**THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY**

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 19, Number 2/2018, pp. 385–392

STRUCTURAL EFFICIENCY ON PLASTIC COMPOSITES

Mihai BEJAN

“Mircea cel Bătrân” Naval Academy
E-mail: `bejan.mihai@gmail.com`

Abstract. Current structural optimization techniques does not provide a structure with all its volume working at allowable stress in order to use the material at its full potential. Even so, the structure is considered optimized. In order to overcome this disadvantage of considering optimized, we will introduce *The Structural Efficiency Percentage* Ξ (Csi/Xi). The above mentioned procent quantifies the working stress level of the material in the whole structure having all stresses below the allowable level. Only according to the result, the engineer will take the decision to improve the structure performing further optimization or not. The Ξ percentage offers an exact value of structure used potential, ideally to achieve fully – stressed design. Calculating Ξ is a necessary step into designing a structure as close as possibile to the ideal structure, with all the volume working at allowable stress level.

Key words: structural efficiency, layered composites, trusses.

1. INTRODUCTION

Presently, it is an increasing concern in using material resources into the most efficient way. Pure mathematical optimization algorithms [1] and numerical finite element method are combined into a new discipline – structural optimization [2]. First development was focused on design, more specific on size and shape. Afterwards, the topological optimization was taken into consideration [3, 4]. The optimization problems can be stated as gaining the best structural behavior using a limited material resource or using a minimum material quantity in order to achieve a required structural response.

Current structural optimization techniques does not provide a structure with all its volume working at allowable stress in order to use the material at its full potential – allowable stress $\sigma_{allowable}$ – and even so the structure is considered optimized [5]. Widely spread objective function to be minimized in structural optimization is structure volume, subjected to allowable stress constraint. This is equivalent to have as much possible from the entire structure with stress less than or equal to (in the ideal case) allowable stress.

In order to overcome this disadvantage of considering optimized, we will introduce *The Structural Efficiency Percentage* Ξ (Csi/Xi). The above mentioned percentage quantifies the working stress level of the material in the whole structure. It offers an exact value of structure used potential, ideally to achieve fully – stressed design [6, 7].

Structural efficiency formulation was already used [8, 9] but the approach was totally different from the concept presented in this paper.

2. STRUCTURAL EFFICIENCY PERCENTAGE

2.1. Concept and Motivation of Using Structural Efficiency Percentage

An ideal optimum structure is that one with all its volume working at allowable stress ($\sigma_{allowable}$) level. This ideal structure has a 100% efficiency percentage ($\Xi = 100\%$). The necessary step from a real structure, representing an feasible solution in the design space, to one as close as possible to the ideal one is to quantify the working stress level of the material in the whole structure. That is the reason why we will use the concept of “*Structural Efficiency Percentage*” Ξ (Csi/Xi) [10]. Only after calculating the Ξ percentage, the engineer will decide to perform further optimization or not. Ideally we will achieve fully – stressed design.

The structural analysis of real structures usually use numerical finite element method. Analytical equations can be used for simple loads and structures and for demonstrative reasons, as it is the present example.

2.2. Homogeneous Structures

Using a fine mesh, the von Mises stresses over each finite element is cvasi-constant σ_i^{elem} and the weighted by volume average element stress of the whole finite element model is

$$\sigma_{avg}^{elem} = \frac{\sum\left(\sigma_i^{elem} \cdot V_i^{elem}\right)}{\sum V_i^{elem}}, \tag{1}$$

where V_i^{elem} is the volume of the i^{th} finite element.

The structural efficiency percentage (Csi/Xi) is defined as

$$\Xi^{elem} = \frac{\sigma_{avg}^{elem}}{\sigma_{allowable}} \cdot 100[\%], \tag{2}$$

and a graphical representation of quantities implied in its definition is given in Fig. 1.

