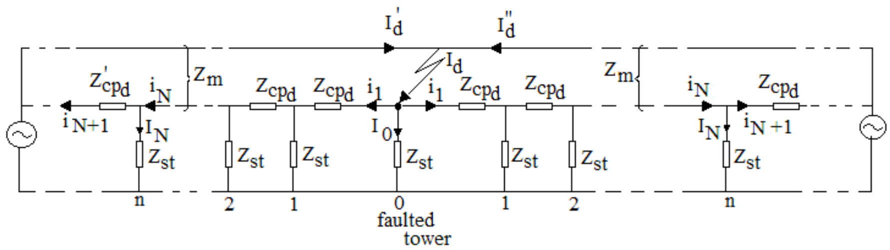

Fig. 1 – Ground fault current distribution on an infinite transmission line with one ground wire.

The current I_n flowing to ground through the n-th tower, counted from the faulted tower, is equal with the difference between the currents i_n and i_{n+1} [14, 15, 16]:

$$I_n = i_n - i_{n+1}. \tag{1}$$

The loop equation for the n-th mesh is given by the next expression:

$$I_n Z_{st} - I_{n-1} Z_{st} + i_n Z_{cp_d} - \nu I_d Z_{cp_d} = 0. \tag{2}$$

In expression (2) ν represents the coupling factor between the overhead phase and ground wire $\left(\nu = \dfrac{Z_m}{Z_{cp_d}}\right)$.The equation (2) could be written in the next form:

$$i_n = \frac{(I_{n-1} - I_n) Z_{st}}{Z_{cp_d}} + \nu I_d. \tag{3}$$

Similarly:

$$i_{n+1} = \frac{(I_n - I_{n+1}) Z_{st}}{Z_{cp_d}} + \nu I_d. \tag{4}$$

Substituting relations (3) and (4) in relation (1), for the current I_n will be obtained the next second order homogeneous difference equation:

$$I_n \frac{Z_{cp_d}}{Z_{st}} = I_{n+1} - 2I_n + I_{n-1}. \tag{5}$$

According to [10], the solution of this equation is the following:

$$I_n = A\mathrm{e}^{\alpha n} + B\mathrm{e}^{-\alpha n}. \tag{6}$$

The parameters A and B could be obtained from the boundary conditions. Parameter α in the solution (6) could be obtained by substituting the solution (6) in equation (5). By applying equation (1) to the (n-1) tower, it will be obtained the following expression:

$$I_{n-1} = i_{n-1} - i_n. \tag{7}$$

By substituting the equations (1) and (7) in equation (2), it will be obtained the next equation:

$$i_n \frac{Z_{cp_d}}{Z_{st}} = i_{n+1} - 2i_n + i_{n-1} + \nu I_d \frac{Z_{cp_d}}{Z_{st}}. \tag{8}$$

Similarly with equation (5), the current in the ground wire is given by the next solution:

$$i_n = a\mathrm{e}^{\alpha n} + b\mathrm{e}^{-\alpha n} + \nu I_d. \tag{9}$$

In expression (9) *a, b* represent parameters. Due to the link between currents i_n and I_n, the parameters *A, B* and *a, b* are not independent. By substituting the solutions (6) and (9) in equation (1), it will be obtained:

$$A\mathrm{e}^{\alpha n} + B\mathrm{e}^{-\alpha n} = a\mathrm{e}^{\alpha n}(1-\mathrm{e}^{\alpha}) + b\mathrm{e}^{-\alpha n}(1-\mathrm{e}^{-\alpha}). \tag{10}$$

Because these relations are the same for every value of *n*, it will be obtained the next expressions:

$$A = a(1-\mathrm{e}^{\alpha}) \tag{11}$$

$$B = b(1-\mathrm{e}^{-\alpha}). \tag{12}$$

The current in the ground wire will be then given by the following expression:

$$i_n = A\frac{\mathrm{e}^{\alpha n}}{1-\mathrm{e}^{\alpha}} + B\frac{\mathrm{e}^{-\alpha n}}{1-\mathrm{e}^{-\alpha}} + \nu I_d. \tag{13}$$

If the line is sufficiently long so that, after some distance, the varying portion of the current exponentially decays to zero (infinite line), then the parameter $A \to 0$. In this case only the parameter *B* must be determined from the boundary conditions [10], [13]. According to (6) and (13), it can be written:

$$I_n = B\mathrm{e}^{-\alpha n} \tag{14}$$

$$i_n = B(\mathrm{e}^{-\alpha n}/1-\mathrm{e}^{-\alpha}) + \nu I_d. \tag{15}$$

The boundary condition at the faulted tower will be the following:

$$I_d = I_0 + 2i_1. \tag{16}$$

Substituting expressions (14) and (15) in (16), with $n=0$ for I_n and $n=1$ for i_n, it could be obtained the expression for the current in the faulted tower.

