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SECOND LAW ANALYSIS AND CONSTRUCTAL DESIGN OF STIRLING ENGINE HEAT EXCHANGER (REGENERATOR) FOR MEDIUM TEMPERATURE DIFFERENCE (MTD)

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Abstract. This paper presents an analysis into the effects that the regenerator length, regenerator matrix choice and dead-volume ratio have on medium temperature difference (MTD) Stirling engine performance. The Stirling engine has potential for use in the renewable energy industry and is suitable for use with low grade heat sources. The aim of the investigation is to give insight into the effects that the choice of regenerator properties and dead-volume ratio have on engine performance. The higher the effectiveness of the regenerator, the higher the thermal efficiency of the Stirling engine. Increasing the length of the regenerator increases the effectiveness, but results in an increased pressure drop. Decreasing the length results in a decreased pressure drop, but the heater and cooler loads increase decreasing engine performance. Therefore, there is an optimal regenerator length that gives optimal engine performance. In this paper, an engine of volume 1 000 cm^3 is analyzed at four different MTD heater inlet temperatures, namely 150°C, 200°C, 250°C and 300°C. Quasi-steady flow conditions and finite source and sink heat capacity rates are assumed and the exergy analysis approach is used to optimize the configuration. Results show that for each source temperature and mesh type there is a regenerator length and dead-volume ratio that give optimal engine work output.

Key words: Stirling engine, Regenerator, Ideal adiabatic, Entropy generation, Optimization.

1. INTRODUCTION

Currently there is major concern over the future availability of fossil fuels and the effect that these fuels have on the environment. Renewable energy sources are currently considered the most effective solution to these problems [1]. The Stirling engine is suitable for use with renewable energy sources, has multi-fuel capabilities, is quiet, and efficient [2]. There are different orders of Stirling engine mathematical models used in the analysis of Stirling cycle machines [3], these approached vary widely in complexity and accuracy. Stirling engines are often categorized in terms of their heater wall temperature. Low temperature difference engines having a heater wall temperature of between 80 °C and 150 °C and medium temperature difference engines having a wall temperature of 150 °C to 400 °C [4].

There have been several efforts to optimize LTD and MTD engines, as these types of engines are of economic interest for use with cheaper non-concentrating solar collectors [5]. An analysis was presented in [6], which looked to analyze and optimize the performance of an LTD engine [7] using a new three component second order model. In [8], an exergetic, energetic and entropic analysis of the Stirling cycle was presented. The cycle was then optimized per these criteria. The same authors conducted a thermodynamic analysis of an LTD Stirling engine at steady state operation [9]. The result of the analysis was the optimal conditions for operation which is the minimum amount of exergy destruction or production of entropy and thus, minimization of operating cost. In [10], the effect of changing the heat transfer coefficients and temperature difference in the engine was investigated. The analysis in [11], looked at the effect of pressure loss and irreversible heat transfer and found that the maximum attainable efficiency is half of Carnot efficiency. The analysis presented in [12], optimized a mean temperature deferential solar Stirling engine with several loss mechanisms incorporated. Several investigations have been conducted into the effect of the regenerator on Stirling engine performance. The analysis presented by de Boer utilizes a first order model to optimize the regenerator, showing that the maximum attainable efficiency is half of the Carnot efficiency [13]. Another analysis looked at a 2-dimensional model of the regenerator, analyzing the different modes of temperature oscillation [14]. Additionally, the constructal law has been applied to regenerators to optimize flow architecture [15], and could be used to minimize temperature oscillations. Furthermore, there have also been several investigations into the effects of dead-volume on Stirling engine performance. The analysis conducted in [16], looked at the effect of dead-volume on the ideal isothermal model. This analysis found that dead-volume decreases the power output and efficiency of the engine, with the optimal dead-volume ratio being zero. However, as established in [17] the dead-volume ratio is an important design parameter that greatly affects the performance of real machines. There have been several authors who have emphasized the great importance of optimizing the global performance of a system by spreading the irreversibility between components [18].