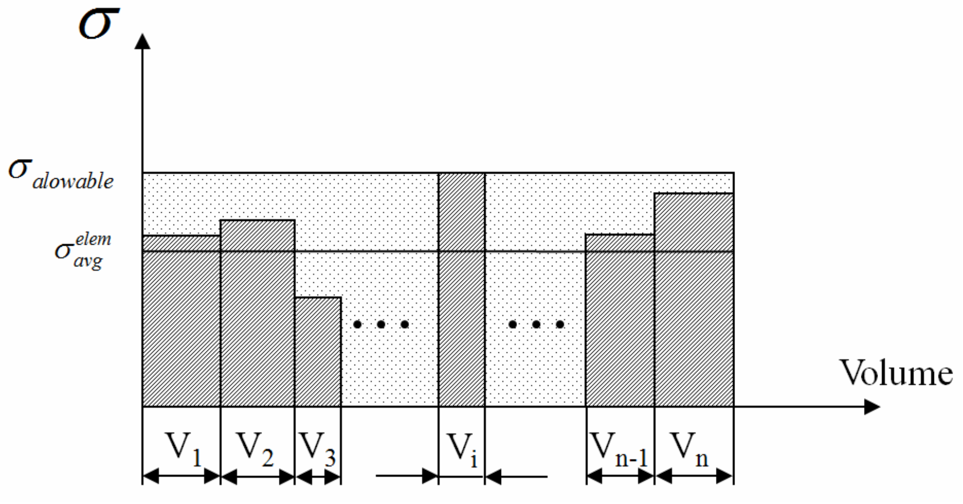

Fig. 1 – Calculus of the structural efficiency percentage.

2.3. Composite Structures

In analyzing composite structure, we must take into consideration a very important aspect: different materials usually do not have the same allowable stress. So, the first step we need to calculate the structural efficiency percentage of each material.

$$\Xi_{(mat\,j)}^{elem} = \frac{\sigma_{avg\,(mat\,j)}^{elem}}{\sigma_{allowable\,(mat\,j)}} \cdot 100[\%]. \tag{3}$$

The next and final step is to combine the effects of constituent materials which compose the structure

$$\Xi^{elem} = \frac{\sum\left(\Xi_{(\mathrm{mat}\,j)}^{elem} \cdot V_{(mat\,j)}^{elem}\right)}{\sum V_{(mat\,j)}^{elem}} \cdot 100[\%]. \tag{4}$$

3. THE PHYSICAL MODEL OF PLASTIC LAYERED COMPOSITE

3.1. The Lamina Model

Studying layered composites, the researcher looking for detailed aspects is focused onto a lamina level. The lamina model used in this paper as an example is revealed in figure 2.

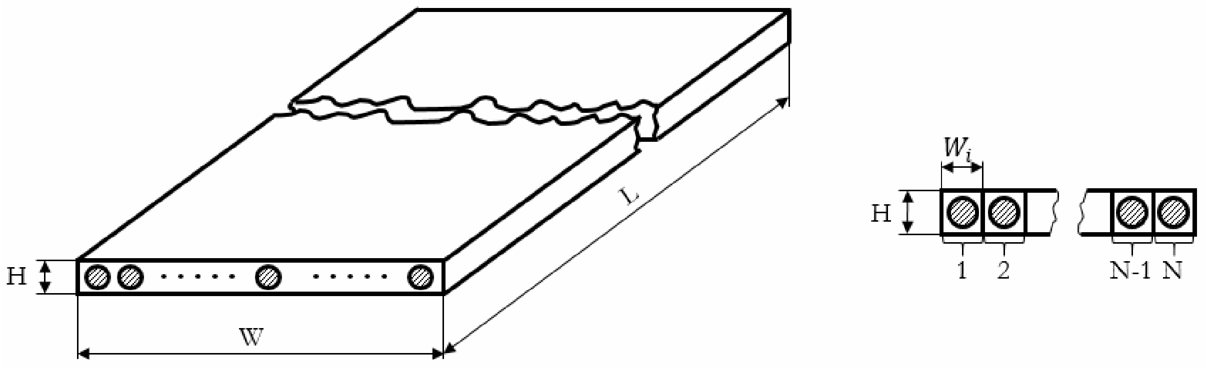

Fig. 2 – The Lamina model.

