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ANALYTICAL-NUMERICAL COUPLING ANALYSIS OF SUBMARINE PIPELINE IN J-LAY PROBLEM

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The article present an analytical-numerical coupling model to deal with the submarine pipeline during its installation in deep water using J-lay method. The pipeline is divided into two segments in this paper: the touchdown segment and the suspended segment. An analytical method was used to analyse the touchdown segment considering the elastic deformation of the seabed as Pasternak foundation and a numerical method for the suspended segment which allows the influence of ocean current to be considered using the method of weighted residuals after the deformation differential equation was proposed to analyse the behaviour of the whole pipeline during its installation. The variation tendency of the longitudinal stress, the hoop stress and the shearing stress of the pipeline as well as the influence of different launching angles and tension forces to them are analysed in this paper which could give good suggestions to the submarine pipeline designers or the pipeline installation managers.

Key words: analytical-numerical coupling analysis, submarine pipeline, *J*-lay installation, touchdown segment, suspended segment, launching angle, tension force.

1. INTRODUCTION

Submarine pipelines, the most efficient cost-effective means of large scale transportation for oil and nature gas form producers to end-users in deep-water by far, have caught much attention in recent decades, with on-gong development of offshore oil and gas exploitation. The offshore industry has been called to improve the laying of submarine pipelines technology for the reason that most pipelines may encounter the highest stresses during the installation process [1, 2] to deal with environmental hazards, typical of such hostile ocean environment in deep-water. Compared with the conventional pipeline installation method such as the S-lay method and the Reel-lay method, the J-lay method, whose name comes after the shape of the pipeline during the laying and consists in lowering the pipe almost vertically into the water by an inclined ramp has been widely accepted as the most suitable and favourable installation method in deep-water because of its most important characteristics [3].

To predict the J-lay process mechanical features, e.g., top angle and tension, pipe bending tension stiffness, ocean currents, soil stiffness and slope, boundary layer phenomena, former researchers have pioneered work through both analytical and numerical methods. Plunkett [4] adopted the singular perturbation method to derive asymptomatic expansion to deal with the behaviour of stiffened catenary, considering the boundary layers. Dixon and Rutledge [5] used the perturbation method to analyze the J-lay method. Some researchers [6, 7] have developed the analytical method in the following years and it proves effective and simple except for the disadvantage that it is not able to consider the effect of ocean currents and pipe embedment. Lenci and Callegari [8] raised some analytical models with linearly elastic seabed to reveal the theoretical basis of the J-lay problem, but the created models are not able to consider the ocean currents effects. Some available numerical tools take the influence of ocean current and seabed embedment into account [9, 10]. Of the numerical tools, mechanical features should be assumed before calculation which makes them inconvenient to use and the commercial numerical tools are often uneconomical. Li [3] created a simple numerical model neglecting the high-order terms of the suspended portion of the laying pipe to analyse the J-lay problem and the model can take into account the influence of ocean currents and pipe embedment. Though years of research on mathematic model of the laying process of submarine pipelines, most researchers regard it as elastic large deflection beam model, considering the non-linearity elasticity large deformation of the pipeline during its installation.

The aim of this paper is to investigate an analytical-numerical coupling method taking the influence of large deformation of the suspended portion of pipeline during J-lay deployment into consideration for the analysis of the mechanical behaviour of the submarine pipeline. The pipeline, in the analytical-numerical coupling model presented in this paper, is divided into two segments: the suspended segment of the pipeline and the touchdown segment of the pipeline. Indeed, one of the main achievements of this paper is proposing an analytical-numerical coupling model taking the influence of currents into consideration using the large deflection theory when analysing the suspended portion of the pipeline and taking into account the pipe embedment during its installation using J-lay method.

2. GOVERNING EQUATIONS

As shown in Fig.1, the submarine pipeline which is to be installed in deep-water, considering its different internal stresses during its installation using J-lay method, is divided into two segments: one segment of the submarine pipeline is the touchdown segment laid on seabed form left boundary point (P_1) where balance the soil frictional force and axial force of the pipeline to touchdown point (TDP) and the other segment of the submarine pipeline is the suspended segment that suspended in the water from touchdown point (TDP) to the lift-off point (P_2) of the laying vessel. In this article, one global coordinate system (*x, y*) is used with its origin at (0, 0) of touchdown point (TDP). An analytical model to analyse the touchdown segment and a numerical model to analyse the suspended segment are proposed in this chapter.

