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3 STAGE HELICAL SPEED REDUCER PARTIAL GEAR RATIOS OPTIMAL DETERMINATION USING GENETIC ALGORITHMS

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Abstract. Designing compact multi-stage speed reducers are challenging demands of nowadays mechanical power transmission manufacturer. In this paper a Genetic Algorithm (GA) was used for obtaining optimal models for partial gear ratios in order to achieve a 3 stage helical speed reducer with minimum mass or length. The objectives were described by a set of 18 design variables of mixed nature and were subjected to a highly non-linear set of 57 engineering constraints. The proposed methodology automates the design process and the results obtained by using GA conduct to expressions for partial gear ratios, which offer an even distribution on all 3 stages for the total transmission ratio with better design solutions for both the objectives as compared with the ones given in literature.

Key words: optimal determination of partial gear ratios, Genetic Algorithms, 3 stage helical speed reducer, automated optimal design.

1. INTRODUCTION

Gears are nowadays widely used in various mechanical engineering systems application, from different fields and working conditions. Their complex shape and geometry require a large number of design variables (typically, well over ten [19] – of different types: integers, discrete and real), resulting a complicated and difficult design process. Furthermore, gearing design is invariably based on iterations and making decisions– which unfortunately are always compromises. Considering all these remarks, it is obviously that the manual design (i.e. a trial and error type method) of gearing is very difficult and there is a need for computer-aided design. Moreover, continuously increasing and challenging demands of compact and reliable gears, force the mechanical designers to consider more and more the optimal design methodology [16]. In the last decades many researchers have paid attention on this problem of gear optimization. Ramamurti et al. in [14] presented a design methodology for two-speed gearbox. Huang et al. developed an interactive physical programming in order to optimize a three-stage spur gear reduction unit [11]. Abersek et al. in [1] developed an expert system to design and manufacture a gearbox. Yokota et al. in [20] solved an optimal weight design problem of a gear with an improved GA. Deb and Sachin, in [6] used a non-dominated sorting genetic algorithm (NSGA-II) in order to solve a multi-objective optimization of a multi-speed gearbox. Thompson et al. [18] presented a generalized optimal design of two stage and three stage spur gear reduction units in a formulation with multiple objectives. Gologlu and Zeyveli in [8] applied GA to minimize the volume of a two-stage helical gear train.

All the above studies were mentioned to highlight the importance of using modern global optimization techniques in mechanical power transmission design. In here the author extends the technology to the broader design space of a 3 stage helical speed reducer gearings (Fig. 2) whose every defining element is subject to change throughout the optimal design process. This discussion represents only the first part from a broader study (in order to build up a generic transmission system design tool based on the evolutionary optimization concepts [5]) at the end of which the author aims to optimize the complete 3 stage speed reducer. The large complexity of the design problem and the author's experience in this field [4,5,19] conducts to a step-by-step procedure which allow finding out which are the dimensional tendencies of the speed reducer's components and how the functional and structural interdependencies affect them. As you will see in Section 4, there is a need of 18 design variables for describing the optimal design problem! Assuming their definition domains (Table 1), there could be obtained a number of possible helical gearings designs of the order of 4×10^{31}! Once this preliminary phase is completed the virtual design space of the helical gearings will be generated and the problem formulation will be extended with the shafts and the housing subsystems. Therefore, a systematic approach [5] in solving this optimal design problem is essential to achieve the desired level of reliability.

Into the next Section, the current procedure for designing speed reducers is introduced, after which, is presented a short description of the general principle of the proposed Genetic Algorithm (Section 3), followed by a detailed discussion regarding the statement of the optimal design problem (Section 4). The fifth Section contains an effective example and a detailed presentation and comparison of the numerical results solutions. Eventually, discussion is concluded with some reflections and suggestions regarding the possible extensions of the present study.

2. THE CURRENT PROCEDURE FOR SPEED REDUCERS DESIGN

Commonly methods (trial and error) for designing mechanical gear transmissions involve some difficulties considering the multiple interactions between its subsystems. Furthermore, design is an iterative and decision-make process [17]. The whole speed reducer design process start from a set of input data with information regarding the input power P_m (kW), the rotational frequency of the pinion rotation n_m (rpm), the total transmission ratio i_T, the working time of the gearings L_h (h), and the layout drawing (Fig. 2). The first phase of the design process (A) consists in making two important preliminary selections, regarding the partial gear ratios for all the stages (A.1) and about the materials (including here the hardness and the thermal treatment) for the gears (A.2). As concerns the total transmission ratio splitting, it is known that represents a key decision which has a major impact over the entire design process with remarkable influence over the mass and the cost of the gear transmission [21]. There are a few studies which tackle this difficult problem. Some of them use different graphics to determine the partial gear ratios. See for example the studies of Kudreavtev et al. [12] (Fig. 1a), and Niemann [13] (Fig. 1b).

