THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 17, Number 1/2016, pp. 76–83

INFORMATION SCIENCE

SIMULATION ANALYSIS OF RESOURCE ALLOCATION PROBLEMS WITH TIME VARYING PARAMETERS

D.N. SHUKAYEV, N.O. YERGALIYEVA, ZH.B. LAMASHEVA

"K.I. Satpayev" Kazakh National Technical University, Information Technology Department, Almaty, Kazakhstan
Corresponding author: Zhanar B. Lamasheva, E-mail: zhanarlb@mail.ru

Abstract. This paper proposes a simulation based approach to the analysis of a class of optimization problems commonly arising in resource allocation systems with parallel structure. The objective of such problem is to find the most economically beneficial way of relaxing binding resource constraints, thus improving most on previously found optimal solutions. We formulate analytically the dependence of the value of the objective function on values of constraining parameters, taking in account possible interaction between several such parameters. We then develop an algorithm for finding optimal expansion areas of feasible sets. The results of this study have been applied successfully to develop a control system for a large scale grain storage and processing company.

***Key words**:* simulation analysis, varying parameters, resource allocation, optimization, extension method, modeling.

1. INTRODUCTION

One of the important problems arising in optimization of various technological processes is identifying conditions and parameters which prevent further optimization, as well as finding the most efficient ways to relax these constraints. The existing methods for analyzing optimization problems for their sensitivity to constraining parameters [1, 2] are quite inflexible and have limited practical applicability. For example, these methods are not applicable to optimization problems with near singular constraint matrices, which arise quite commonly in processes "disturbed" by small parameter variations [3]. Such optimization problems are often complicated by instability of obtained solutions and were considered by a number of researchers [4–7]. In their previous work [8–10], the principal researcher of this project and his colleagues suggested a powerful method for solving resource allocation problems in systems with parallel processing units, with possible near singularity of the constraint matrix. This method finds the solution of the original optimization problem by implementing a directed transition starting from a solution of a similar problem with an expanded admissible set.

The aim of this study is to develop simulation methods and procedures which will help to improve obtained solutions in technological processes with time varying parameters, by finding the most economically efficient ways to expand the admissible set. We will start by stating the general problem in the next section.

2. THE GENERAL PROBLEM

Suppose the optimal regime of a certain technological process is determined by the solution of the following standard mathematical programming problem [11]:

$$F = \max F(x), \tag{1}$$

$$g(x) \le b, \tag{2}$$

$$x \geq 0. \tag{3}$$

where **x** is *n*-dimensional vector, **b** is *m*-dimensional vector, $g(x)$ is a set of *m* real valued functions defined on the space of *n*-dimensional vectors.

Assume that the vector **b** on the right-hand side of (2) can be deliberately changed in order to relax some of the constraints within a given time interval (*e.g.* work shift, work day, decade, etc). The change Δb can be deterministic, or in a more general case, have stochastic component. The objective is to develop an algorithm for identifying the most costly binding constraints in (2) and then finding the optimal relaxation increments Δb^*. The next section presents the general structure of our algorithm for simulating and solving such a problem.

3. THE BASIC STRUCTURE OF OUR SIMULATION PROCEDURE

The general algorithm for improving optimal solutions of technological processes must fulfill the following basic steps:

1. Solving the problem (1)–(3).
2. Modeling deterministic or stochastic changes in constraining parameters of the optimization problem [8, 12].
3. Determining the areas of the parameter space which lead to the improvement in the value of the objective function of the (1)–(3).

In accordance with the above structure, we outline the following algorithm of finding Δb^*.

Step 1. Solve the problem (1)–(3).

Step 2. Compute the value b^0 of the vector on the right-hand side of constraint (2), corresponding to the optimal solution of the problem (1)–(3).

Step 3. Identify binding constraints.

Step 4. Simulate deterministic or partially stochastic relaxation changes Δb of the constraining parameters on the right-hand side of binding constraints.