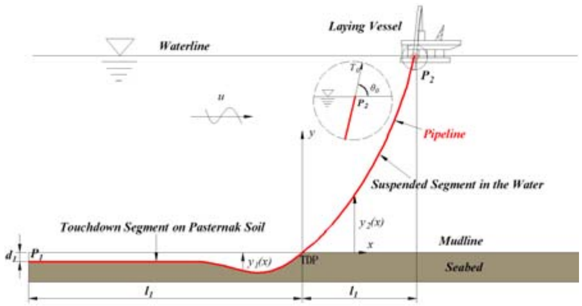

Fig. 1 – Scheme of the model.

2.1. Touchdown segment

The embedment of the touchdown segment of pipeline into the seabed during installation resulting from special soil surrounding environment on the seabed introduce stresses and strains which have significant influence on the success of pipe laying. The influence of current on the touchdown segment could be ignored in this model for the ocean current velocity is small.

Analysing the mechanical model of the touchdown segment of the pipeline shown as in Fig. 2a and considering the application force of a differential touchdown pipe segment of stretched length dx shown as in Fig. 2b, the equilibrium equations for the touchdown segment of the pipe can be derived:

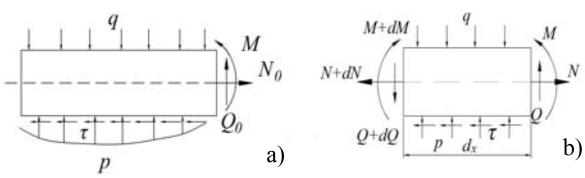

Fig. 2 – a) Fore sketch of the touchdown segment; b) fore sketch of a differential touchdown segment.

$$\begin{cases} \sum X = \mathrm{d}N - \tau \mathrm{d}x = 0 \\ \sum Y = \mathrm{d}Q + \left(w - p\right)\mathrm{d}x = 0 \\ \sum M = \mathrm{d}M - N\mathrm{d}y - Q\mathrm{d}x = 0, \end{cases} \tag{1}$$

where: *N* is the axial force and *Q* is the shearing force of the pipeline, *w* is the submerged weight of the touchdown pipe per length, *p* is the soil resistance to a touchdown pipe of a unit length, τ is the soil frictional force to the pipe per length, *X, Y* and *M* are the touchdown segment pipe's vertical force, horizontal force and bending moment, respectively.

Considering the seabed is not rigid in this model but behaves like a Pasternak foundation, and the bending moment of the touchdown segment could be obtained by the small deflection theory. Then the governing equation of the touchdown segment of pipeline can be obtained as:

$$EIy_1^{(4)}(x) - (N_0 + G_p D)y_1''(x) + \tau y_1' + kDy_1(x) - w = 0 \quad (x \le 0), \tag{2}$$

where dN is the differential axial force on the touchdown point, and it can be obtained as N_0=τl_1, here l_1 is the length left boundary point (P_1) to touchdown point (TDP), k is the seabed stiffness, D is the outer diameter of the pipeline, EI is the bending stiffness of the pipeline, G_P is the shear modulus of seabed and it can be obtained as G_P= 0.5E_s/(1+υ_s), where E_s and υ_s are the Young's modulus and Poisson ratio of the seabed.

The general solution of the governing equation can be obtained by

$$y_1(x) = \mathrm{e}^{\alpha_1}\left[C_1\cos(\beta_1 x) + C_2\cos(\beta_1 x)\right] + \mathrm{e}^{\alpha_2}\left[C_3\cos(\beta_2 x) + C_4\cos(\beta_2 x)\right] - \frac{w}{kD} \quad (x \le 0), \tag{3}$$

where C_1, C_2, C_3, C_4 are unknown coefficients and α_1, α_2, β_1 and β_2 are constant parameters calculated by EI, N_0, E_s, υ_s, τ and k. Then the shearing force of the pipeline can be obtained as $Q(x) = -EIy'''_1(x)$.

Considering the inclination slop, bending moment and shear force at left boundary point, and assuming the embedment depth of the touchdown segment at left boundary point is d_1, we face a system of 4 boundary equations with 4 unknown coefficients (C_1, C_2, C_3, C_4): $y_1(-l_1) = -d_1$; $y'_1(-l_1) = 0$; $y''_1(-l_1) = 0$; $y'''_1(-l_1) = 0$.