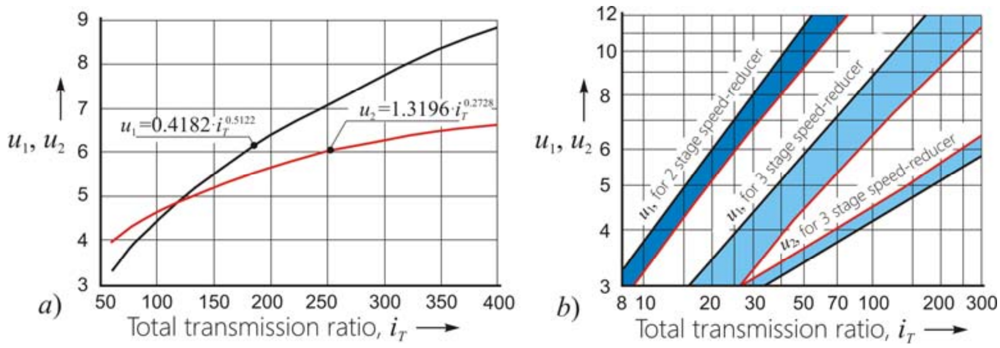

Fig. 1 – Determination of partial ratios of 3 stage gearboxes: a) from to Kudreavtev et al. [12]; b) according to Niemann [13].

Another category uses modelling methods [21]. From here, there should be reminded the study of Romhild and Linke [15], which developed the following equations:

$$u_1 = 0.4643\cdot i_T^{0.609}, \quad u_2 = 1.205\cdot i_T^{0.262}. \tag{1}$$

Also, from this class two other interesting studies are those made by Vu, firstly described in [21], where are shown the following expressions for the gear ratio:

$$u_2 \approx 1.3104\cdot i_T^{0.2533}\cdot k_{c2}^{0.3714}\big/k_{c3}^{0.0977}, \quad u_3 \approx 2.3417\cdot i_T^{0.088}\cdot k_{c2}^{0.3455}\big/k_{c3}^{0.2492}, \tag{2}$$

and, secondly, the study from [22]:

$$u_1 = 0.314\cdot\sqrt[3]{i_T^2}, \quad u_2 = 1.33\cdot\sqrt[4]{i_T}. \tag{3}$$

Next, the number of teeth on pinion and helical gear on each stage are determined (*A*.3). Then the error of the actual gear ratio is checked (according to National Romanian Standard, i.e. STAS 6012 the error should be ±2.5% when $u<4$; or ±3% when $u>4$). Once this preliminary phase is completed the process continues with the helical gears design phase – *B*. In a briefly description, at this level of design process there should be made several computations regarding the estimated allowable contact ($\sigma_{HP1,2}$, σ_{HP}) and bending stresses ($\sigma_{FP1,2}$), the preliminary center distance a_w, the normal module m_n, the elementary center distance a, the elements of the helical gears and of the equivalent spur gears. Finally, the helical gearing is checked on bending and contact stresses. The last two sections concern with the shafts subassembly (section *C*) and housing design (section *D*). They will be included into a further study, when the complete optimization of the entire speed reducer will be considered. However, until then, let us return to our present discussion whereas already was pointed out, a simple GA is proposed for developing appropriate models for determine the values of the partial gear ratios for achieving speed reducers with minimum mass or length.

3. GENETIC ALGORITHM

Genetic Algorithms (GAs) are a subclass of Evolutionary Algorithms (EAs). They are a computer based search technique, which mimics the biological evolution as a problem-solving strategy. The basic concepts of GAs were developed by John Holland [10]. Algorithmically, the basic GAs is outlined as below: (a) the genotype (i.e. the search space of coded solutions) of every chromosome (individual) in the population is randomly initialized. (b) The phenotype (i.e. collections of parameters such as: the number of teeth on pinions and wheels, the standardized center distances, etc.) of every chromosome from the initial population is evaluated using the fitness function (i.e. the mass and the total length of the helical gearings). Next, (c) two parent chromosomes are randomly selected (using the roulette wheel method) for reproduction according to their fitness (the higher the fitness, the more chances of selection). Offsprings are created (d, e) by applying the genetic operators: crossover (merges information from two parent chromosomes into one or two offsprings) and mutation (acts on a single offspring and works by applying some variation to one or more genes in the offspring's chromosome). The new generated individuals are then evaluated (f) using the fitness measure. After the evaluation, the offspring replaces some/all of the chromosomes in the current population (g). This full process of evaluation and reproduction continues until either a satisfactory solution emerges or the GA has run for a specified number of generations.

4. STATEMENT OF THE OPTIMAL DESIGN PROBLEM

4.1. The 'genotype' of the 3 stage helical speed reducer

The 18 genes that uniquely describe the optimization problem (Fig. 2) are detailed in Table 1.