Step 5. Compute new relaxed values of the constraining parameters $b=b^0+\Delta b$.

Step 6. Find the new solution of the problem (1)–(3) and compute the minimum sufficient values $b^1 \leq b$ of the constraining parameters on the right-hand side of (2).

Step 7. Compute the optimal parameter increment $\Delta b^*=b^1-b^0$.

Step 8. End of the algorithm.

On step 4, if changes Δb in parameters on the right-hand side of binding constraints are stochastic, we can have two different simulation strategies. In case if Δb is a continuous random variable with a known PDF function $f(\Delta b)>0$ we can use its inverse to simulate Δb according to proposition 1 [8]:

PROPOSITION 1. *A random variable* Δb*, realizations of which are determined from the expression*

$$F(\Delta b) = \int f(\Delta b)ds = U \ \text{ or } \ \Delta b = F^{-1}(U),$$

where U is the uniform distribution defined on [0, 1] interval, has its PDF given by $f(\Delta b)$.

In an alternative case, to model discrete values of Δb, with possible realizations Δb_j, $(j=\overline{1,m})$, happening with corresponding probabilities $\Delta b = \begin{pmatrix} \Delta b_1 \ \Delta b_2 \ldots \Delta b_m \\ p_1 \quad p_2 \quad \ldots \quad p_m \end{pmatrix}$, one can use proposition 2 [8].

PROPOSITION 2. *A random value realizes with probability* p_k *if* $U \in \Delta_k$*, where* $\Delta_k = p_k$.

The proofs of propositions 1 and 2 can be found in [8, 12].

Thus we can further specialize the sub-algorithms realizing the step 4 above for continuous and discrete values of Δb as follows:

Sub-algorithm C

Step 4.1. Take draw from U.

Step 4.2. Compute $\Delta b = F^{-1}\left(U\right)$.

Sub-algorithm D

Step 4.1. Take a draw from U. Set k = 1.

Step 4.2. Check the condition $U > \sum_{j=1}^{k} p_j$. If it is not satisfied, transition to step 4.4.

Step 4.3. Set k = k +1. Transition to step 4.2.

Step 4.4. Set $\Delta b = \Delta b_k$ Transition to step 4.2.

When this optimization problem (1)–(3) deals specifically with optimal resource allocation [9], specific features of such problems (existence of equality constraints) simplifies somewhat the process of finding optimal Δb^* . Let us consider two specific examples: a linear problem and a quadratic problem of resource allocation.

4. FINDING OPTIMAL Δb^* IN PROBLEMS OF RESOURCE ALLOCATION

Allocation of physical resources or information flows in systems with parallel structures is a common practical problem. For example, distribut ion of a product from a warehouse to various retailers; or allocation of repair orders among various repair units, etc. (see [3–6] for other applications). Finding an optimal solution to such problems often requires costly computational procedures and methods. These high computational costs arise because the parallel structures are typically, at least partially homogeneous, which leads to ill-conditioned constraint matrixes with near-singular determinants. As a result, optimization problems with such ill-conditioned constraint sets become highly unstable and hard to solve. There are methods that were developed to tackle these kinds of complications. For example, [7] studied a problem of finding stable solutions in systems with singularities in the constraint matrix by using “stabilizing functionals”. This idea has been also applied to problems with near singular constraint matrixes. For instance, [8] proposes a method which first disregards small differences between nearly collinear constraints in order to extract a so called “characteristic system” of the problem, and then uses this system to assess the impact of small differences between constraints on optimal solution. The aforementioned computational methods have an important theoretical significance. However their applicability is conditional on a set of fairly stringent assumptions regarding the nature of singularity in the constraint set. Also these methods can only provide approximate solutions. An alternative solution method has been proposed in [13–15]. These studies develop an “Extension method,” for solving problems of resource allocation in systems with parallel objects with possible (but not required) near singularity of the constraint matrix. The main idea of this method is to start from a solution of a simpler problem with an expanded (i.e. relaxed) constraint set, and then perform a directed transition to the optimal solution by re-introducing the original constraints, which happen to be binding at the solution of the relaxed problem. Such differentiation between binding and non-binding constraints not only eliminates the sensitivity of the proposed method to near-singularity of the constraint set, but also allows obtaining exact solutions.