The lamina consists in one single layer having a $H = 0.5$ mm height, a $W = 10$ mm width and a $L = 100$ mm length. There are 20 fibers ($N = 20$), each having a diameter of 0.3989422804 mm. For this particular geometry results

$$V_{fibers} = A_{fibers} \cdot L \tag{5}$$

$$A_{fibers} = A_{fiber} \cdot n_{fibers} = \frac{\pi \ d^2}{4} \cdot 20 = 2.5 \text{ mm}^2 \tag{6}$$

$$V_{total} = A_{total} \cdot L \tag{7}$$

$$A_{total} = W \cdot H = 10 \cdot 0.5 = 5 \text{ mm}^2 \tag{8}$$

These values produce a 50% fiber volume ratio.

$$\text{Fiber Volume Ratio} = \frac{V_{fibers}}{V_{total}} \cdot 100[\%] = \frac{A_{fibers}}{A_{total}} \cdot 100[\%] = 50\%. \tag{9}$$

3.2. The Materials

The material is polyester resin (*Nestrapol 220*) for matrix, with Young's modulus $E_{matrix} = 4 \cdot 10^3$ MPa and Poisson's ratio $\nu_m = 0.4$ and fiber glass for reinforcement fibers with $E_{fibers} = 8 \cdot 10^3$ MPa and $\nu_m = 0.25$ [11].

The ultimate tensile strength of the matrix (*Nestrapol 220*)

$$\sigma_{u,\,matrix} = 50 \text{ MPa}. \tag{10}$$

The ultimate strength of the fiber glass is

$$\sigma_{u,\,fibers} = 4\,000 \text{ MPa}. \tag{11}$$

Usually, in a composite the layers have different orientation but this is a particular case used just in order to introduce the structural efficiency percentage for composite structures.

4. THE ADOPTED ANALYTICAL MODEL

Corresponding to the physical model considered above, it is very convenient to assume a truss modell for both, matrix and fibers. All analysis will be performed below the proportionality limit where the stress – strain curve is caracterized by the Hooke's low.

Starting from Hooke’s law, $\sigma = E \cdot \varepsilon$ and considering only the axial loaded structure as being composed from two trusses forced to have the same displacements we will have the same strain (ε).

$$\begin{aligned} \sigma_{matrix} &= E_{matrix} \cdot \varepsilon \\ \sigma_{fibers} &= E_{fibers} \cdot \varepsilon \end{aligned} \tag{12}$$

$$\frac{\sigma_{\text{matrix}}}{E_{matrix}} = \frac{\sigma_{\text{fibers}}}{E_{\text{fibers}}} \Rightarrow \frac{\sigma_{\text{fibers}}}{\sigma_{\text{matrix}}} = \frac{E_{\text{fibers}}}{E_{\text{matrix}}} = \frac{80 \cdot 10^3}{4 \cdot 10^3} = 20$$

$$\sigma_{\text{fibers}} = 20\,\sigma_{\text{matrix}} \,. \tag{13}$$

Considering the present structure a non – critical one, we will adopt a factor of safety $c = 2$ for both materials. This means that allowable stresses for the above mentioned matrix and , both in traction, are:

$$\begin{aligned} \sigma_{a,matrix} &= 25 \text{ MPa} \\ \sigma_{a,\,fibers} &= 2\,000 \text{ MPa} \end{aligned} \tag{14}$$

When the matrix reaches the allowable stress of 25 MPa, according to (13), the stress in fibers reach a 500 MPa level.

5. STRUCTURAL EFFICIENCY PERCENTAGE CALCULUS FOR ANALYTICAL MODEL

The material efficiency will be calculated with the structural efficiency calculus Ξ for composite materials.

In this particular case equation (3) becomes:

$$\Xi^{elem}_{fibers} = \frac{\sigma^{elem}_{avg\ fibers}}{\sigma_{allowable\ \ fibers}} \cdot 100 \ \ [\%] = \frac{25}{25} \cdot 100\% \ = 100\% \tag{15}$$

$$\Xi^{elem}_{matrix} = \frac{\sigma^{elem}_{avg\ matrix}}{\sigma_{allowable\ \ matrix}} \cdot 100 \ \ [\%] = \frac{500}{2000} \cdot 100\% \ = 25\%. \tag{16}$$

For the above mentioned lamina, using equation (4) we have

$$\Xi^{elem} = \frac{100 \cdot V^{elem}_{fibers} + 25 V^{elem}_{matrix}}{V^{elem}_{fibers} + V^{elem}_{matrix}} \cdot 100\% \ = 62.5\%.. \tag{17}$$

6. NUMERICAL MODEL USING FINITE ELEMENT METHOD

The first intuitive approach is a beam reinforced shell model. It has a great advantage of being simple and requires a reduced calculus volume. In this model, the shell elements would occupy the whole matrix volume V_{matrix} and also the fiber volume V_{fibers}. As $V_{matrix} = V_{fibers} = V_{reference}$ results that $V_{shell} = 2V_{reference}$. The entire structural active model volume would be $V_{total} = V_{shell} + V_{fibers} = 3V_{reference}$. In this case, the real fiber volume ratio would be

$$\text{Shell Model Fiber Volume Ratio} = \frac{V_{fibers}}{V_{total}} 100\% = \frac{1}{3} 100\% = 33.4\%. \tag{18}$$

Considering the above mentioned model data, the shell model produces a significant fiber volume ratio alteration due to volume overlaps. So, the only model entierly scientific correct is the one using solid elements. It must provide accurate results.