Table 1

The 18 genes describing the multi-stage helical speed reducer

Symbol	Range	Description
z_1, z_3, z_5	{14 ,…, 21}	Number of teeth of the pinions. Integer values.
z_2, z_4, z_6	{45 ,…, 172}	Number of teeth of the wheels. Integer values.
$a_{w\{1\}}$, $a_{w\{2\}}$, $a_{w\{3\}}$ (mm)	{56 ,…, 315}	Center distance of each stage. Standardized, discrete, real values
x_{n1}, x_{n3}, x_{n5}	{–0.5,…,1}	Normal tooth addendum coefficients of the pinions. Discrete, real values.
$\beta_{\{1\}}$, $\beta_{\{2\}}$, $\beta_{\{3\}}$ (°)	[4, 19.75]	Helix angles measured at the pitch diameters. Discrete real values.
$\psi_{\{1\}}$, $\psi_{\{2\}}$, $\psi_{\{3\}}$	[0.2 ,…, 0.5]	Gear width to center distance ratio coefficients. Real values.

4.2. The objective functions

Two separately objectives were considered for minimization, i.e. the mass (Eq. 4) and the length (Eq. 5) of the helical gearings in order to obtain optimal models for partial gear ratios. The expressions for the objectives functions are:

$$\text{Obj 1:} \quad F_1(x) = \sum_{i=1}^{6} \left(v_{i1} + v_{i2} \right) \rho \rightarrow \min \,, \tag{4}$$

$$\text{Obj 2:} \quad F_2(x) = 0.5 \cdot d_{a1} + a_{w\{1\}} + a_{w\{2\}} + a_{w\{3\}} + 0.5 \cdot d_{a6} \rightarrow \min \,, \tag{5}$$

where: v_{i1}, v_{i2} are the components of the volumes corresponding to the helical gears; ρ is the density of steel (i.e. $7.85 \cdot 10^{-6}$ mm^3/kg); d_{a1} and d_{a6} are the outside diameters of the helical gears 1 and 6 (see Fig. 2); $a_{w\{1\}}$, $a_{w\{2\}}$ and $a_{w\{3\}}$ are the standardized center distances.

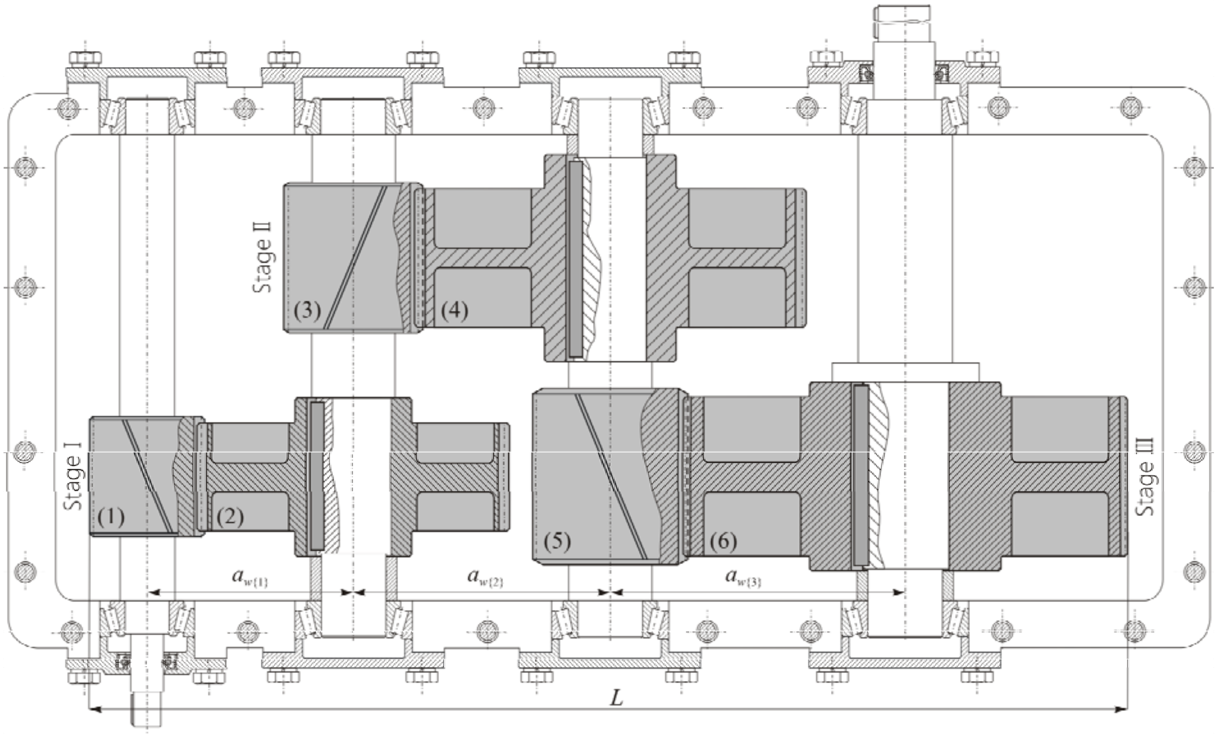

Fig. 2 – The helical gearings of a 3 stage speed-reducer.