Let us state a linear problem of resource allocation:

$$F = \max \sum_{j=1}^{n} c_j x_j, \tag{4}$$

$$\sum_{j=1}^{m} a_{ij} x_j \le b_i, \qquad i = 1,...,m-1, \tag{5}$$

$$\sum_{j=1}^{n} x_j = b_m, \tag{6}$$

$$x_j \geq 0, \qquad j = 1,...,n. \tag{7}$$

In case we would like to stress that parameters of the constraints (5) are different from each other only by small amounts we could also decompose them into common and specific parts: $a_{ij} = a_{i0} + \varepsilon t_{ij}$, where ε is a small scalar parameter.

The extended problem is obtained by disregarding the "inconvenient" constraints (5) which often lead to instabilities due to small perturbations in parameters a_{ij} relative to each other:

$$F = \max \sum_{j=1}^{n} c_j x_j, \tag{8}$$

$$\sum_{j=1}^{n} x_j = b_m, \tag{9}$$

$$x_j \geq 0, \qquad j = \overline{1,n}. \tag{10}$$

The general structure of the extension solution method has the following steps:

1) Solve the extended problem (8–10).
2) Verify the obtained solution for its admissibility with respect to constraints (5). If the solution is admissible, it is optimal.
3) Otherwise transition to a new solution which satisfies the constraints (5).

The articles [14, 15] in the reference section provide mathematical foundations of the extension method and detailed algorithms for solving the problem (4–7) above, as well as the problem (14–17) below, with a quadratic objective function.

Suppose that in the problem (4)–(7) we can change some of the parameters (resources) $b_i (i = 1,...,m-1)$. Write the objective function (4) as a Taylor's series of order 1

$$F = F^{ex} - \frac{\Delta c}{\Delta a}(b^{ex} - b), \tag{11}$$

around the solution x^{ex} of an expanded problem (8)–(10), where x^{ex} and F^{ex} are the optimal solution and the value of the objective function of the extended problem (8–10) and b^{pc} stands for the vector of values equal to the left-hand side expressions of the constraints (5) evaluated at x^{ex}.

Let X and X^{ex} stand for admissible sets of the original (4)–(7) and expanded (9)–(11) problems correspondingly. Suppose that the original problem has a unique solution. The following two lemmas relate admissible sets of these two problems and of their solutions.

LEMMA 1 [9]. *The admissible set X of the original problem (4)–(7) is always a subset of the admissible set of the expanded problem (9)–(11), $X \in X^{pc}$.*

LEMMA 2 [9]. *Optimal solution of the original problem coincides with the optimal solution of the expanded problem only if:*

1) *the admissible sets of these two problems are identical, or*

2) *the optimal solution of the expanded problem belongs X*, i.e. $x^{ex} \in X$.

The formal proofs of these perhaps self-evident lemmas is stated in [9].

The essence of the extension method [9, 10] consists in that the solution of the original problem (4)–(7) is obtained by a directed transition from the solution of the extended problem (9)–(11).

We will analyze the problem (4)–(7) on its sensitivity to resource constraints $b_i, (i = 1,...,m-1)$ while holding all the other parameters $c_j (j = 1,...,n)$ and $a_{ij} (i = 1,...,m-1, j = 1,...,n)$ as well as b_m fixed. As a result, the values F^{ex} and b^{ex} will also be unchanged. Consequently this leads to a liner dependence of the objective function value on the vector **b**, as shown in Fig. 1 for a one-dimensional example. b^0 is a binding right-hand side value of constraint (5) corresponding to the optimal solution of the original problem (4)–(7).