3. GROUND FAULT CURRENT DISTRIBUTION ON A FINITE TRANSMISSION LINE WITH ONE GROUND WIRE

Next, it is treated the case when those two sections of the line are divided by the faulted tower into two finite sub-lines. In Fig. 2 the fault occurs at the tower number *0*.

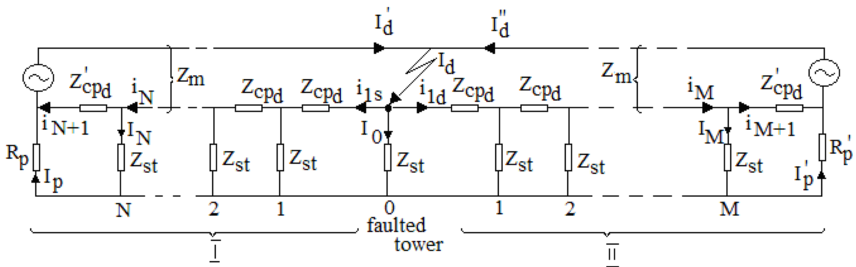

Fig. 2 – Ground fault current distribution on a finite transmission line with one ground wire.

There are *N* towers between this tower and the left station and respectively *M* towers between tower number 0 and the right station. The resistances of the grounding systems from the left, respectively from the right stations (Fig. 2) are R_p, respectively R'_p. The impedance of the ground wire in the last span is Z'_{cp_d}, and $Z_p = R_p + Z'_{cp_d}$, respectively $Z'_p = R'_p + Z'_{cp_d}$. The total fault current I_d is given by the sum between the current I'_d from one side, and I'_d from the other side of the transmission line.

Considering the left part from the faulted tower, the situation is identically with that presented in Fig. 1. The solutions for the current in the towers, respectively in the ground wire are:

$$I_{n-s} = A_s \mathrm{e}^{\alpha n} + B_s \mathrm{e}^{-\alpha n}, \tag{17}$$

$$i_{n-s} = A_s \frac{\mathrm{e}^{\alpha n}}{1-\mathrm{e}^{\alpha}} + B_s \frac{\mathrm{e}^{-\alpha n}}{1-\mathrm{e}^{-\alpha}} + \nu I'_d . \tag{18}$$

The subscript s is used for the left part from the faulted tower [3]. I_{n-s} represents the current in the tower number n, counted from the faulted tower to the left part of the transmission line. The parameters A_s, B_s will be obtained from the boundary conditions. For the faulted tower, the following formulas can be written:

$$\begin{cases} I_0 Z_{st} - I_{1s} Z_{st} - i_{1s} Z_{cp_d} + \nu Z_{cp_d} I'_d = 0 \\ I_{1s} Z_{st} - I_{2s} Z_{st} - i_{2s} Z_{cp_d} + \nu Z_{cp_d} I'_d = 0. \end{cases} \tag{19}$$

Also it can be written the next expression:

$$I_{1s} = i_{1s} - i_{2s} . \tag{20}$$

Substituting i_{1s} and i_{2s} from equations (19) in equation (20), it will be obtained:

$$I_{1s}(2 + Z_{cp_d} / Z_{st}) = I_0 + I_{2s} . \tag{21}$$

According to [3] for the left terminal it can be written:

$$I_N + i_{N+1} = i_N , \tag{22}$$

$$I_N Z_{st} + I_p R_p - i_{N+1} Z'_{cp_d} + Z'_m I'_d = 0 , \tag{23}$$

$$(I_{N-1} - I_N) Z_{st} - i_N Z_{cp_d} + \nu Z_{cp_d} I'_d = 0 , \tag{24}$$

$$i_{N+1} = I'_d - I_p , \tag{25}$$

where Z'_m represents the mutual coupling between the ground wire and the faulted phase, in the last span. Replacing i_{N+1} and i_N from expressions (23), (24) in (22), by taking into account expression (25), it will be obtained the following expression:

$$I_N \left(1 + \frac{Z_{st}}{Z_{cp_d}} + \frac{Z_{st}}{Z'_{cp_d} + R_p} \right) = I_{N-1} \frac{Z_{st}}{Z_{cp_d}} + I'_d \left(\nu - \frac{R_p + Z'_m}{Z'_{cp_d} + R_p} \right). \tag{26}$$

The expressions (21) and (26) represent the boundary conditions. A_s and B_s will be obtained by replacing I_{1s}, I_{2s}, I_N and I_{N-1} from the solutions (17) and (18) in (21) and (26).