This paper presents a dynamic model with finite source and sink heat capacity rates, pressure drops, irreversible heat transfer, and thermal conduction between the hot and cold components. The exergy analysis methodology is used to optimize the different variables. A 1 000 cm^3 engine is optimized for maximum power output at 4 different MTD heater inlet temperatures with four different regenerator mesh types.

2. ANALYSIS

The following section briefly discusses the mathematical model of the irreversible Stirling engine which was used in the analysis, and the formulation of the objective function.

2.1. Stirling engine model

The Stirling engine model used is the ideal-adiabatic model, developed by Urieli and Berchowitz [19]. The quasi-steady flow assumption is made and empirical relations are used to calculate the heat transfer and flow losses in the different heat exchangers. The relations proposed by [20] are used to calculate the regenerator effectiveness and pressure drop and the standard steady turbulent flow relations are used to calculate the heat transfer coefficients and frictions factors in the heater and cooler. These values are then used to calculate the rate of entropy generation which is an input into the objective function.

2.2. Objective function

The objective function was formulated using the exergy analysis approach, where exergy is defined the maximum amount of energy available to be converted to work. Combining the mathematical expressions for the first and second laws of thermodynamics gives equation 1, the exergy equation

$$\dot{W} = -\frac{d}{dt}(E + P_0 V - T_0 S) + \sum_{i=1}^{n}\left(1 - \frac{T_0}{T_i}\right)\dot{Q}_i + \sum_{out}\dot{m}(h^0 - T_0 S) - \sum_{in}\dot{m}(h^0 - T_0 S) - T_0 \dot{S}_{gen}\,. \tag{1}$$

Rewriting equation 1 to describe the maximum net-work output of the Stirling engine yields equation 2, this equation incorporates the entropy generation rate in each component in the Stirling engine which is calculated using the empirical relations describing heat transfer and fluid flow in the engine. This equation is the objective function used to optimize the geometry, volume ratios and speed of the engine.

$$\begin{aligned}\dot{W}_{net} = \dot{Q}_{net} &- T_0\left[\frac{R}{2\pi}\oint\left|\dot{m}_h(\theta)\right|\left|\ln\left(\frac{P_{eb}(\theta)}{P_{hb}(\theta)}\right)\right|d\theta + \frac{\dot{Q}_{h,loss}}{T_0}\right]_{heater} \\ &- T_0\left[\frac{C_p}{4\pi}\ln\left(\frac{T_{kb}T_{hb}}{T_k T_h}\right)\oint\left|\dot{m}_r(\theta)\right|d\theta + \frac{R}{2\pi}\oint\left|\dot{m}_r(\theta)\right|\left|\ln\left(\frac{P_{hb}(\theta)}{P_{kb}(\theta)}\right)\right|d\theta\right]_{regenerator} . \\ &- T_0\left[\frac{R}{2\pi}\oint\left|\dot{m}_k(\theta)\right|\left|\ln\left(\frac{P_{kb}(\theta)}{P_{cb}(\theta)}\right)\right|d\theta + \frac{\dot{Q}_{k,loss}}{T_0}\right]_{cooler}\end{aligned} \tag{2}$$

2.3. Method of solution

In the case of the ideal adiabatic model no closed form solution exists and it is therefore necessary to find the solution using an iterative method. To quickly and effectively find the solution, the 4th order Runge-Kutta method is used for the first 4 steps, following this the 4th order Adams-Bashforth method is used. The iterative scheme is run until convergence is reached between the start and end temperatures in the expansion and compression spaces. The heat transfer rates are then obtained, and using the empirical relations the effectiveness's of the heat exchangers are calculated and used to calculate new heater and cooler temperatures. This is run continuously until convergence between the previous and current temperature is achieved.

3. NUMERICAL OPTIMISATION

3.1. Model parameters

The working fluid in the engine is assumed to be air and model parameters that are assumed to be constant are seen in table 1.