The whole model would have a high number of finite elements – 960 000. This model will require considerable hardware and time resources. In order to overcome these inconveniences we will use the symmetry advantages and analyze an eighth of the real structure – Fig. 3. The real structure has three symmetry planes and the corresponding displacements of the model are zero in order to simulate the real behaviour.

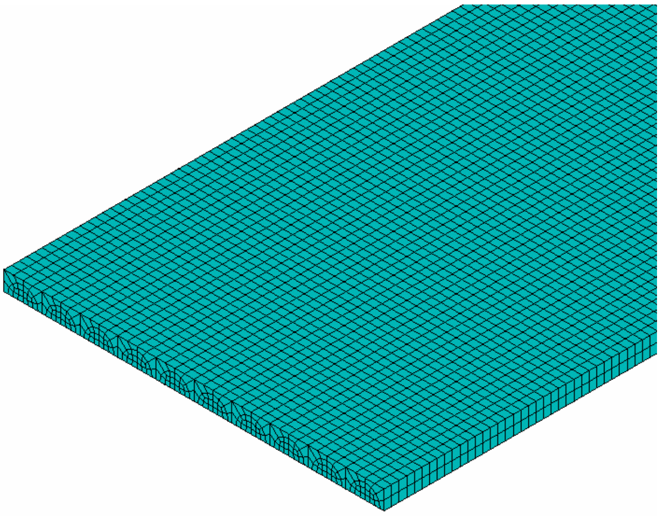

Fig. 3 – The finite element model.

The finite element type used for mesh is eight node brick – SOLID 185 in ANSYS code [12]. This model has 120 000 elements and 151 803 nodes. The dimensions of the elements are between $4.8462626906 \cdot 10^{-2}$ mm and 0.1540822504 mm.

6.1. The Loads

Considering equations (12), (13) and (14), the common strain is

$$\varepsilon = \frac{\sigma_{matrix}}{E_{matrix}} = \frac{\sigma_{fibers}}{E_{fibers}} = \frac{25}{4 \cdot 10^3} = \frac{500}{80 \cdot 10^3} = 0.00625. \tag{19}$$

The imposed displacements are

$$\delta = \varepsilon \cdot L = 0.00625 \cdot 100 = 0.625\text{mm}. \quad . \tag{20}$$

We will apply the displacements as loads of the finite element model. Because we used the symmetry, the applied displacements are

$$\delta_{FEM\ \mathrm{model}} = \frac{\delta}{2} = 0.3125\ \mathrm{mm}. \tag{21}$$

6.2. Results

The stress results are presented in figure 4. Being modeled highly detailed, the adopted stress criteria is von Mises stress on element.