4. SIMILAR EXPRESSIONS COULD BE OBTAINED FOR THE CURRENTS FROM THE RIGHT PART OF THE FAULTED TOWER. TRANSMISSION LINE WITH TWO GROUND WIRES

So far it was considered that the transmission line has only one ground wire. When there are two ground wires, if they are identical and disposed in a mutually symmetrical position, the expressions presented for single ground wire can still be used. In this case, those two ground wires are represented by an imaginary single wire, but the geometric mean radius of the equivalent wire and the geometric mean distance relative to the faulted phase must be substituted [4]. In the general case, presented in Fig. 3, those two ground wires could be modelled as un equivalent ground wire. In this case, the self impedance of the equivalent ground wire it is noted with Z_{cpe}, respectively the mutual impedance between equivalent ground wire and the faulted phase it is noted with Z_{cpme}. The equivalent impedances Z_{cpe} and Z_{cpme} will represent the group of the ground wires. Expressions for the self impedance of a ground return circuit and for the mutual impedance between two ground return circuits are available from the literature [2], [4]. In case of a single ground wire, these expressions could be directly applied in order to find the mutual impedance between the ground wire and the faulted phase conductor. When there are multiple ground conductors, in order to find the equations for the self and mutual impedances of the group, it must be taken into consideration the mutual coupling between the individual ground wires.

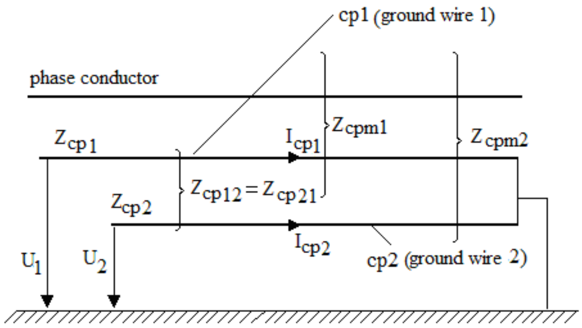

Fig. 3 – Transmission line with two ground wires.

Considering Fig. 3, the following equations could be written:

$$\begin{aligned} U_1 &= Z_{cp11} I_{cp1} + Z_{cp12} I_{cp2} \\ U_2 &= Z_{cp21} I_{cp1} + Z_{cp22} I_{cp2}. \end{aligned} \tag{27}$$

In equation (27), Z_{cp11}, Z_{cp22} represent the self impedance of the ground wires, and Z_{cp12}, Z_{cp21} represent the mutual impedance between the ground wires. These two ground wires, being interconnected at both ends, thus $U_1 = U_2$, and the self impedance of the group can be defined as [4]:

$$Z_{cpe} = \frac{U}{I_{cp1} + I_{cp2}}. \tag{28}$$

The mutual impedance between the equivalent ground wire and the faulted phase conductor is given in the following equation by particularising the corresponding formula presented in [4]:

$$Z_{cpme} = \frac{I_{cp1} Z_{cpm1} + I_{cp2} Z_{cpm2}}{I_{cp1} + I_{cp2}}. \tag{29}$$

In equation (29) Z_{cpm1}, Z_{cpm2} represent the mutual impedances between the ground wires and the faulted phase. From these three expressions, it will be determined the expressions for the self impedance of the equivalent ground wire, respectively for the mutual impedance between equivalent ground wire and the faulted phase.

$$Z_{cpe} = \frac{Z_{cp1} Z_{cp2} - Z_{cp12}^2}{Z_{cp1} + Z_{cp2} - 2Z_{cp12}}, \tag{30}$$

$$Z_{cpme} = \frac{Z_{cpm1}(Z_{cp2} - Z_{cp12}) + Z_{cpm2}(Z_{cp1} - Z_{cp12})}{Z_{cp1} + Z_{cp2} - 2Z_{cp12}}. \tag{31}$$

5. OBTAINED RESULTS

In order to illustrate the theoretical approach outlined in the sections above, we are considering that the line who connects two stations is a 220kV transmission line. The line (Fig. 4) has two aluminium-steel ground wires 160/95 mm^2 [7].