4.3. The constraints

In here, a set of **57 engineering design constraints** (involving strength, geometrical and structural considerations) typically encountered in practical design of a power transmission were considered. All these constraints are of inequality type, defined as $g_i = a_i / b_i - 1 \leq 0$, where a constraint is of the form $a_i \leq b_i$, with a_i, $b_i > 0$. It is obviously that the values of all these constraints have to be negative or at last zero (case in which the solution is feasible). For the sake of conciseness, we shall not dwell on the details regarding their calculation. It should be mentioned here that all the details of the gearings calculations may be founded in the relevant industrial standard document DIN 3990 [7]. These 57 constraints should be with reference to the sketch presented in Fig. 2. **C1–4** The relative error of the total and partial gear ratios (on each stage) should be ±2.5% when $u<4$; or ±3% when $u>4$. **C5–7** *The numbers of teeth on all stages must be relative primes*. **C8–10** *The Hertzian contact pressure on the teeth of gears on each stage must not exceed the allowable Hertzian contact pressure*. **C11–16** *The bending stress on the teeth of helical gears on all three stages must not exceed a specified value*. **C17–22** *The teeth of all helical gears must not be undercut*. **C23–28** *The top land on the teeth on gears 1 through 6 must not vanish*. **C29–31** *The contact ratio of each stage must be greater than a specified value* (i.e. $\varepsilon_{\alpha\,\min} = 1$). **C32–34** *The addendum coefficient of the gears* (2), (4) and (6) should be in the range of [–0.6,...,1]. **C35–52** *A set of measurability constraints for all the helical gears*. **C53–55** *The shearing stresses on the key and keyway for mounting the wheels* (2), (4) *and* (6) *must not exceed a specified value* (*i.e. the allowable shearing stress 70 MPa; corresponding to the material of the keys i.e. steel grades E*355 [9]). Is worth noting here, that there are a few papers [1,6,8,16,18,21,22] where, at this stage of problem formulation (when only the gears are considered-without shafts sub-assembly, housing, etc.) the authors do not consider the shaft–gear hub connections. The key joints have a major impact over the shape and the mass of the driven helical gears. Depending on the key length (computed from bearing strength condition) a gear could or not to have a salient hub [9]. In the following rows a short explanation on how this problem was tackled in here. Firstly, were computed the diameters of the shafts on which are mounted the helical gears (2), (4) and (6) (Fig. 3). These dimensions are determined with: $d = (T/0.2\cdot\tau_t)^{1/3}$, where: T is the torque acting in the rated shaft cross section, in [N·mm]; τ_t is the allowable stress on the torsion, in [MPa] [9]. Next depending on the values obtained with the above equation, from standard tables are chosen the key cross-section ($b\times h$) dimensions. Once these values are known, the rated key length from the strength condition ($l_c = 4\cdot T/(\sigma_{st}/d/h)$ [9], where σ_{st} is the allowable bearing stress [MPa]; h is the height of the key cross-section) is determined. Now, that the key length is known a decision can be made as to whether the helical driven gears will have or not a salient hub. At this moment all the dimensions of the driven helical gears (2), (4) and (6) are complete defined. **C56** *Gear* (2) *and* (5) *must not interfere*. **C57** *Gear* (4) *must not interfere with the output shaft*.

5. A 3 STAGE HELICAL SPEED REDUCER OPTIMAL DESIGN EXAMPLE

This is probably a good time to consider the practical example of a 3 stage helical speed reducer optimal design (Fig. 2). In order to obtain optimal models for the partial gear ratios for which the speed reducer has a minimum mass or a minimum length were considered 7 input data sets (InDS).

Table 2

The 7 input data sets (InDS)

InDS – 1			**InDS – 2**			**InDS – 3**			**InDS – 4**			**InDS – 5**			**InDS – 6**			**InDS – 7**		
i_T	n_m (rpm)	P_m (kW)	i_T	n_m (rpm)	P_m (kW)	i_T	n_m (rpm)	P_m (kW)	i_T	n_m (rpm)	P_m (kW)	i_T	n_m (rpm)	P_m (kW)	i_T	n_m (rpm)	P_m (kW)	i_T	n_m (rpm)	P_m (kW)
40	750	6.5	**50**	750	5.2	**63**	750	4.5	**80**	750	3.3	**100**	750	2.6	**125**	750	2.2	**160**	750	1.8
	1000	8		1000	7		1000	6		1000	4.5		1000	3.5		1000	3		1000	2.5
	1500	12.5		1500	10.5		1500	9		1500	6.7		1500	5.2		1500	4.5		1500	3.7

Each InDS [23] is defined by a total transmission ratio – i_T, an input speed – n_m (rpm) and the corresponding input power – P_m (kW). The helical gears of the speed reducer should be based on an ISO 53 basic rack profile ($\alpha_n = 20°$, $h_{an} = 1$, $c_{sa} = 0.4$) with the pinions and wheels made of case hardened alloy steel 17CrNiMo6 and 17Cr3, respectively. The values of all considered genes, after optimization, for each InDS are given in Table 3 (for minimum mass) and Table 4 (for minimum length of the helical gears).