Table 1

Table of assumed model values

Symbol	Description	Value
V_{tot}	Total engine volume (cm^3)	1000
α	Phase difference (°)	90
N_h	Number of heater and cooler tubes (-)	250
N_k	Number of cooler tubes (-)	250
C_h	Heat capacity rate of source (kJ/K)	0.25
C_k	Heat capacity rate of sink (kJ/K)	0.25
P_{mean}	Mean engine pressure (bar)	25

Four different regenerator mesh types are used and their respective properties are seen in Table 2.

Table 2

Table of regenerator wire netting dimensions [20]

Symbol	Diameter (mm)	Porosity (-)
WN50	0.23	0.645
WN100	0.1	0.711
WN150	0.06	0.754
WN200	0.05	0.729

3.2. Optimization procedure

The model has five variables that are optimized at each specified dead-volume ratio. These variables are the length of regenerator to total heat exchanger length (L_r/L), the cooler and heater tube diameter (D), the ratio of heater length to cooler length (L_h/L_k), the swept volume ratio (V_c/V_e), and the engine speed. The objective function, which is the maximum net-work output is optimized in terms of the input variables at different dead-volume ratios using the Nelder-Mead algorithm.

4. RESULTS AND DISCUSSION

The following section presents the results of the numerical optimization: the maximum net-work output, optimal thermal efficiency, regenerator length and regenerator effectiveness are all plotted versus dead-volume ratio. Each figure has four plots, each plot representing a different heater inlet temperature which is in the range of medium temperature difference (MTD) source temperatures.

Figures 1 and 2 are plots of maximum net-work output, and thermal efficiency versus dead-volume ratio.

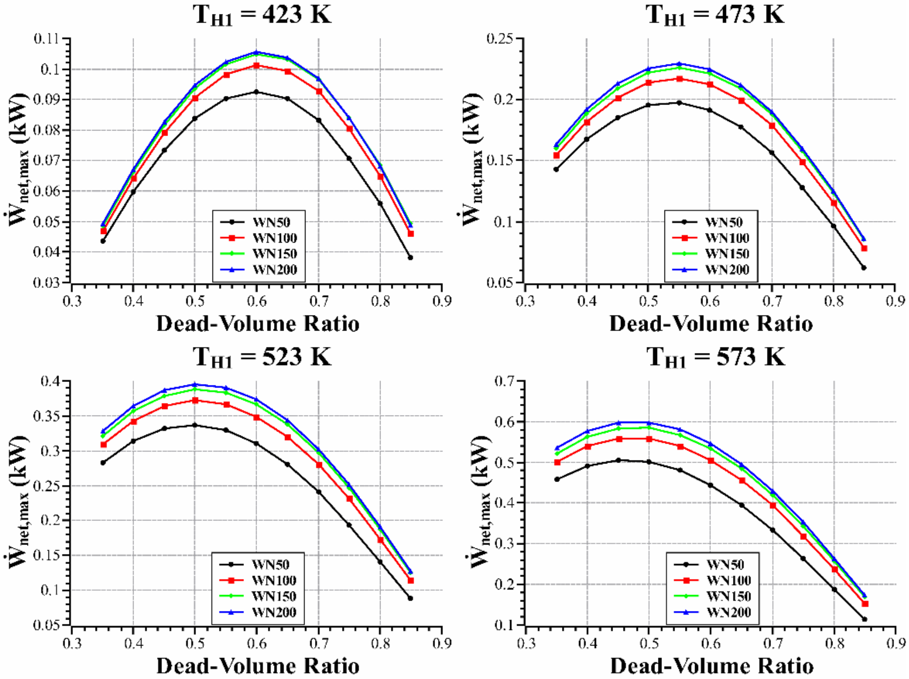

Fig. 1 – Network output versus dead-volume ratio at the four different source temperatures.