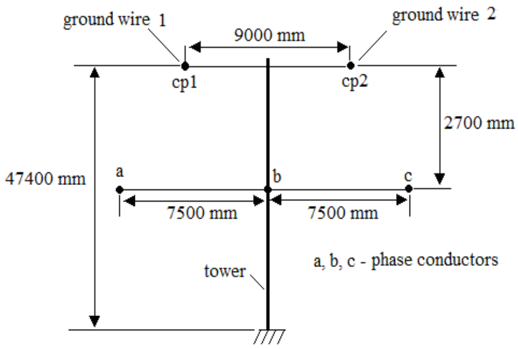

Fig. 4 – Disposition of the transmission line conductors.

Line impedances per one span are determined on the basis of the following assumption: average length of the span is 250 m. Impedance Z_m is calculated only in relation to the faulted phase conductor, because it could not be assumed that a line section of a few spans is transposed. Ground wires impedances per one span and the mutual impedance between the ground wires and the faulted phase are calculated for different values of the soil resistivity with formulas based on Carson's theory of the ground return path [1]. The fault was assumed to occur on the phase which is the furthest from the ground conductors, because the lowest coupling between the phase and ground wire will produce the highest tower voltage. The total fault current from both stations was assumed to be $I_d = 15\,000\ A$. Those values are valid for a soil resistivity of 100 $\Omega\cdot$m. All the further quantitative results are based on the theoretical approaches presented during the previous sections. In order to do this there were developed some numerical intensive programs written in MATLAB 7.0 software frame.

Figure 5 presents the obtained mutual impedance between the two ground wires as a function of the soil resistivity based on Carson's formulas [1], for different values of the horizontal distance between those wires.

Figure 6 presents on the same theoretical basis the mutual impedance between the two ground wires as a function of the horizontal distance between ground wires, for different values of the soil resistivity.

Figure 7 presents, based on expressions (14), the currents flowing through transmission towers as a function of the number of spans, considering, respectively neglecting the coupling factor between the ground wires and the faulted phase.

Figure 8 presents currents through transmission line towers as a function of the number of spans, for different values of the tower impedances. It was considered the case of an infinite transmission line.

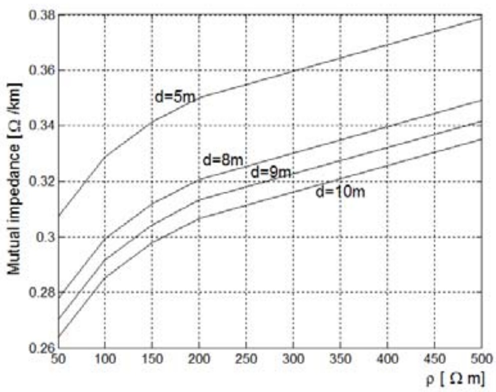

Fig. 5 – The mutual impedance between the two ground wires as a function of the soil resistivity.

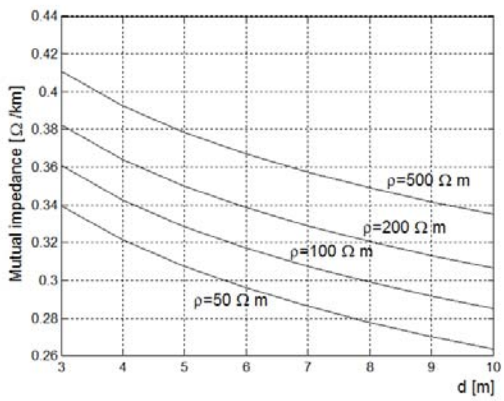

Fig. 6 – The mutual impedance between the two ground wires as a function of the horizontal distance between ground wires.

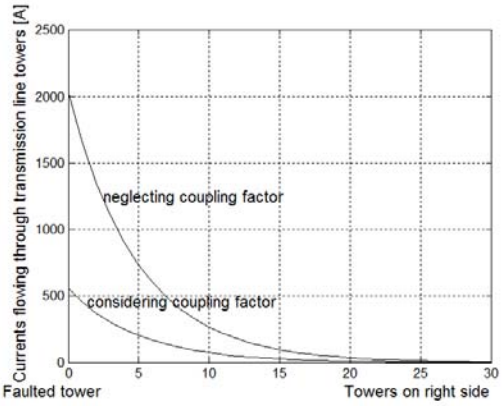

Fig. 7 – Currents flowing through transmission towers as a function of the number of spans.