Table 3

The values of the genes obtained after optimization – **obj. 1 the mass of the helical gearings**

z_1	z_3	z_5	z_2	z_4	z_6	$a_{w\{1\}}$ (mm)	$a_{w\{2\}}$ (mm)	$a_{w\{3\}}$ (mm)	x_{n1}	x_{n3}	x_{n5}	$\beta_{\{1\}}$ (°)	$\beta_{\{2\}}$ (°)	$\beta_{\{3\}}$ (°)	$\psi_{a\{1\}}$	$\psi_{a\{2\}}$	$\psi_{a\{3\}}$	**Mass (kg)**
***InDS* – 1: (1.1): i_T = 40:** n_m = 750 rpm, P_m = 6.5 kW; **(1.2):** n_m = 1000 rpm, P_m = 8 kW; **(1.3):** n_m = 1500 rpm, P_m = 12.5 kW																		
19	19	21	66	67	67	80	125	160	0.5758	0.9224	0.7431	16.5	11	14.5	0.3875	0.3445	0.4975	24.666
17	19	21	60	68	65	80	125	180	0.937	0.763	0.7615	4	13.25	16	0.4025	0.3425	0.3275	23.933
17	19	21	60	68	65	80	140	160	0.58	0.9935	0.7885	13.5	16	18.75	0.4025	0.2325	0.4325	23.268
***InDS* – 2: (2.1): i_T = 50:** n_m = 750 rpm, P_m = 5.2 kW; **(2.2):** n_m = 1000 rpm, P_m = 7 kW; **(2.3):** n_m = 1500 rpm, P_m = 10 kW																		
18	16	21	71	65	65	90	125	160	0.586	0.5545	0.781	7.75	12	18.75	0.2915	0.3775	0.4325	23.837
18	16	21	71	65	65	90	125	160	0.586	0.586	0.799	7.75	10	18.75	0.3025	0.368	0.4425	23.954
17	21	21	67	85	65	90	140	200	0.6176	0.6953	0.5758	19.75	18.25	14.25	0.245	0.2425	0.2075	23.446
***InDS* – 3: (3.1): i_T = 63:** n_m = 750 rpm, P_m = 4.5 kW; **(3.2):** n_m = 1000 rpm, P_m = 6 kW; **(3.3):** n_m = 1500 rpm, P_m = 9 kW																		
18	19	19	79	77	66	100	140	180	0.6116	0.5459	0.779	13.5	16	16.25	0.2	0.33	0.3285	26.203
18	19	19	79	77	66	100	140	180	0.4024	0.4383	0.6176	14.5	16.5	18.75	0.2	0.3295	0.3125	25.446
18	19	19	79	77	66	100	140	180	0.6116	0.5459	0.689	13.5	16	17	0.2	0.3295	0.3245	26.049

Table 3 (continued)

***InDS* – 4: (4.1): i_T = 80:** n_m = 750 rpm, P_m = 3.3 kW; **(4.2):** n_m = 1000 rpm, P_m = 4.5 kW; **(4.3):** n_m = 1500 rpm, P_m = 6.7 kW																		
18	15	19	83	73	66	80	140	160	0.7	0.829	0.7825	17	13.75	19.75	0.3025	0.2175	0.4202	22.411
17	16	19	83	73	67	80	140	160	0.661	0.571	0.727	19.75	17.25	19	0.3375	0.215	0.4505	23.338
17	16	19	83	73	67	80	140	160	0.76	0.3775	0.7345	15.25	18	18.5	0.3255	0.225	0.4485	23.28
***InDS* – 5: (5.1): i_T = 100:** n_m = 750 rpm, P_m = 2.6 kW; **(5.2):** n_m = 1000 rpm, P_m = 3.5 kW; **(5.3):** n_m = 1500 rpm, P_m = 5.2 kW																		
19	19	19	104	84	77	80	125	200	0.6893	0.9344	0.9762	14.75	18.5	8.5	0.35	0.2485	0.2875	24.995
21	17	19	103	83	77	90	140	180	0.8626	0.5638	0.7551	16	10.25	19.75	0.2275	0.2385	0.3225	23.506
18	19	17	83	88	78	90	125	200	0.4675	0.598	0.604	9.25	14.75	17.75	0.2075	0.275	0.2435	22.839
***InDS* – 6: (6.1): i_T = 125:** n_m = 750 rpm, P_m = 2.2 kW; **(6.2):** n_m = 1000 rpm, P_m = 3 kW; **(6.3):** n_m = 1500 rpm, P_m = 4.5 kW																		
21	20	17	118	101	77	90	125	200	0.9941	0.7312	0.9284	9	12	14.5	0.256	0.3836	0.265	25.487
19	18	17	104	89	77	80	125	200	0.8965	0.64	0.9205	10.5	15	14.25	0.325	0.3175	0.2775	25.019
19	18	17	105	89	77	112	140	180	1	1	0.7855	4.5	10	19.75	0.21	0.2295	0.3355	24.869
***InDS* – 7: (7.1): i_T = 160:** n_m = 750 rpm, P_m = 1.8 kW; **(7.2):** n_m = 1000 rpm, P_m = 2.5 kW; **(7.3):** n_m = 1500 rpm, P_m = 3.7 kW																		
17	17	19	108	93	86	112	160	200	0.767	0.9463	0.8985	4.25	19	18	0.2025	0.2	0.2405	26.128
17	17	19	107	93	86	112	140	200	0.8029	0.5578	0.9284	8	19.25	11.5	0.2025	0.2505	0.2585	26.121
19	16	17	120	87	77	90	125	200	0.6594	0.8686	0.8746	18	14	19	0.21	0.28	0.2825	24.876

