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CHEBYSHEV RECURSION IN DESIGN OF LINEAR PHASE LOW-PASS FIR FILTER WITH EQUIRIPPLE STOP-BAND

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Abstract. The objective of this letter is to generate new low-pass FIR filters with linear phase response and equiripple stop-band characteristic. The Chebyshev orthogonal polynomials of the first kind are used to generate the design forms of the new non-recursive filters with all poles at zero. Comparative analyses of frequency response characteristics of these filters with filters suggested earlier in the literature are given for the same design parameters in a few examples.

Key words: low-pass filter, FIR filter, Chebyshev recursion.

1. INTRODUCTION

Extensively used type of digital filters in digital signal processing applications is the Finite Impulse Response (FIR), mainly for its guaranteed stability (they have no poles). The order of these filters directly affects the required number of adders, multipliers and memory for the filter. Cascaded-Integrator-Comb (CIC) filter [1] is widely used as decimation filter due to its simplicity; it requires no multiplication but rather only additions/substractions. Generally, the classical CIC filter does not provide enough attenuation of aliasing, has passband droop that increase with the increase of its comb parameter. An example of design and analysis of new FIR filter functions based on classical CIC filters is proposed in [2]. The basic CIC section, in conjuction with seven other sections repeated several times, makes the new filter functions presented in [2]. Compared with existing filter functions in the literature, the proposed functions not only have improved insertion loss but also a multiplierless architecture, better passband characteristics and lower impulse response coefficients. The Chebyshev polynomials are special class of polynomials especially suited for approximating other functions. In [3], Christoffel-Darboux formula for Chebyshev orthogonal polynomials of the first kind is used for generation of a linear phase digital FIR filter function in compact explicit form by using an analytical method. New class of extremely economic linear phase symmetric selective FIR filters is obtained by the proposed approximation technique. Generally, several methods for improving the digital filter efficiency have been described in [4–7]. In the modified filter designs presented in [6] and [7], the designs of compensation filters for comb decimators are introduced in order to improve magnitude characteristics in passband.

The Chebyshev polynomials of the first kind are used here to generate integer coefficients of new selective low-pass filters. The motivation behind the use of Chebyshev polynomials of the first kind is to design filters with appropiate characteristics in passband and stopband without additional compensation filters. This paper proposes non-recursive filters with all poles at zero which show equiripple characteristics in stop-band. Linear phase characteristic is preserved which is very important in realization of decimator for large conversion factors.

The recursion for generation of the Chebyshev polynomial of the first kind [8–10] denoted as $T_N(x)$ is

$$T_N(x) \overset{\Delta}{=} \begin{cases} 1 & , \ N=0 \\ 2 & , \ N=1 \\ 2x \cdot T_{N-1}(x) - T_{N-2}(x), & N>1 \end{cases} \tag{1}$$

The first few Chebyshev polynomials of the first kind can be found in [8].

The Chebyshev polynomials of the first kind are used to generate Coleman filter form given in [9]. There, the frequency response is $G(f) = 2^{-20} \cdot T_7(F(f))$, where $T_7(x)$ is the Chebyshev polynomial of the first kind of degree seven and the frequency dependent function is defined as $F(f) = 2 + 2 \cdot \cos(2\pi f)$. In that paper, the exact functions with Chebyshev polynomials of degree N are obtained by following relation

$$G_N(f) = G(0) \cdot T_N(F(f)), \tag{2}$$

where the constant is exactly calculated as $G(0) = T_N(F(0))$, i.e as the Chebyshev polynomilas of degree N for argument being function $F(f) = 2 + 2 \cdot \cos(2\pi f)$ and frequency chosen to be zero.

The authors' idea is to try to design new filter functions which will keep simplicity of the given structure by avoiding the multipliers, being low complexity structure with linear phase characteristic and equiripple stopband. The idea is to change the argument function used in Chebyshev polynomials. In this approach, the desired high stopband attenuation can be achieved directly, without the need for combination with some compensation filter.