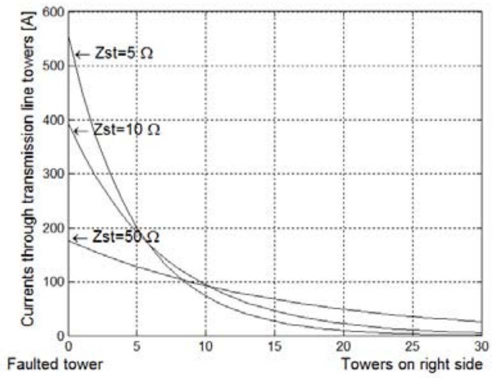

Fig. 8 – Currents through transmission line towers as a function of the number of spans.

Figure 9 presents the currents flowing through transmission towers for different values of the faulted tower impedance. It was considered the case of a finite transmission line and that the fault appears in the middle of the transmission line, and there are 20 towers of each side of the faulted tower.

Figure 10 presents the current through the faulted tower, for a finite line in both directions, as a function of the line length. When the line length exceed a limit, the current remains practically constant with the further increase of the line length.

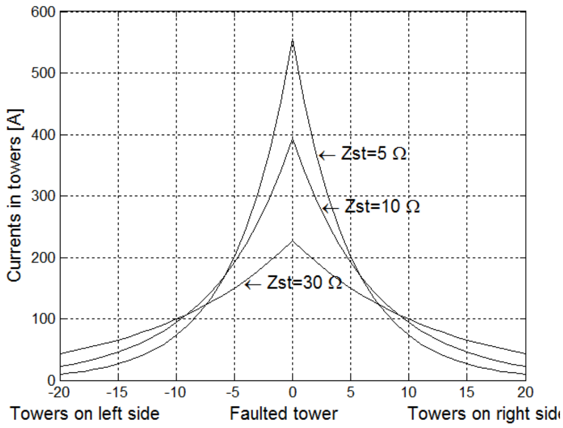

Fig. 9 – Currents flowing through transmission towers for different values of the faulted tower impedance.

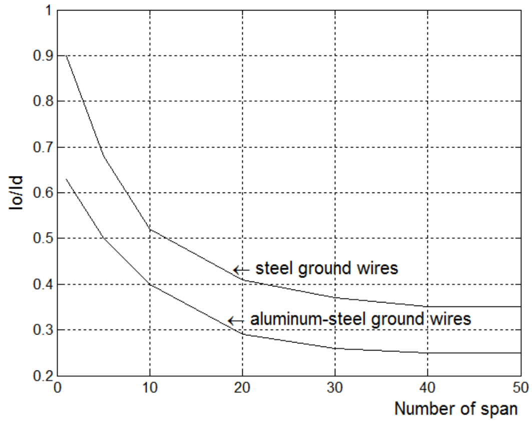

Fig. 10 – The current through the faulted tower as a function of the line length.

In order to validate the analytical expressions, their corresponding results were compared with those measured in similar conditions using real fault tests [3] or with those obtained using other recognized analytical methods [5, 8, 12, 13]. The obtained computations results are in a good agreement with those obtained from real measurements, respectively with those analytically obtained by other researchers. As important advantages, the presented method is simpler, easily understandable and far less time consuming than others.

6. CONCLUSIONS

At first was considered an overhead transmission line with one ground wire, connected to the ground at every tower of the line. But, quite often there are more than one ground wire installed on the transmission line. Additional ground wire reduces the overall series impedance. As a consequence the tower voltages during a ground fault will be lower than in the case of a single ground wire. Considering two ground wires, the method and equations presented for single ground wire can be still applied, but in equations for the self and mutual impedances of the two ground wires, the mutual coupling between the two ground wires must be taken into consideration [4]. The mutual impedance between the ground conductor and the faulted phase conductor, reduces the total circuit impedance. In these conditions, the fault current will be higher if the mutual impedance is neglected. It also can be seen that in the absence of mutual coupling, the fault current will flow through the ground, through a smaller number of towers then in the mutual coupling presence. The method can be used for the design of new transmission lines in order to select the size of the ground wire, for evaluating safety conditions near transmission towers, etc.

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Received August 1, 2012