Table 4

The values of the genes obtained after optimization – **obj. 2 the length of the transmission**

z_1	z_3	z_5	z_2	z_4	z_6	a_{w1} (mm)	a_{w2} (mm)	a_{w3} (mm)	x_{n1}	x_{n3}	x_{n5}	$\beta_{\{1\}}$ (°)	$\beta_{\{2\}}$ (°)	$\beta_{\{3\}}$ (°)	$\psi_{a\{1\}}$	$\psi_{a\{2\}}$	$\psi_{a\{3\}}$	**Length (mm)**
***InDS* – 1: (1.1): i_T = 40:** n_m = 750 rpm, P_m = 6.5 kW; **(1.2):** n_m = 1000 rpm, P_m = 8 kW; **(1.3):** n_m = 1500 rpm, P_m = 12.5 kW																		
19	20	19	68	71	59	80	125	160	-0.1952	0.1096	0.6475	14.75	6.75	11.75	0.455	0.5	0.5175	506.13
19	21	16	69	74	51	80	112	160	-0.0458	0.0259	0.6953	11	18.75	15.75	0.465	0.4975	0.3625	494.47
19	20	19	69	71	59	80	140	160	-0.0996	0.7252	0.6475	11.5	4.75	11.75	0.495	0.295	0.5025	521.05
***InDS* – 2: (2.1): i_T = 50:** n_m = 750 rpm, P_m = 5.2 kW; **(2.2):** n_m = 1000 rpm, P_m = 7 kW; **(2.3):** n_m = 1500 rpm, P_m = 10 kW																		
19	16	19	78	63	59	90	125	160	-0.1952	0.2351	0.6475	17	18.75	11.75	0.465	0.4575	0.5285	516.33
20	16	19	81	65	59	80	125	160	-0.2968	0.6774	0.6535	16.5	5	10.75	0.4325	0.4425	0.5765	504.36
18	16	15	71	63	47	80	125	160	0.0259	-0.0697	0.51	6.25	19.5	14.25	0.495	0.5685	0.4875	506.27
***InDS* – 3: (3.1): i_T = 63:** n_m = 750 rpm, P_m = 4.5 kW; **(3.2):** n_m = 1000 rpm, P_m = 6 kW; **(3.3):** n_m = 1500 rpm, P_m = 9 kW																		
18	19	19	79	78	66	90	125	160	-0.1952	0.5339	0.7909	17.25	4	19.5	0.47	0.5625	0.485	518.31
18	19	19	79	77	66	90	125	160	-0.0996	-0.0458	0.9284	16.5	15.75	17.5	0.45	0.5	0.475	518.31
18	19	19	79	77	66	90	125	160	-0.1952	0.528	0.779	17.25	13.25	19.75	0.445	0.3765	0.5075	518.3
***InDS* – 4: (4.1): i_T = 80:** n_m = 750 rpm, P_m = 3.3 kW; **(4.2):** n_m = 1000 rpm, P_m = 4.5 kW; **(4.3):** n_m = 1500 rpm, P_m = 6.7 kW																		
20	16	17	99	71	61	80	125	160	0.5578	0.1275	0.6714	19	16	6	0.43	0.48	0.505	507.22
19	17	18	83	78	71	80	112	160	-0.1892	0.4144	0.4443	14	13.5	13.5	0.3425	0.465	0.4875	497.39
17	20	17	84	91	59	80	125	160	-0.0996	-0.0458	0.6415	16	6.75	16.5	0.375	0.48	0.4025	505.53
***InDS* – 5: (5.1): i_T = 100:** n_m = 750 rpm, P_m = 2.6 kW; **(5.2):** n_m = 1000 rpm, P_m = 3.5 kW; **(5.3):** n_m = 1500 rpm, P_m = 5.2 kW																		
17	18	17	83	91	67	71	125	160	0.002	0.1574	0.8806	9.5	10	19.5	0.4675	0.4275	0.4125	498
18	18	17	89	91	67	71	125	160	-0.1832	0.4264	0.8806	17.25	11.25	19.5	0.4275	0.425	0.415	497.61
17	18	17	83	91	69	71	125	160	0.002	0.1933	0.6475	9.5	11.75	19.25	0.4825	0.4625	0.4825	498.86
***InDS* – 6: (6.1): i_T = 125:** n_m = 750 rpm, P_m = 2.2 kW; **(6.2):** n_m = 1000 rpm, P_m = 3 kW; **(6.3):** n_m = 1500 rpm, P_m = 4.5 kW																		
16	16	15	91	89	59	100	125	160	-0.0816	0.4228	0.5789	19.25	17.75	17.5	0.3225	0.345	0.4255	531.64
16	16	15	91	89	59	100	125	160	-0.0816	0.3248	0.8029	19.25	18.75	17.75	0.385	0.38	0.5125	530.7
15	14	20	83	79	79	71	125	180	0.1036	0.5459	0.9762	14.5	18	8.25	0.4555	0.375	0.5575	532.79
***InDS* – 7: (7.1): i_T = 160:** n_m = 750 rpm, P_m = 1.8 kW; **(7.2):** n_m = 1000 rpm, P_m = 2.5 kW; **(7.3):** n_m = 1500 rpm, P_m = 3.7 kW																		
17	16	16	105	103	63	80	140	180	0.0498	0.5997	0.4024	1405	13.25	9.75	0.36	0.295	0.3785	558.38
17	19	19	105	118	77	80	140	180	-0.0996	0.2291	0.7551	15.5	10.75	19.75	0.3425	0.34	0.3555	557.64
17	19	19	105	119	77	80	140	180	-0.0896	0.1291	0.6551	15.5	11	19.75	0.3425	0.4525	0.355	558.01