The rest of the paper is structured as follows. In the Section II, a new filter form based on the Chebyshev polynomials of the first kind is generated. Also, implementation form of the proposed filter is presented. Designs of a few examples with even and odd filter orders are described in Section III. Section IV presents comparative analysis of filters' characteristics designed for different filter orders. The paper is concluded in Section V.

2. THE NEW FILTER FORM AND ITS IMPLEMENTATION

The Chebyshev polynomials of the first kind are also used here to generate new low-pass filters. In this case, the function of new filter $G_{N,\mathrm{new}}(f)$ is generated by equation

$$G_{N,\mathrm{new}}(f) = G_{\mathrm{new}}(0) \cdot T_N(F_{\mathrm{new}}(f)), \tag{3}$$

where the normalized constant is calculated as $G_{\mathrm{new}}(0) = T_N(F_{\mathrm{new}}(f))$, i.e. as the Chebyshev polynomials of degree and frequency chosen to be zero ($f = 0$) for applied new cosinuse function

$$F_{\mathrm{new}}(f) = 1 + 2 \cdot \cos(2\pi f). \tag{4}$$

Here, a new cosinuse function with reduced DC component is used. These equations allow one to predict how the filter will respond to varying frequency.

The non-recursive nature of FIR filter offers the opportunity to create implementation schemes that significantly improve the overall efficiency of the filter. Using the complex exponential form of the cosine function

$$2 \cdot \cos(k\omega) = e^{jk\omega} + e^{-jk\omega}, \tag{5}$$

and setting

$$z = e^{j\omega}, \tag{6}$$

it follows

$$2 \cdot \cos(k\omega) = z^k + z^{-k} \tag{7}$$

which will be used in forming digital filter function. Generally, the FIR filter function of the corresponding non-recursive implementation has the form [11]

$$G_N(z) = \frac{z^N}{G(0)} \cdot G_N^*(z), \tag{8}$$

where the filter function $G_N^*(z)$ is defined as

$$G_N^*(z) = \sum_{r=0}^{2N} h(N,r) \cdot z^{-r} , \tag{9}$$

and $G(0)$ is the normalized constant for the unit magnitude response at frequency $f = 0$. The vector of impulse response coefficients $h(N,r)$ is defined as $\mathbf{h}(N,r) = \{ h(N,0), h(N,1),\ldots, h(N,2N) \}$. The coefficients satisfy the following symmetry condition, $h(N,r) = h(N,2N-r)$.

The non-recursive structure can achieve computational simplicity through polyphase decomposion. The polyphase decomposition can be applied to non-recursive form by grouping the odd and even numbered coefficients. The transfer function can be written in form of two polyphase componenets $E_0(z^2)$ and $E_1(z^2)$

$$G_N^*(z) = \sum_{r=0}^{2N} h(N,2r) \cdot z^{-2r} + z^{-1} \cdot \sum_{r=0}^{2N} h(N, 2r+1) \cdot z^{-2r} = E_0(z^2) + z^{-1} \cdot E_1(z^2) . \tag{10}$$

In case of a higher order filters, the polyphase decomposition of the transfer function can be also used. The FIR transfer function is decomposed into several lower order transfer function called polyphase components, which are added together to compose the original transfer function. The polyphase components are combined to form parallel structure.

3. DESIGN EXAMPLES

Designs of a few examples are carried out in MATLAB. The function proposed by J. Coleman [9], $G_N(\omega)$ from Eq. (2), and functions of new FIR filters, $G_{N,\text{new}}(\omega)$ from Eq. (3), for filter order N are arranged here differently as the following unique function of ω

$$G_N(\omega) = \left[a(0) + a(1) \cdot \cos(\omega) + a(2) \cdot \cos(2\omega) + \ldots + a(N) \cdot \cos(N\omega) \right] / G(0) , \tag{11}$$

$$G_{N,\text{new}}(\omega) = \left[a(0) + a(1) \cdot cos(\omega) + a(2) \cdot \cos(2\omega) + \ldots + a(N) \cdot \cos(N\omega) \right] / G_{\text{new}}(0) , \tag{12}$$

which are normalized with the constant $G(0)$ and $G_{\text{new}}(0)$, respectively. Filter coefficients $a(i)$, $i = 1, 2, \ldots, N$ are given in tabular form in Table 1 for solution of FIR filters given by J. Coleman and Table 2 for new FIR filters. Filter coefficients of new FIR filters given in Table 2 have lower values versus coefficients values of Coleman filter functions given in Table 1.