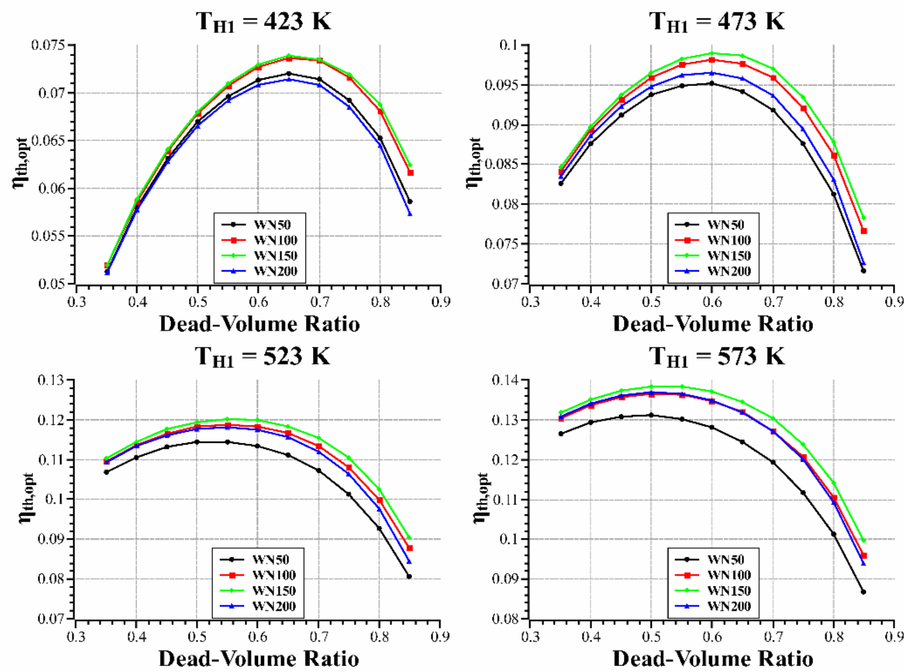

Fig 2 – Thermal efficiency *versus* dead-volume ratio at the four different source temperatures.

Figures 1 and 2 show that there is an optimal dead-volume ratio for maximum net-work output and one for maximum thermal efficiency. Looking at the plots it is seen that the optimal dead-volume ratio for maximum work output is slightly lower than the optimal dead-volume ratio for maximum thermal efficiency. This is because the ideal model shows that the optimal dead-volume ratio for maximum thermal efficiency asymptotically approaches 1, whereas for maximum work output it is less than one and decreases with increasing heater temperature. Although, due to the other losses included the difference is not as pronounced.

It is also seen that for maximum net-work output the WN200 mesh performs best which is not the case for the maximum thermal efficiency. This is due to the conductive thermal bridging loss which is large in the case of the WN200 mesh as it gives the smallest optimal regenerator length of all the mesh types analyzed.

Figures 3 and 4 are plots of optimal regenerator length, and effectiveness versus dead-volume ratio.