Then, the longitudinal stress σ_x, the hoop stress σ_y and the shearing stress τ_{xy} of the touchdown segment of the pipe can be obtained as: $\sigma_x = (N_0-\tau x)/\delta + EIy''_1(x)/W$; $\sigma_y = 0.5\gamma_w hD/\delta$; $\tau_{xy} = 2\,EIy'''_1(x)/B$, where B is the cross section area of the pipe, W is the bending modulus of the pipe, γ_w is specific weight of ocean water, h is the depth of ocean, δ is the thickness of the pipeline.

2.2. Suspended segment

Considering the equilibrium of the vertical force V, horizontal force H and the bending moment M of a differential suspended pipe segment of stretched length d_s subjected to gravity, external hydrodynamic loading and internal structural response loading, shown as in Fig. 3. The influence of the ocean current to the suspended segment of the submarine pipeline is considered in this model during its installation.

The equilibrium equations for the suspended pipe segment are set up as:

$$\begin{cases} \mathrm{d}H + p_H \sin\theta \mathrm{d}s = 0 \\ \mathrm{d}V + (p_V - w)\cos\theta \mathrm{d}s = 0 \\ \mathrm{d}M - H\sin\theta \mathrm{d}s + V\cos\theta \mathrm{d}s = 0, \end{cases} \tag{4}$$

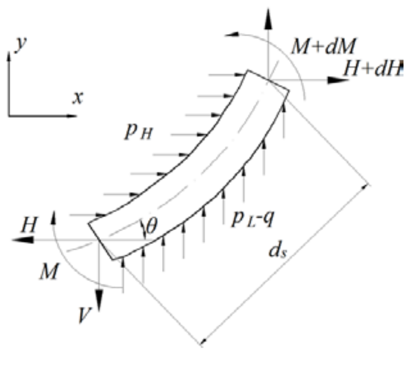

Fig. 3 – Fore sketch of a differential suspended segment.

where: θ is the inclination slop of the submarine pipeline, d_s is the length of a differential suspended segment, w is the submerged weight of the suspended pipe per length, p_H is the horizontal force obtained as $p_H = 0.5\rho_w C_D D u|u|$, and p_V is the vertical force obtained as $p_V = 0.5\rho_w C_L D u^2$ acted on the suspended pipe by the ocean current, here ρ_w is the density of ocean water, u is the current velocity, C_D and C_L are the drag coefficient and lift coefficient of the pipeline, respectively [11].

The geometric relationship shown as below can be obtained as cosθds = dx, sinθds = dy_2, dx/ds = dx/((dx)^2+(dy_2)^2)$^{-0.5}$ = 1/(1+(dy_2/dx)^2)$^{-0.5}$, and the bending moment of the suspended pipeline can be obtained by multiplying the bending stiffness EI and the curvature as $M = EI$dθ/ds. Assuming the length of horizontal suspended segment of the pipeline was l_2, we obtain:

$$\frac{\mathrm{d}M}{\mathrm{d}x}=EI\frac{\mathrm{d}^2\theta}{\mathrm{d}s\cdot\mathrm{d}x}=EI\left(\frac{y_2'''(x)}{1+\left(y_2'(x)\right)^2}-\frac{y_2''(x)}{1+\left(y_2'(x)\right)^2}\cdot\frac{y_2'(x)\cdot y_2''(x)}{1+\left(y_2'(x)\right)^2}\right)\quad(0\le x\le l_2).\tag{5}$$

The governing equation of the pipeline suspended in water could be obtained by means of the Eq. (4), and Eq. (5):

$$EI\left[y_2'''(x)+y_2'''(x)\cdot\left(y_2'(x)\right)^2-\left(y_2''(x)\right)^2\cdot y_2'(x)\right]+\left(V-Hy_2'(x)\right)\left[1+\left(y_2'(x)\right)^2\right]^2=0\quad(0\le x\le l_2),\tag{6}$$

where $H=N_0-\int_0^{y_2}p_H\mathrm{d}y_2$, $V=Q_0-\int_0^{x}\left(p_L-w\right)\mathrm{d}x$ .The suspended segment's governing equation belongs to the nonlinear higher-order differential equation and the accurate analytical solution could not be obtained. Here we assume $x=0.5l_2+0.5l_2t$ $(-1\le t\le 1)$, then the governing equation of the pipeline suspended in water could be expressed as follows:

$$EI\left[\frac{8\mathrm{d}^3y_2}{l_2^3\mathrm{d}t^3}+\frac{8\mathrm{d}^3y_2}{l_2^3\mathrm{d}t^3}\cdot\left(\frac{2\mathrm{d}y_2}{l_2\mathrm{d}t}\right)^2-\frac{2\mathrm{d}y_2}{l_2\mathrm{d}t}\left(\frac{4\mathrm{d}^2y_2}{l_2^2\mathrm{d}t^2}\right)^2\right]+\left(V-\frac{2H\mathrm{d}y_2}{l_2\mathrm{d}t}\right)\left[1+\left(\frac{2\mathrm{d}y_2}{l_2\mathrm{d}t}\right)^2\right]^2=0\quad(-1\le t\le 1).\tag{7}$$

The approximate solution of Eq. (7) could be found by using the method of weighted residuals (henceforth abbreviated as MWR) and a trial function is taken in the form using Chebyshev polynomial:

$$y=0.5C_5+C_6t+C_7\left(2t^2-1\right)+C_7\left(4t^3-3t\right)+C_9\left(8t^4-8t^2+1\right)+C_{10}\left(15t^5-20t^3+5t\right)\ (-1\le t\le 1),\tag{8}$$

where C_5, C_6, C_7, C_8, C_9 and C_{10} are parameters to be determined in the solution. Substituting Eq. (8) into Eq. (7), we can obtain the residuals within the field as $R_1=\sum_{i=0}^{20}K_it^i\quad(-1\le t\le 1)$, where K_i is a parameter calculated by C_5, C_6, C_7, C_8, C_9 and C_{10}. The weighting functions can be chosen in many ways and each choice corresponds to a different criterion of the MWR [12]. The weighting functions in this article can be obtained using the subdomain method so that $\int_{-1}^{1}R_I\mathrm{d}t=0$ .There are six unknown parameters (C_5, C_6, C_7, C_8, C_9 and C_{10}) in the equation, and we could get boundary conditions respectively at lift-off point (P_2) and touchdown point (TDP) considering the displacement and slope angle at P_2 and the continuity of displacement, bending moment and shear force at TDP:

P_2:

$$\begin{cases}y_2(x)|_{x=l_2}=y_2(t)|_{t=1}=0.5C_5+C_6+C_7+C_8+C_9+C_{10}=h\\ y_2'(x)|_{x=l_2}=y_2'(t)|_{t=1}=2l_2^{-1}\left(C_6+4C_7+8C_8+16C_9+25C_{10}\right)=\tan\theta\end{cases}\tag{9}$$

TDP:

$$\begin{cases}y_1(x)|_{x=0}=y_2(x)|_{x=0}=y_2(t)|_{t=-1}\\ y_1'(x)|_{x=0}=y_2'(x)|_{x=0}=y_2'(t)|_{t=-1}\\ y_1''(x)|_{x=0}=y_2''(x)|_{x=0}=y_2''(t)|_{t=-1}\\ y_1'''(x)|_{x=0}=y_2'''(x)|_{x=0}=y_2'''(t)|_{t=-1}\end{cases}\tag{10}$$

where h is the depth of the ocean where vessel located, θ is the slope angle at P_1.Then the tension force that the pipeline need at lift-off point (P_2) can be calculated as $T_2=B\,(N_2^2+Q_2^2)^{0.5}$, where N_2, Q_2 are the axial stress and the shear stress of the pipeline at lift-off point (P_2), B is the cross-sectional area of the pipeline.

The longitudinal stress σ_x, the hoop stress σ_y and the shearing stress τ_{xy} of the suspended segment of the pipe can be obtained as: $\sigma_x=(H\cos\theta+V\sin\theta)/B+EIy''_2(x)/W$; $\sigma_y=0.5\gamma_w\,(h-y_2)\,D/\delta$; $\tau_{xy}=2\,EIy'''_2(x)/B$.

3. SOLUTIONS

The proposed mathematical model to analyse the submarine pipeline's behaviour during its installation using J-lay method is so complicated that no explicit solution of the model can be found. In this article, a numerical method is used. At the beginning of the solutions, basic parameters such as parameters of pipeline, depth of ocean, ocean currents, and parameters of soil are input for both touchdown segment of pipeline laid on the seabed and pipeline suspended in the water. To start the calculation of the touchdown segment and suspended segment of pipeline, it is necessary to assume an initial d_1 and an initial l_1. And then, the two initial parameters can be input into the 4 boundary equations of touchdown segment to determine the four unknown parameters C_1, C_2, C_3 and C_4. Eventually, the axial force, the displacement, the first-order derivative, the second derivative, the third derivative and the shear force on the touchdown point can be obtained.