Table 1

Coefficients $a(i)$, $i = 1, 2, \ldots, N$ of FIR filters (Eq. (2)) with function $F(f) = 2 + 2 \cdot \cos(2\pi f)$

N	5	6	7	8	9
$G(0)$	15124	119071	937444	7380481	58106404
$a(0)$	3642	26315	192530	1421825	10576370
$a(1)$	6130	45456	339010	2540800	19122498
$a(2)$	3600	29028	230048	1805504	14089824
$a(3)$	1400	13312	118160	1008128	8390256
$a(4)$	320	4128	44352	432448	3976128
$a(5)$	32	768	11424	137216	146275
$a(6)$		64	1792	30208	401664

Table 1 (continued)

$a(7)$		128	4096	77148
$a(8)$			256	9216
$a(9)$				512

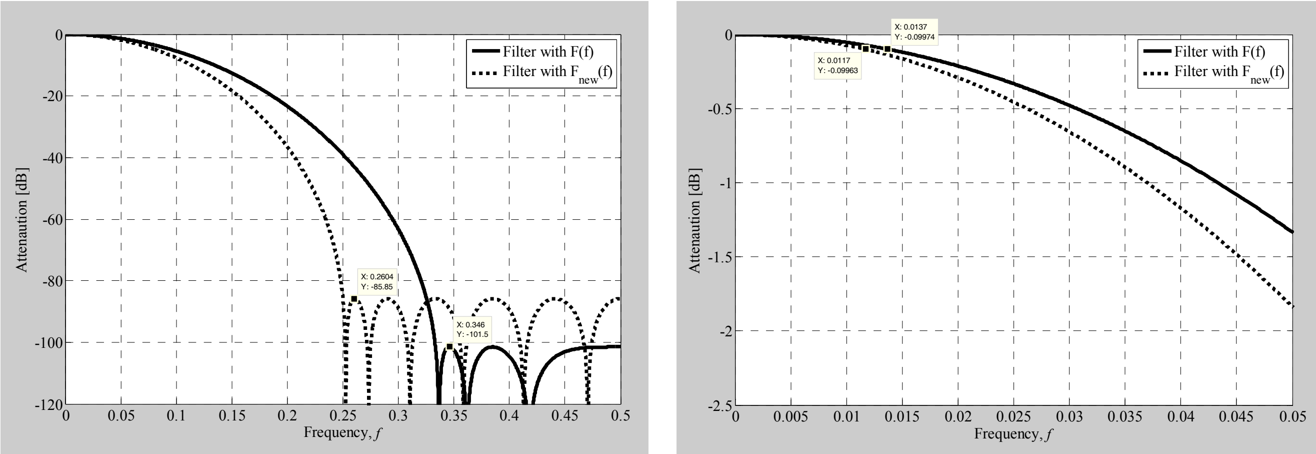
Table 2

Coefficients $a(i)$, $i = 1, 2, ..., N$ of FIR filters (Eq. (3)) with new function $F_{\text{new}}(f) = 1 + 2 \cdot \cos(2\pi f)$

N	5	6	7	8	9
$G_{\text{new}}(0)$	3363	19601	114243	665857	3880899
$a(0)$	681	3653	19825	108545	598417
$a(1)$	1210	6600	36274	200576	1114578
$a(2)$	840	4836	27664	157760	898416
$a(3)$	440	2816	17360	104704	622896
$a(4)$	160	1248	8736	57664	367200
$a(5)$	32	384	3360	25600	180576
$a(6)$		64	896	8704	71808
$a(7)$			128	2048	21888
$a(8)$				256	4608
$a(9)$					512