The proposed GA (as it could be seen from the above tables) led to helical gearings which weights from ~ **22 kg** (for InDS 4 – (4.1)) to ~ **26 kg** (for InDS 7 (7.1)) and lengths between ~ **490 mm** (for InDS 1 (1.2)) and ~ **558 mm** (for InDS 7 (7.3)). The following remark on speed reducers with minimum volume/length generated by genetic simulations refers to the chaotic behaviour of the value of the helix angle measured at the pitch diameter for each stage. Despite this fact, the objective functions have very similar values. This means that $\beta_{\{1,2,3\}}$ does not affect significantly the overall goal of the optimization. Another important note about the optimization result emerges by analysing the values of the helical gear width to center distance ratio coefficient $\psi_{a\{1,2,3\}}$. When the objective function was the **mass**, the values of these coefficients vary as follows: **0.2–0.4** (for the 1st stage); **0.2175–0.33** (for the 2nd stage); **0.2–0.49** (for the 3rd stage). For the second case when was minimized the **length** of the gearings, the values of $\psi_{a\{1,2,3\}}$ ranged between: **0.34–0.49** (for the 1st stage); **0.29–0.56** (for the 2nd stage); **0.29–0.57** (for the 3rd stage). It is obviously that the genetic simulations for this case had generated optimal solutions with significantly higher values of $\psi_{a\{1,2,3\}}$. Now, once these preliminary conclusions were pointed out, based on these result sets models for determine the optimum values for gear ratios for each stage of the mechanical transmission will be further on developed. Firstly, the values of the gear ratios are computed (Table 5). Next, the Curve Fitting Toolbox™ (Fig. 3) from MATLAB is used to perform an exploratory data analysis. From this toolbox is selected the '**cftool**' function for conducting the regression analysis.

The regression analysis conducts to the following expressions for the helical gear ratios:

a) for **minimum mass**:

$$u_{\{1\}} = 0.8184 \cdot i_T^{0.3996}, \quad u_{\{2\}} = 1.302 \cdot i_T^{0.2809}, \quad u_{\{3\}} = 0.9194 \cdot i_T^{0.3208} \tag{6}$$

b) for **minimum length**:

$$u_{\{1\}} = 0.9126 \cdot i_T^{0.3731}, \quad u_{\{2\}} = 0.7414 \cdot i_T^{0.4188}, \quad u_{\{3\}} = 1.486 \cdot i_T^{0.2023}. \tag{7}$$

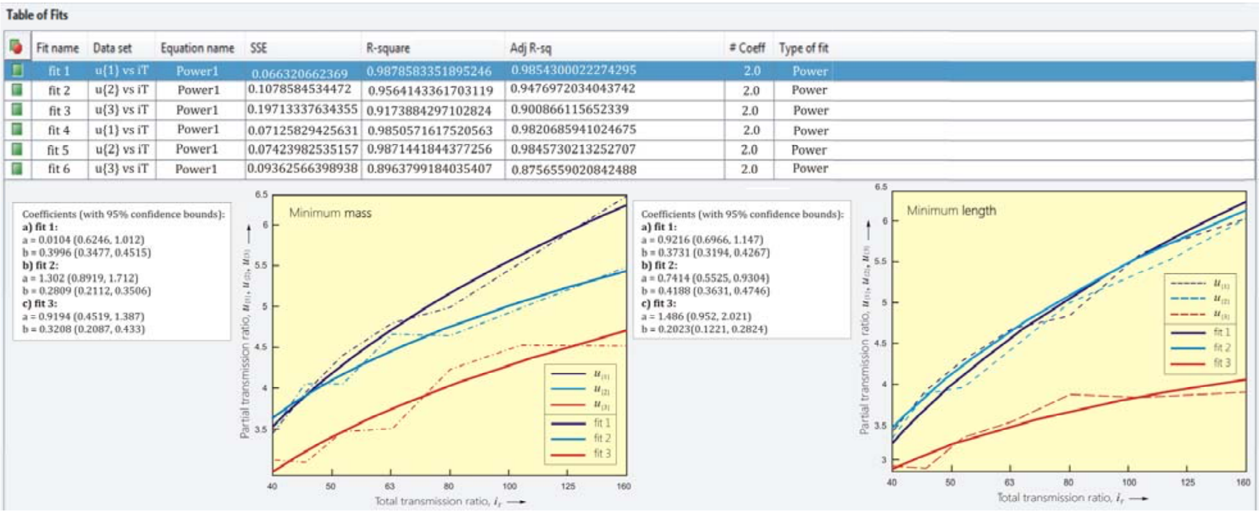

Fig. 3 – Optimal partial gear ratios vs total transmission ratio (for helical gears with minimum **mass** or **length**); and the corresponding fitting curves (MATLAB Fitting Toolbox™).