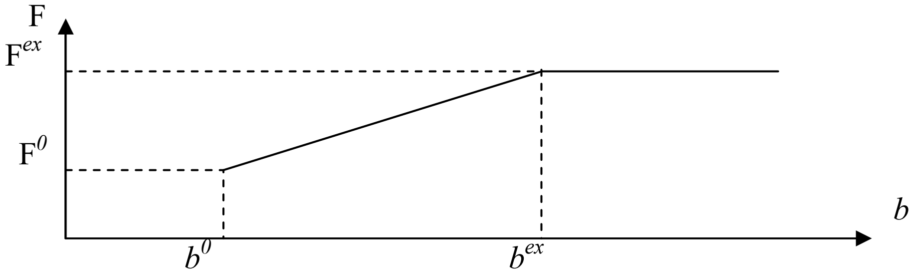

Fig. 1 – Area of changes of resource for a linear problem.

From Fig. 1 it is clear that the interval $[b^0, b^{ex}]$ *is the only* area of possible changes in the amount of resource b, which could bring about an improvement in the objective function. That is when choosing b we should not violate the following condition

$$b = b^0 + \Delta b \le b^{ex}. \tag{12}$$

This condition must be satisfied by every binding element of the vector **b**:

$$b_i = b_i^{\,0} + \Delta b_i \le b_i^{\,ex}, \forall i \in I_n. \tag{13}$$

where I_n is a set of indexes of binding constraints in (5).

In a quadratic problem of resource distribution stated as:

$$F = cx + x^t Gx, \tag{14}$$

$$\sum_{j=1}^{m} a_{ij} x_j \le b_i, \quad i = 1,...,m-1, \tag{15}$$

$$\sum_{j=1}^{n} x_j = b_m, \tag{16}$$

$$x_j \ge 0, \quad j = 1,...,n \tag{17}$$

the above state conclusions still remain valid with the exception of the fact that the dependence *F(b)* becomes nonlinear (Fig. 2).

The special features of resource distribution problems allow not only finding the optimal value (b) of deficit resources as was described above, but also characterize mutual interdependence between optimal amounts of several deficit resources.

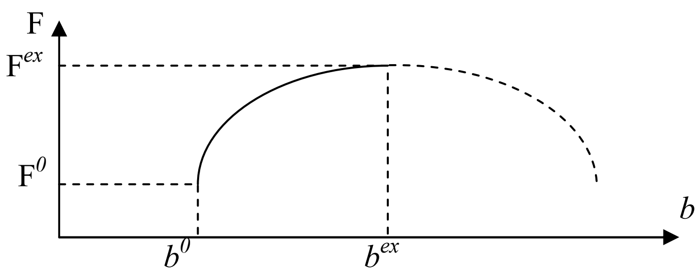

Fig. 2 – Area of changes of resource for a nonlinear problem.

5. ACCOUNTING FOR INTERDEPENDENCE OF RESOURCES

In practice only a subset of inequality constraints of type (5) or (15) are finding at the optimal solutions. For concreteness let us consider the case where there are two effective constraints. Let b_k^0 and b_e^0 be the elements of b^0 corresponding to amounts of binding resources. Moreover suppose that one of these resources, say b_k is more important in the sense that it is harder to increase its available quantity. Let us change the amounts of these resources as: $b_k^0 \to b_k^1, b_e^0 \to b_e^1$ (Figs. 3 and 4).

Such a change of resources is shown as a shift to point *A*. The value of the objective function has improved to F^{-1}.