To guarantee the continuity of displacement at the touchdown point, verification is needed to see if $|y_1(0)| < \varepsilon$ is satisfied, where $y_1(0)$ is the displacement of the pipeline on the touchdown point, and ε is a small specified quantity. If $y_1(0) > \varepsilon$ appears, l_1 must be too large and should be decreased. If $y_1(0) < -\varepsilon$ appears, l_1 must be too small and should be increased. After $|y_1(0)| < \varepsilon$ is satisfied, the solution of the touchdown segment of pipeline is done. The output of the displacement, the first-order derivative, the second derivative and the third derivative on the touchdown point calculated in the touchdown segment can then input into Eq. (9) and Eq. (10) to determine the six unknown parameters C_5, C_6, C_7, C_8, C_9, C_{10} and the parameter of T_2 can be obtained too. Considering the accuracy of the calculation, $|T_2 - T_{vessel}| < \varepsilon$ must be satisfied when the calculation of the suspended segment of pipeline is completed, where T_{vessel} is the total real tension at lift-off point (P_2) during the installation of pipeline. If $|T_2 - Tvessel| > \varepsilon$ appears, d_1 must be modified. After $|T_2 - T_{vessel}| < \varepsilon$ is satisfied, the analysis of the behaviour of the touchdown segment and suspended segment of the submarine pipeline during its installation using J-lay method are done and the detailed process is shown in Fig. 4.

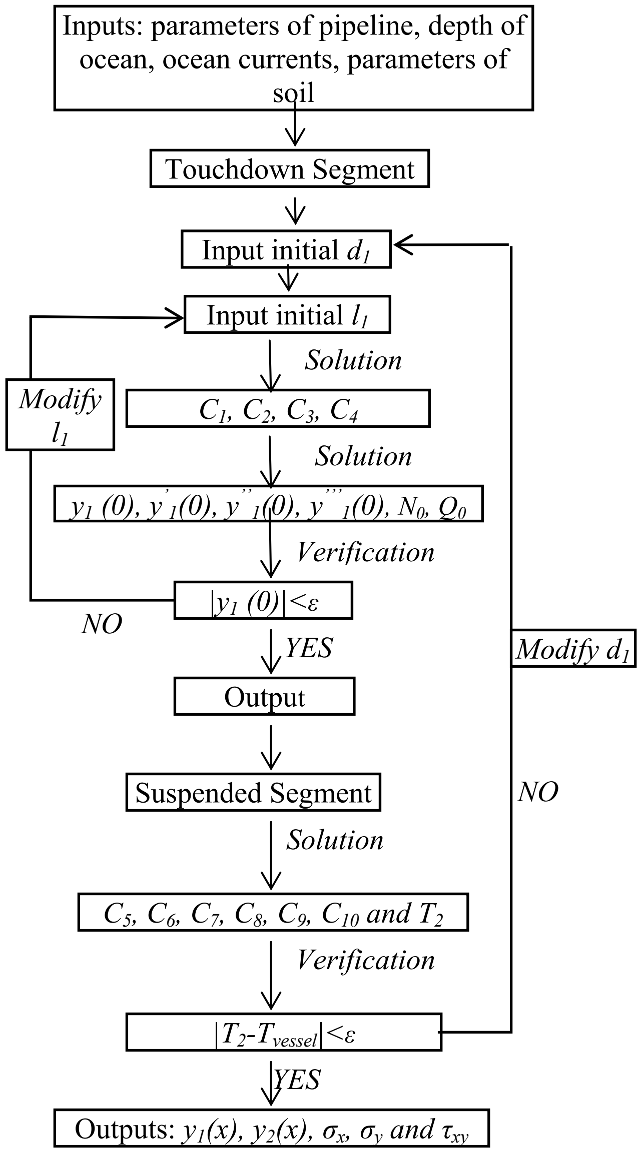

Fig. 4 – Flow scheme for J-lay method calculation.