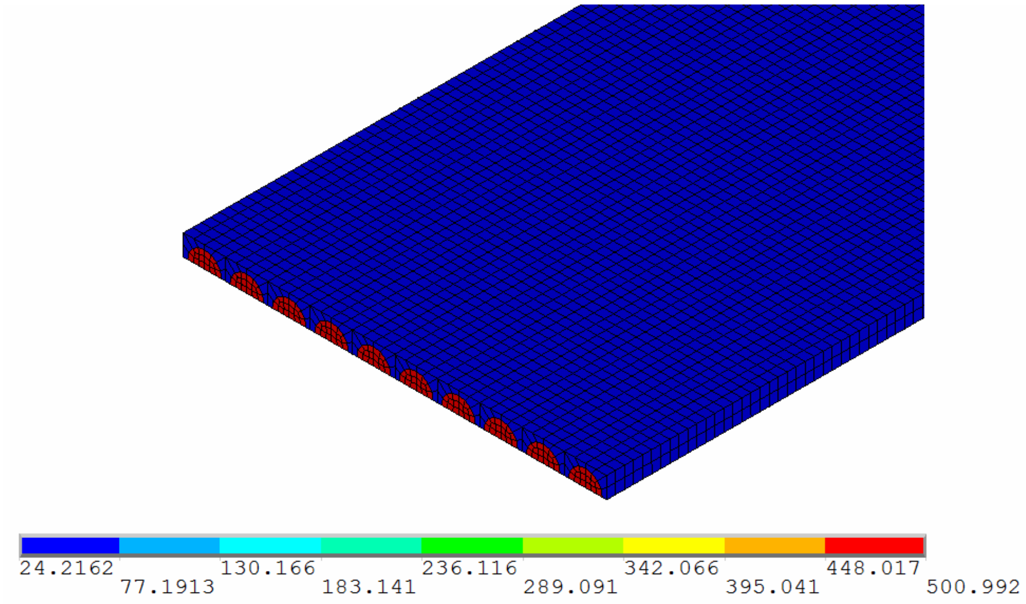

Fig. 4 – The von Mises stresses on elements.

5. STRUCTURAL EFFICIENCY PERCENTAGE CALCULUS FOR NUMERICAL MODEL

In Fig. 5 it is presented the calculus of structural efficiency percentage algorithm for compozite structures. In this algorithm *NM* is *number of materials*, *NE* is *number of elements* and *mat j* is *material number*.

Based on the above mentioned algorithm we developed a software using MATLB.

We extract the elements volumes and the corresponding von Misses stresses. Processing these data with our proprietary software we also obtain a 62.5 % structural efficiency.

6. CONCLUSIONS

Using the analytical calculus the result is 62.5 % structural efficiency.

Processing the data from finite element model we obtain the same result of 62.5 % structural efficiency.

The analytical model validates the numerical calculus using finite element method. It is important because in future calculus and developments it is supposed that we will use finite element method combined with the structural efficiency software developed by us.

We must notice the strange situation. Using classical approach, we achieved structural optimum, with all matrix volume at allowable stress level, which is desirable,but considering the fibers too, we observe that in reality the structure use only 62.5 % from its theoretical potential. This is far away from the ideal situation. The real evaluation using structural efficiency percentage is the first step to improve further structures.