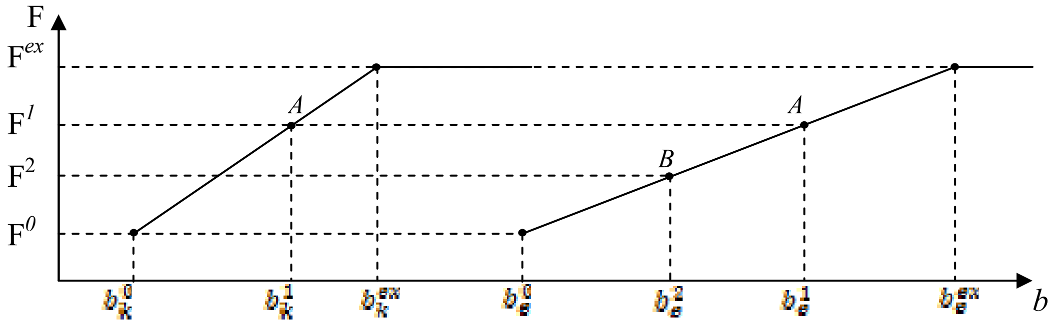

Fig. 3 – The dependence of the objective function value of the resource for a linear problem.

Now let $b_k^0 \to b_k^1$, while $b_e^0 \to b_e^2$, where $b_e^2 < b_e^1$.

In that case (point *B* on the figures) $F^2 (< F^1)$, and the constraint corresponding to more important resource b_k^1 becomes nonbinding. We can state the following proposition:

PROPOSITION 3. *The largest benefit of resource constraints relaxation will be achieved if at the new solution previously binding constraints remain binding.*

It is not hard to show that given a change in resources (Δb_k) the formula

$$b_e = b_e^{ex} - \frac{\Delta a_e}{\Delta a_k} \Delta b_k \tag{18}$$

allows to find the amounts of other binding resources which yield the largest benefit. We sum up the algorithm in the next section.

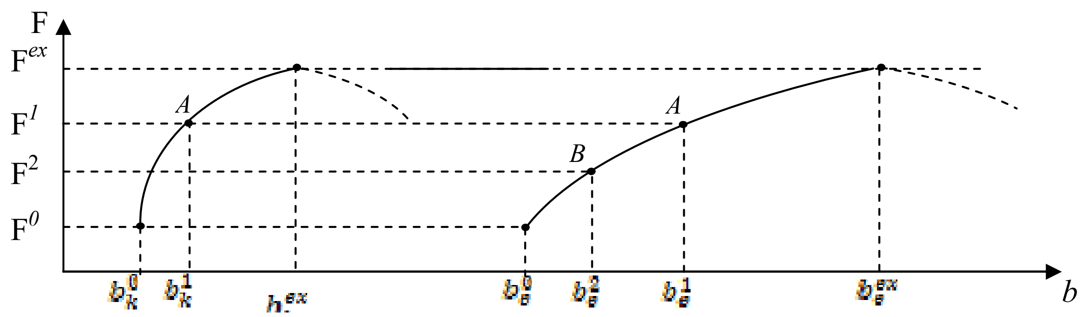

Fig. 4 – The dependence of the objective function value of the resource for a nonlinear problem.

6. THE ALGORITHM OF FINDING OPTIMAL Δb^* IN PROBLEMS OF RESOURCE ALLOCATION

Step 1. Solve the extended problem (9)–(11).

Step 2. Transition to the solution of the original problem (4)–(7) using the extension method.

Step 3. Identify binding constraints in (5). If there is only one binding constraint, then determine the optimal area of resource changes according to formula $\Delta b^* = b^{pc} - b^0$ and transition to the end of algorithm, step 6. If however there are several binding constraints, go to the next step.

Step 4. Change the more important resource within: $\Delta b_k = b_k{}^{pc} - b_k{}^0$.

Step 5. Compute optimal new values of other resources according to formula (18) and return to step 4.

Step 6. End of the algorithm.

7. CONCLUSIONS

The results of this study were used in the development of a control system of a large grain storage and processing company Tcesna-Astyk. The production structure of this company is dominated by an extended network of sequential and parallel production processes. Each production phase of grain processing is performed on several parallel processing units with slightly different technological parameters. Because of this structure, the model of grain processing system had a system of constraints with small parametric differences among them. Experimental realization of our computable simulation model for this production structure, allowed us to identify and redeploy underused resources thus increasing production capacity by 5 per cents. Numerical computation of this resource allocation model has shown the effectiveness of the extension method [9, 10] for finding solutions of the problem with small parametric differences among its constraints.