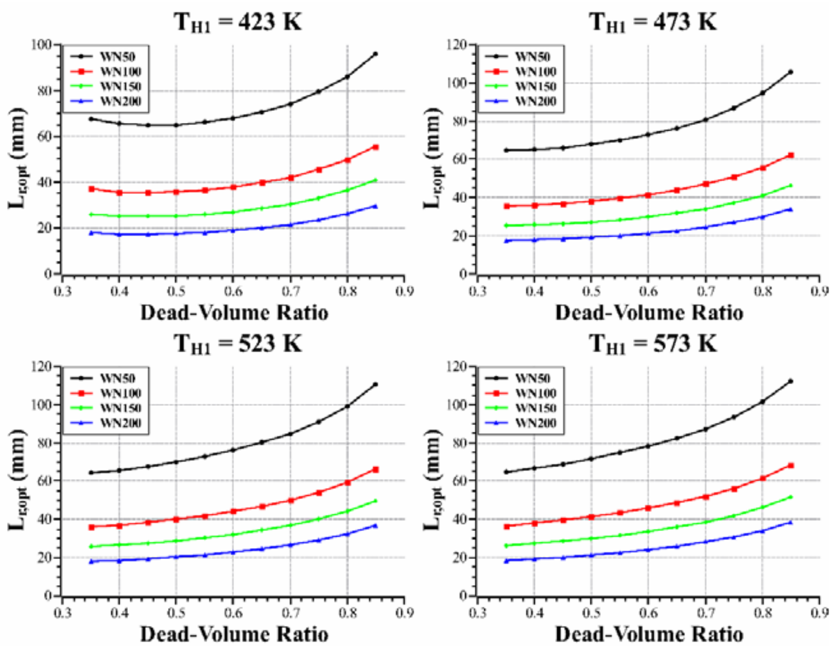

Fig. 3 – Regenerator length versus dead-volume ratio at the four different source temperatures.

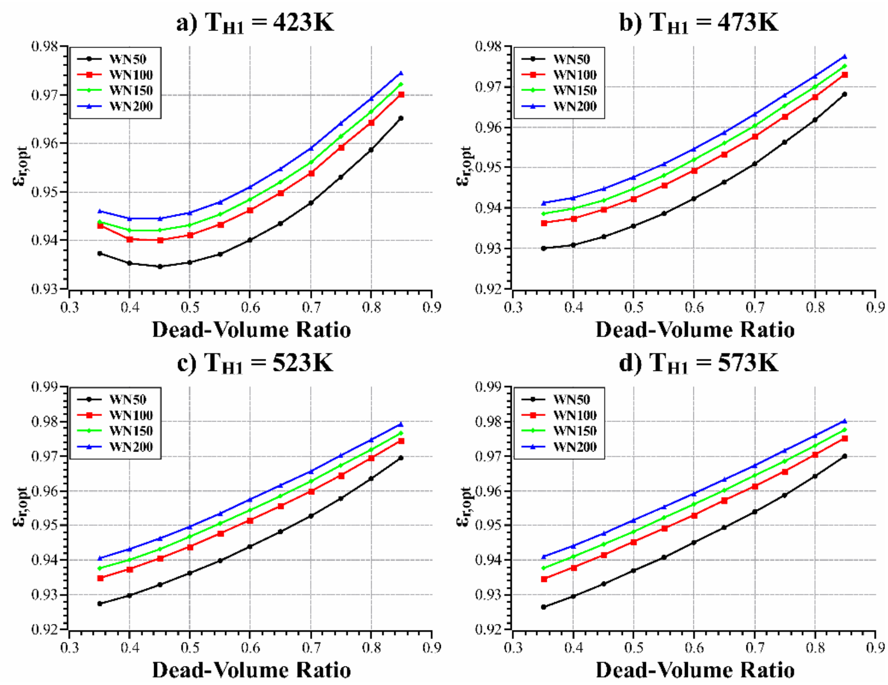

Fig. 4 – Regenerator effectiveness versus dead-volume ratio at the four different source temperatures.

Figures 3 and 4 show that as the heater inlet temperature increases the optimal regenerator length and optimal regenerator effectiveness both increase. From the plots, it is seen that the WN200 mesh type gives the shortest optimal regenerator with the highest effectiveness. Figure 1 confirms this, as the WN200 gives the highest net-work output this is because it gives the highest effectiveness and shortest regenerator (resulting in a lower pressure drop) and therefore the lowest irreversibility rate at a fixed engine speed.

5. CONCLUSION

The optimization of a 1 000 cm^3 MTD Stirling engine is presented and it illustrates the effect of regenerator mesh type on engine performance. In the analysis, the input energy is not fixed which makes the solution space highly complex. The analysis shows that in terms of performance the regenerator has a significant effect. The WN200 mesh type is seen to result in the greatest net-work output, greatest effectiveness and the shortest regenerator. However, it is not seen to give optimal thermal efficiency. This shows that the thermal bridging loss is an important factor to consider in Stirling engine design.

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