Table 6

Gear ratios comparison ($u_{\{1\}} \times u_{\{2\}} \times u_{\{3\}}$ and $i_{\{1\}} \times i_{\{2\}} \times i_{\{3\}}$)

	Buiga	Vu Ngoc Pi [21]	Vu Ngog Pi et al. [22]	Romhild&Linke [15]
40	3.574×3.6697×3.0022	3.5729×3.424×3.2696	3.6726×3.3448×3.2563	4.3776×3.1676×2.8847
	3.55×3.55×3.15	3.55×3.55×3.15	3.55×3.55×3.15	4.5×3.15×2.8
50	3.9073×3.9071×3.225	4.1387×3.6231×3.3345	4.2616×3.5367×3.3174	5.0289×3.3583×2.9606
	4×4×3.15	4×3.55×3.55	4×3.55×3.55	5×3.55×2.8
63	4.2853×4.1691×3.4732	4.8192×3.8415×3.403	4.9715×3.747×3.3819	5.7889×3.5679×3.0502
	4.5×4×3.55	5×4×3.55	5×4×3.55	5.6×3.55×3.15
80	4.7146×4.4589×3.7499	5.6405×4.0812×3.4753	5.8298×3.9776×3.4499	6.6955×3.7984×3.1457
	5×4.5×3.55	5.6×4×3.55	5.6×4×3.55	6.3×4×3.15
100	5.1543×4.7469×4.0281	6.5336×4.3185×3.5442	6.7649×4.2058×3.5147	7.6701×4.0271×3.2375
	5×4.5×4	6.3×4.5×3.55	7.1×4.5×3.55	8×4×3.15
125	5.635×5.054×4.3271	7.5681×4.5696×3.6145	7.85×4.4471×3.5806	8.7865×4.2695×3.3321
	5.6×5×4.5	8×4.5×3.55	8×4.5×3.55	9×4×3.55
160	6.2192×5.4169×4.6837	8.9044×4.8645×3.6939	9.2543×4.7302×3.6551	10.2119×4.5548×3.4399
	6.3×5.6×4.5	9×5×3.55	9×4.5×3.55	10×4.5×3.55

Next, for validating these results the values of the gear ratios computed with the above equations are compared with the ones determined with the current models from literature (Section 2, Eqs. 1-3). For a better analysis and interpretation, the traditional and optimal designs gear ratios are compared side-by-side in Table 6. Also, in here are suggested the standardized values – $i_{\{1,2,3\}}$ that should be selected from the STAS 6012 for the actual gear ratios – $u_{\{1,2,3\}}$. It can be seen from Table 6 that all 3 models from literature, conduct to greater values of the gear ratios for the first stage and to an uneven distribution of these on speed reducer's stages. Furthermore, when the total transmission ratio increases (above 80) current models offer for the first stage larger values (i.e. 8, 9 or 10) than the recommended ones. The results obtained by using GA show significant improvement over the results obtained by traditional design.

6. CONCLUSIONS

Speed reducers are the most common ways of transmitting power. Designing mechanical power transmissions is not an easy task, considering the iterative nature of the whole process. Furthermore, designing compact multi-stage speed reducers are challenging demands of nowadays mechanical power transmission manufacturer. Considering these aspects in this paper a GA was used to solve the complex structural design problem of multi-stage helical speed reducer. In here two objectives (the mass and the length of the helical gearings) and a set of 7 input data sets (InDS) [23] were used for obtaining appropriate optimal values for the partial gear ratios. The design variables considered in the optimization problem are of mixed nature i.e., integer (e.g. the gears number of teeth), discrete (e.g. normal tooth addendum coefficients) and real (e.g. gears width), in a total of 18. The objective functions were subjected to a highly non-linear set of 57 constraints. The results obtained by using GA conduct to mathematical expressions for optimal values of the partial ratios (for achieving minimum mass Eq. 6, and minimum length of the helical gearings Eq. 7), which offer an even distribution of the gear ratios on all 3 stages as compared with the values given by [15,21,22]. Once this preliminary phase is completed the virtual design space of the helical gearings will be generated and the problem formulation will be extended with the shafts and the housing subsystems. After that, when the complete optimization of the speed reducer will be done a generic transmission system design tool based on the evolutionary optimization concepts [5] will start to be developed. This optimization example illustrates the effectiveness of the proposed approach and also serves as further evidence of the power and versatility of GAs in designing mechanical power transmissions. The proposed GA could be easily modified to suit multi-objective design optimization.

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