REFERENCES

1. GOLSHTEIN E.G., YUDIN D.B., *New direction in linear programming methods*, M.: Soviet Radio, 1992.
2. DONTCHEV A.L., *Perturbations, approximations and sensitivity analysis of optimal control systems*, Springer Verlag, 1983.
3. MARTINEZ J.A., DE LEÓN F., DINAVAHI V. *Simulation tools for analysis of distribution systems with distributed resources*, Present and future trends IEEE PES General Meeting, Minneapolis, July 2010.
4. KERSTING W.H., *Distribution System Modeling and Analysis,* CRCPress, 2nd Edition, 2007.
5. STRUCHENKOV V.I., *Combined algorithms of optimal resource allocation*, Applied Mathematics, *3*, pp. 78-85, 2012.
6. IZUMI K., YAMASHITA T., KURUMATANI K. *Analysis of complexity and time restriction in resources allocation problems*, (Conference Paper) Lecture Notes in Economics and Mathematical Systems, **567**, pp. 265-278, 2006.
7. TIKHONOV A.N., ARSENIN V.Y., *Methods for solving illposed problems*, M.: Science, 1976.
8. PERVOZVANSKYI A.A., *Optimization of system with weak coupling*, Systems Science, **2**, pp. 23-32, 1979.
9. NIKOLOUZOU E.G., SAMPATAKOS P.D., VENIERIS I.S., *Evaluation of an algorithm for dynamic resource distribution in a differentiated services network*, Proceedings of the IEEE International conference on networking, Colmar, France, July 9-13, 2001.
10. WEIKARD H-P., SEYHAN D. *Distribution of phosphorus resources between rich and poor countries: the effect of recycling, Ecological economics*, *8*, pp. 1749-1755, 2009.
11. CANNON S., REHMAN S., MENDEZ A., VO V., ORDONEZ A., SINGH A., *Optimization of resource distribution in the George Mason university parking system*. http://seor.gmu.edu/projects/Capstone_2002/Parking.pdf
12. PERVOZVANSKYI A.A., GAYZGORI V.G., *Decomposition, aggregation and approximate optimization*. M.: Science, 1979.
13. SHUKAYEV D.N., *Computer Simulation*, Almaty: KazNTU, 2004.
14. SHUKAEV D.N., KIM E.R., *Extension method in location problem with discrete objects*, Proceedings of the 21st IASTED International Conference “Modeling and Simulation (MS 2010)”, Banff, Alberta, Canada, pp. 270-274, 2010. http://www.actapress.com/Abstract.aspx?paperId=40843

15. SHUKAYEV D.N., KIM E.R., SHUKAYEV M.D., ERGALIEVA N.O., MEREKE A.A., *Modeling resource flows and allocations in systems with parallel structure*, Proceedings of the 23rd IASTED International Conference Applied Simulation and modeling (ASM 2012), Napoli, Italy, 2012, pp. 57-63.
16. LANCASTER K., *Mathematical economics*, New York, 1968.
17. KELTON W.D., LAW A.M., *Simulation modeling and analysis*, St.-Pb: Piter; Kiev BHV Publishing Group, 2004.
18. GENTLE J.E., *Random Number Generation and Monte Carlo Methods*, 2nd ed., Springer, Berlin/Heidelberg, pp. 696-030, 2003.
19. OMMANI A.R., CHIZARI M., SALMANZADEH C., HOSSAINI J.F. *Extension methods and organizational characteristics for supporting sustainable water resource management in agriculture of Iran*, Journal of Applied Sciences, **9**, *3*, pp. 567-572, 2009.

Received January 5, 2015