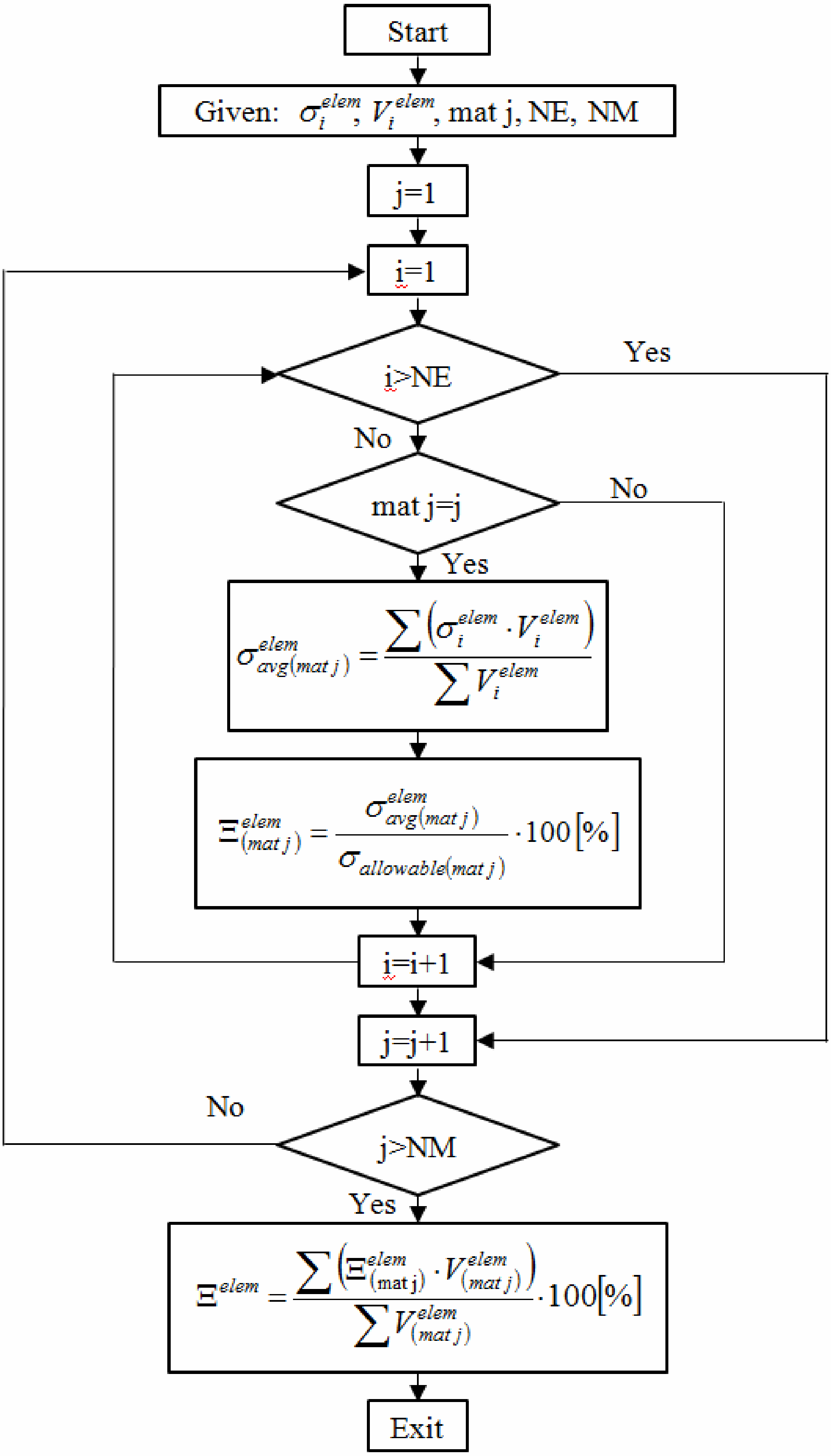

Fig. 5 – Calculus of structural efficiency percentage algorithm for compozite structures.

REFERENCES

1. Harish GARG, *Solving structural engineering design optimization problems using an artificial bee colony algorithm*, Journal of Industrial and Management Optimization, **10**, *3*, pp. 777–794, 2014.
2. Garret N. VANDERPLAATS, *Multidiscipline Design Optimization,* Vanderplaats Research & Development, Inc., 2007.
3. Ming ZHOU & Raphael FLEURY, *Fail-safe topology optimization*, Structural and Multidisciplinary Optimization, **54**, *5*, pp. 1225–1243, 2016.
4. Navid CHANGIZI, Hamid KABOODANIAN & Mehdi JALALPOUR, *Stress-based topology optimization of frame structures under geometric uncertainty*, Computer Methods in Applied Mechanics and Engineering, **315**, pp. 121–140, 2017.
5. Diana Lilia BANDRABUR, Mihai BEJAN, Angela MUNTEAN, Corneliu MOROIANU & Gh. SAMOILESCU, *Improving Structural Behaviour of a Medical Rod*, The 19th International Symposium "Intelligent Manufacturing & Automation: Focus on Next Generation of Intelligent Systems and Solutions" Trnava Slovakia ISI Proceedings, **19**, *1*, pp. 071–072, 2008.
6. Ali AHRARI & Kalyanmoy DEB, *An improved fully stressed design evolution strategy for layout optimization of truss structures*, Computers and Structures, **164**, *C*, pp. 127–144, 2016.
7. Avanti PATRIKAR & K.K. Pathak, *Fully Stressed Design of Fink Truss Using STAAD.Pro Software*, Open Journal of Civil Engineering, **6**, pp. 631–642, 2016.
8. Stuart C. BURGESS, *The Ranking of Efficiency of Structural Layouts Using Form Factors*, Journal of Mechanical Engineering Science, **212**, *2*, pp. 117–128, 1998.
9. Zhou JIANLONG, Bao LIANJIN & Qian PENG, *Study on Structural Efficiency of Super-Tall Buildings*, International Journal of High-Rise Buildings, **3**, *3*, pp. 185–190, 2014.
10. Mihai BEJAN, *Structural Efficiency*, Annals of DAAAM for 2010 & Proceedings of the 21st International DAAAM Symposium, **21**, *1*, pp. 141–142, 2010.
11. Elena ALĂMOREANU & Dan Mihai CONSTANTINESCU, *Proiectarea plăcilor compozite laminate*, Editura Academiei Române, 2005.
12. ANSYS User Guide, ANSYS Inc., 2013.

Received September 19, 2017