4. EXAMPLES

To illustrate the previous model with some examples, a typical pipeline is selected with its basic parameters including the outer diameter $D = 0.60$m, thickness $\delta = 0.025$m, bending stiffness $EI = 3.92\times10^8$Nm^2, bending modulus $W = 4.869\times10^{-3}$m^3, submerged weight of the touchdown pipe per length w = 632.5N/m, lift coefficient C_L = 1.3 and drag coefficient C_D = 1.5. An ultra-deep water ocean is selected in this section with its depth h = 2 000m, density of ocean water ρ_w = 1 030kg/m^3 and current velocity u = 0.93m/s. We assumed the touchdown segment of the pipeline is supposed to be rest on a Pasternak deformable seabed with its soil frictional force to the pipe per length τ = 24 500N/m, seabed stiffness k = 78.6kN/m^2, young's modulus E_s = 80MN/m^2 and Poisson ratio υ_s = 0.2.

The parameters of launching angle at lift-off point which adjusted by hinged ramp and tensioner of the laying vessel and tension force which adjusted by tensioner of the laying vessel, could controlled by pipe installation managers. Once the ocean environment and seabed condition and the pipeline which to be installed are confirmed, managers could choose suitable launching angle and tension force to ensure the stress of the pipeline is less than the allowable stress of the pipeline. It's important to choose suitable launching angle and tension force to installation managers under the consideration of different sea state, pipeline parameters, ocean depth and soil parameters during the J-lay deployment. The launching angle and tension force are investigated in detail in this article to predict their influence on the pipe configuration, longitudinal stress, hoop stress and shearing stress of the pipeline.

4.1. Influence of launching angle

The influence of different launching angles to configuration and stresses of pipeline predicted by the previous model derived in this article with the tension force T_{vessel} = 3MN adjusted by the tensioner of the laying vessel at lift-off point are depicted in Fig. 5. Three different launching angles are chosen for the analysis: 85°, 75° and 65°.

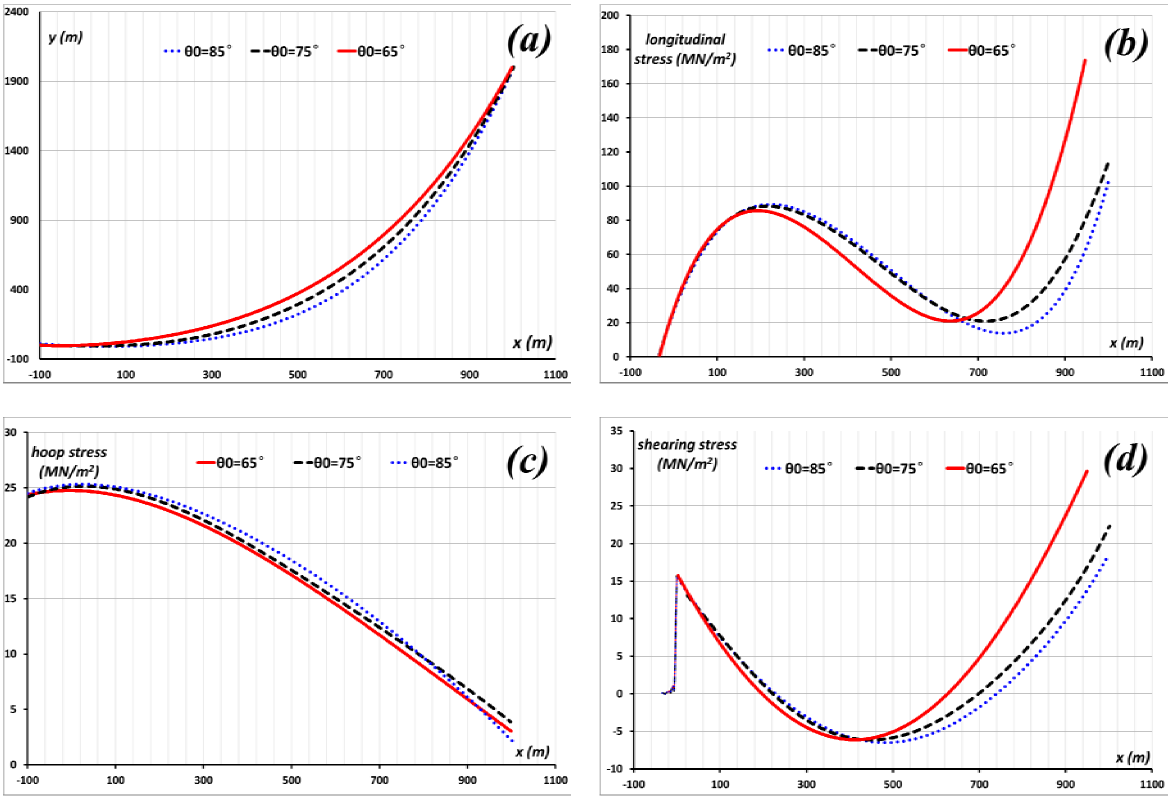

Fig. 5 – a) Configuration of pipeline for different launching angles; b) the longitudinal stresses for different launching angles; c) the hoop stresses for different launching angles; d) the shearing stresses for different launching angles.