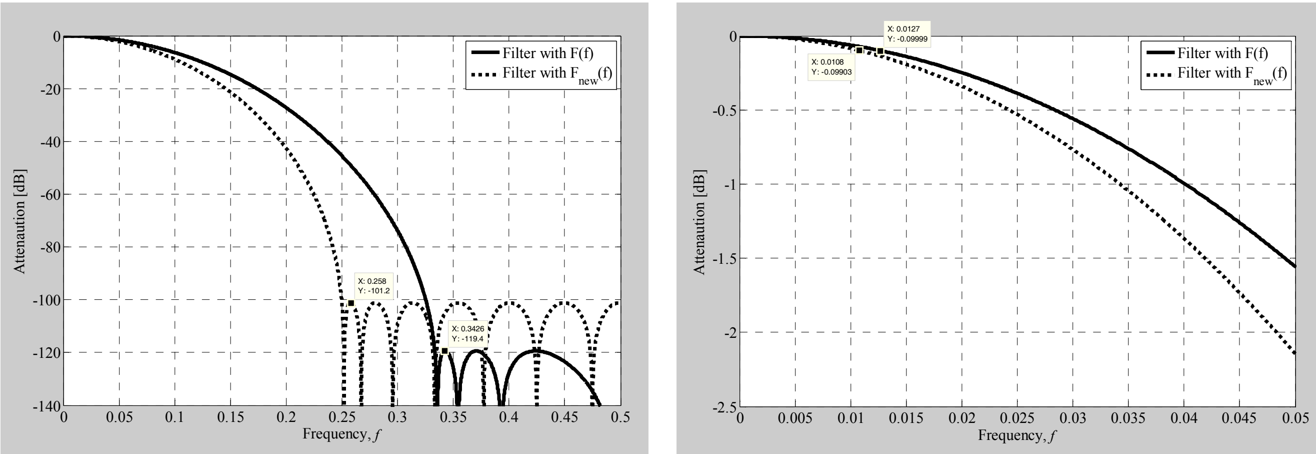
4. COMPARATIVE ANALYSIS OF FILTERS

Normalized curves of designed low-pass filters with functions $F(f)$ and $F_{\text{new}}(f)$, versus normalized frequency $f = \omega/(2\pi)$, are summarized in Figs. 1 and 2. Examples of even and odd orders are generated in MATLAB. The generated filter characteristics by Eq. (11) for the function $F_{\text{new}}(f)$ and different filter order N show higher selectivity in the transient area in comparison with filters generated by Eq. (11) for the function $F(f)$. The both filters give equiripple stop-band characteristics where little better stop-band performances are obtained by filters designed with function $F(f)$. That filter has an stop-band down about 100 dB for $N = 6$, and about 120 dB for $N = 7$, which is depicted in Table 3. The suppression in the stop-bands for new filters are about 85 dB and 100 dB, for $N = 6$ and $N = 7$, respectively, Table 3. In order to achieve the same attenuation in stop-band, the new filters require higher order. Table 3 gives also some other FIR filter parameters, such as pass-band cut-off frequencies f_{cp} at $\alpha_{\max} = 0.1\,\text{dB}$, and minimum attenuation $\alpha_{\min}$ at stop-band cut-off frequency f_{cn}. For the filters with function $F(f)$, the normalized stop-band cut-off frequency is $f_{cn} = 0.33$, and in case of filters with the function $F_{\text{new}}(f)$ it is $f_{cn,\text{new}} = 0.25$. These values of stop-band cut-off frequencies indicate higher selectivity of new filters. For the designed new filters, the normalized stop-band cut-off frequency is $f_{cn,\text{new}} = 0.25$ independently of the filter order. Also, passband drop is very small and passband edge frequency differs very little for different values of filter order, Table 3. Deep, equiripple stopband is obtained for all filter orders, but the suppression in the stop-bands differs depending of filter order as shown in Table 3.



a) filter characteristics b) pass-band detail

Fig. 1 – Normalized curves of filters in dBs for case $N = 6$.



a) filter characteristics b) pass-band detail

Fig. 2 – Normalized curves of filters in dBs for case $N = 7$.

Table 3

Parameters of designed FIR filters

	N	5	6	7	8
$\alpha_{max} @ f_{cp}$	Ref. [9]	0.0150	0.0137	0.0127	0.0119
$\alpha_{max} = 0.1\text{dB}$	New filter	0.0129	0.0117	0.0108	0.0102
$\alpha_{min} @ f_{cn}$	Ref. [9] @0.33	83.59	101.50	119.40	137.40
	New filter @0.25	70.53	85.85	101.20	116.50

5. CONCLUSION

New lowpass filters proposed here are FIR filters with an equiripple stop-band characteristic and a linear phase response. Design forms include the Chebyshev orthogonal polynomials of the first kind which are frequency dependant according to suggested function. Digital filters are commonly employed in signal processing applications. The suggested design keeps the good features of previously designed FIR filters such as linear phase characteristic which is very important for large factor conversions. Verifycation of the design equations of new filters and FIR filter parameters are provided for even and odd order. Using new function in filter design, filter performances are improved in a manner that greater selectivity of magnitude response characteristics and reduced values of filter coefficients are obtained.

The proposed filter functions have potential applications in certain applications of signal processing where it is desire to avoid aliasing phenomenon, i.e. to eliminate high-frequency components. Since the signals usually have high frequency tails, one has to filter out this tail and limit the maximum frequency of the signal such that one can lower the sampling frequency and avoid aliasing. High-frequency components can be suppressed very well by use of these filter functions because of their high stopband suppression.

APPENDIX

Designs of a few examples presented in the paper are carried out in MATLAB. Here, MATLAB code for one design example of new filter functions based on Chebyshev polynomial of the first kind of degree six is shown. The design of other examples can be done easily by use of recursion for generation of the Chebyshev polynomial of the first kind given in Eq. (1).

```
% Design of filter functions based on Chebyshev polynomial of the first kind of degree six
clear;clc;
format long
f = 0:0.0001:0.5;
omega = 2*pi.*f;
z = exp(j.*omega)';
%% Filter functions from Jeffrey O. Coleman ISCAS 2014 [9]
Ff = 2+2.*cos(omega);
Ff_0 = 2+2.*cos(0);
G0 = 32*power(Ff_0,6)-48*power(Ff_0,4)+18*power(Ff_0,2)-1,
G = (32.*power(Ff,6)-48.*power(Ff,4)+18.*power(Ff,2)-1)./G0;
alphaG = 20*log10(G);
%% New filter functions
Ffnew = 1+2.*cos(omega);
Ffnew_0 = 1+2.*cos(0);
G0_new = 32*power(Ffnew_0,6)-48*power(Ffnew_0,4)+18*power(Ffnew_0,2)-1,
G_new = (32.*power(Ffnew,6)-48.*power(Ffnew,4)+18.*power(Ffnew,2)-1)./G0_new;
alphaGnew = 20*log10(G_new);
%% Plot options
figure(1);
plot(f,alphaG,'k-',f,alphaGnew,'r:');grid;
legend('Filter with F(f)','Filter with F_{new}(f)')
axis([0 0.5 -150 0]);
xlabel('Frequency, {\itf}','FontSize',12); ylabel('Attenaution [dB]','FontSize',12);
pause
figure(2);plot(f,G,'k-',f,G_new,'r:');grid;
legend('Filter with F(f)','Filter with F_{new}(f)')
axis([0 0.5 0 1]);
xlabel('Frequency, {\itf}','FontSize',12); ylabel('Magnitude]','FontSize',12);
pause;close('all')
```

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