Shown as in Fig. 5, the stresses of the pipeline have different variation tendency. The minimum longitudinal stress appears at the touchdown segment of the pipeline and the value of the longitudinal stress has decrease after a short increase to the right of the touchdown point, and then the value of the longitudinal stress increases to the maximum at the lift-off point (Fig. 5b). The maximum hoop stress appears at the touchdown segment of the pipeline and then the value of hoop stress decline to the minimum at lift-of point of the laying vessel (Fig. 5c). The variation curve of the shearing stress, shown as in Fig. 5d, present a *u* shaped trend: the shearing stress values of the pipeline at touchdown point and lift-off point are much bigger than the shearing stress values of other part of the pipe and the suspended segment even have negative shearing stress values. The maximum shearing stress appears at the lift-off point of the laying vessel.

The configuration and stresses of pipeline vary with the launching angles. Minor differences appear that the radius of curvature of the pipeline becomes smaller when choosing a larger launching angle at lift-off point, although the configurations of pipeline for different launching angles are almost coincide (Fig. 5a).The stresses including the longitudinal stress, the hoop stress and the shearing stress of the pipeline for different launching angles near the touchdown point are almost the same when the launching angles changing. On the other hand, the stresses near the lift-off point vary with the launching angles change: the value of the stresses decrease when choosing a bigger launching angle while installation of the pipeline. The reason for the change is that: the height of the lift-off point will gets lower as the launching angle at the lift-off point increase, causing the support force applied to the pipeline by the tensioner of the laying vessel increase. Then, the stresses of the pipeline will be much smaller. Therefore, the installation manager should choose a larger launching angle of the pipeline if conditions are satisfied.

4.2. Influence of tension force

Same as the launching angle of the pipeline at the lift-off point, the parameter of tension force at the lift-off point which could adjusted by managers is also important. Fig. 6 shows the configuration and stresses of pipeline for different tension force predicted by the previous model derived in this article with the launching angle $\theta_0 = 75°$. Three different tension forces are chosen for the analysis: 4MN, 3MN and 2MN.

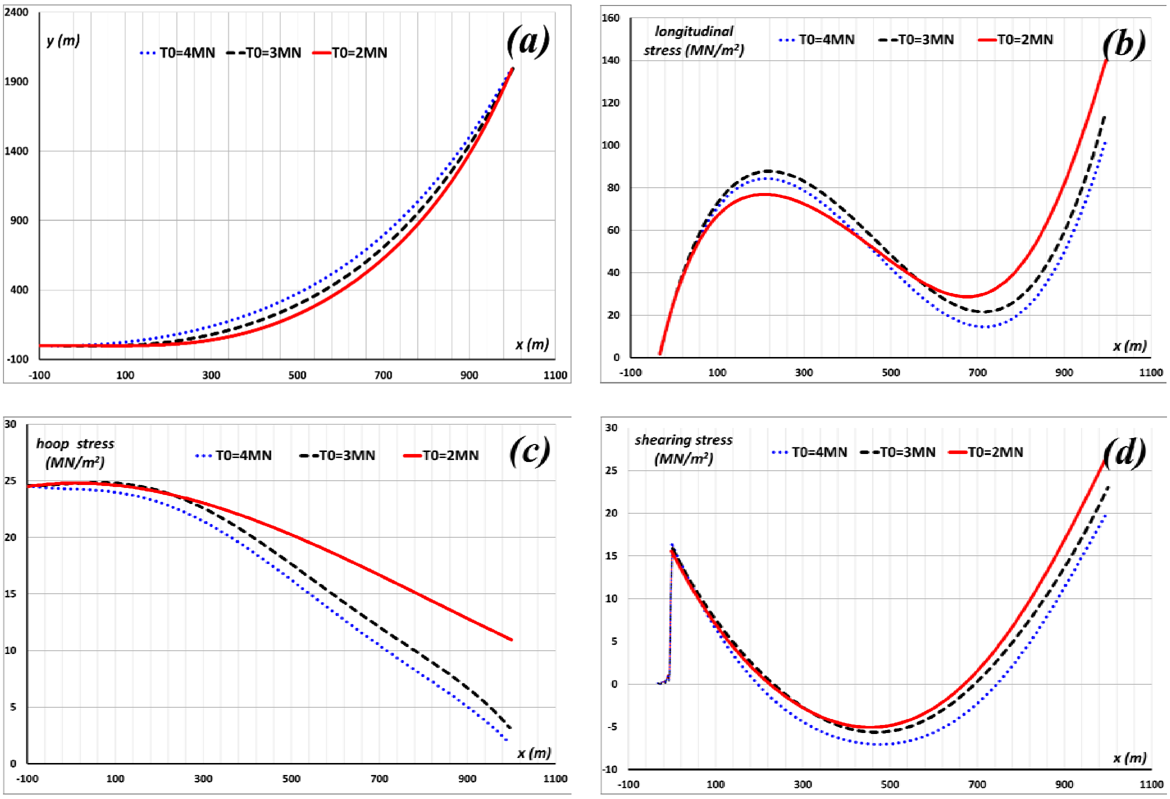

Fig. 6 – a) Configuration of pipeline for different tension force; b) the longitudinal stresses for different tension force; c) the hoop stresses for different tension force; d) the shearing stresses for different tension force

Same as the influence of launching angles, the radius of curvature of the pipeline becomes larger when choosing a larger tension force at lift-off point, although the configurations of pipeline for different tension forces are almost coincide (Fig. 6a).The stresses including the longitudinal stress (Fig. 6b), the hoop stress (Fig. 6c) and the shearing stress (Fig. 6d) of the pipeline for different tension forces near the touchdown point are almost the same when the tension force changing. On the other hand, the value of the stresses decrease when choosing a bigger tension force while installation of the pipeline. The reason for the change is that: the effect of gravity to the pipeline will be reduced and the height of the lift-off point will gets lower as the tension force at the lift-off point increase, causing the support force applied to the pipeline by the tensioner of the laying vessel increase. Then, the stresses of the pipeline will be much smaller. Therefore, the installation manager should choose a larger launching angle of the pipeline if conditions are satisfied.

5. CONCLUSIONS

A simple analytical model for predicting the submarine pipeline's touchdown segment and a numerical model for the submarine pipeline's suspended segment which allows the influence of ocean current to be considered are proposed to analyse the behaviour of the pipeline during its installation. The general solution for the touchdown segment of the pipeline is obtained considering the elastic deformation of the seabed using Pasternak foundation in the analytical model and the numerical solution for the suspended segment of the pipeline is obtained using the method of weighted residuals after the deformation differential equation of the suspended segment was proposed.

The presented model is applied to analyse the variation tendency of the longitudinal stress, the hoop stress and the shearing stress of the pipeline as well as the influence of different launching angles and tension forces to them:

1. The minimum longitudinal stress appears at the touchdown segment of the pipeline and the value of the longitudinal stress has a decrease after a short increase to the right of the touchdown point, and then the value of the longitudinal stress increases to the maximum at the lift-off point. The maximum hoop stress appears at the touchdown segment of the pipeline and the value of hoop stress decline to the minimum at lift-of point of the laying vessel. The variation curve of the shearing stress presents a "u" shaped trend: the shearing stress values of the pipeline at touchdown point and lift-off point are much bigger than the shearing stress values of other part of the pipe and the suspended segment even have negative shearing stress values. The maximum shearing stress appears at the lift-off point of the laying vessel.

2. The radius curvature of the pipeline becomes smaller when choosing a larger launching angle or a smaller tension force at lift-off point of laying vessel. The stresses of the pipeline near the touchdown point are almost the same when the launching angles or tension force are changing while the stresses near the lift-off point vary much: the value of the stresses decrease when choosing a bigger launching angle or tension force while installation of the pipeline. The reason for the change is that: the height of the lift-off point will gets lower as the launching angle or tension force at the lift-off point increase, causing the support force applied to the pipeline by the tensioner of the laying vessel increase. Then, the stresses of the pipeline will be much smaller. Therefore, the installation manager should choose a larger launching angle or tension force at the lift-off point of the laying vessel if conditions are satisfied.

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