

## BLIND SOURCE SEPARATION BASED ON SOURCE NUMBER ESTIMATION AND FAST-ICA WITH A NOVEL NON-LINEAR FUNCTION\*

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**Abstract.** Blind source separation (BSS) has already become a hot field of signal processing and neural network, and independent component analysis (ICA) is an effective means to resolve the BSS problem. In this paper, we first adopt an estimation method for the number of the sources based on the clustering approach named as communication with local agents (CLAs). A new non-linear function ('sin') of the Fast-ICA algorithm is further introduced. It has the ability to replace the commonly used classical functions (such as 'tanh', 'gauss' and 'pow3') and it is no longer necessary to select different non-linear functions according to the Gaussian characteristics of the signals. The experiments presented to validate the new proposal are run using music tunes and images and communication signals and white Gaussian noise. Experimental evaluation concludes that the proposed new function accelerates the computation while maintaining the performance.

**Key words:** blind source separation, clustering, Fast-ICA, non-linear function, independent component analysis, signal de-noising.

### 1. INTRODUCTION

Blind source separation (BSS) was first considered in 1982, when the BSS problem was formulated by Ans, Héroult, and Jutten for a biomedical problem [1,2]. The problem is to extract the potential source signals from a group of linear mixed signals, where the mixing matrix is unknown. That means BSS seeks to recover or separate original source signals from the mixed signals without knowing any prior information on the sources or the parameters of the mixtures. Independent Component Analysis (ICA) is the commonest and basic method to solve BSS problem, and it has been widely used in speech and image signal processing. Jutten, Héroult, Comon and Sorouchyari published three classical papers on BSS (an adaptive algorithm based on neuromimetic architecture, problems statement and stability analysis) in 1991 and proposed the concept of ICA, which made the research of BSS developed greatly. BSS of instantaneous mixed signals is systematically analyzed, which provides a more accurate explanation for independent component analysis (ICA) [3]. ICA-based technologies are widely used, for example, in constructing pseudo-MIMO mixtures for communication signals, sources can be separated by applying ICA on the artificial mixtures [4]. Hyvarinen proposed a fixed-point training algorithm based on the non-Gaussian nature of the source signal. He first proposed Fast-ICA algorithm based on kurtosis [5] in 1997, and then proposed Fast-ICA algorithm based on negative entropy [6] in 1999. In recent years, some progress has been made in the research on the selection of non-linear functions in Fast-ICA algorithm [7–10].

In order to avoid choosing different non-linear functions in Fast-ICA algorithm according to the Gaussian characteristics of the signal, we try to find a new general non-linear function, which can be used as another choice to perform BSS tasks.

In this paper, we adopt a clustering method to estimate the number of unknown source signals and propose one new non-linear function to replace the commonly used non-linearities in the Fast-ICA algorithm to improve the convergence property. The outline of the rest of this paper is organized as follows. The method of source number estimation and the improved Fast-ICA Algorithm with the proposed non-linear

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function is briefly introduced in Section II. In Section III, Matlab simulations validate the performance and the runtime of the improved algorithms. The final section concludes this paper.

## 2. METHODOLOGY

The noise-free model of basic ICA is as follows:  $x = As$ , where  $s$  denotes the original independent sources  $x$  denotes mixtures of original sources and  $A$  is the unknown constant mixing matrix.

### 2.1. Source number estimation

Generally, blind source separation algorithms do not have the ability to estimate the number of unknown signal sources. They can only be calculated on the premise that the number of signal sources has been determined in advance. Otherwise, signal separation is not possible. So, it will be helpful for the implementation of BSS to determine the exact number of source signals before separation. Here we use cluster analysis to estimate the number of source signals from the mixed signal  $x$ . The objective of cluster analysis is to partition a set of data points into several groups based on analysis of the relationship between data points and their neighborhoods.

From the latest literature, we select a clustering approach named as communication with local agents (CLAs), which can attain satisfactory clustering results using only one parameter [11]. In the first phase, CLA computes the local resultant force and centrality values of all the data points. Next, each data point tries their best to find a local agent. After that, the local agents communicate with each other and connect as clusters.

The sources are three common waveforms ('square wave', 'sine wave' and 'cosine wave'). For each source signal, the number of samples  $N=200$ . The mixing square matrix  $A$  was randomly generated. The clustering results of mixed signals with CLA algorithm are shown in Fig. 1.

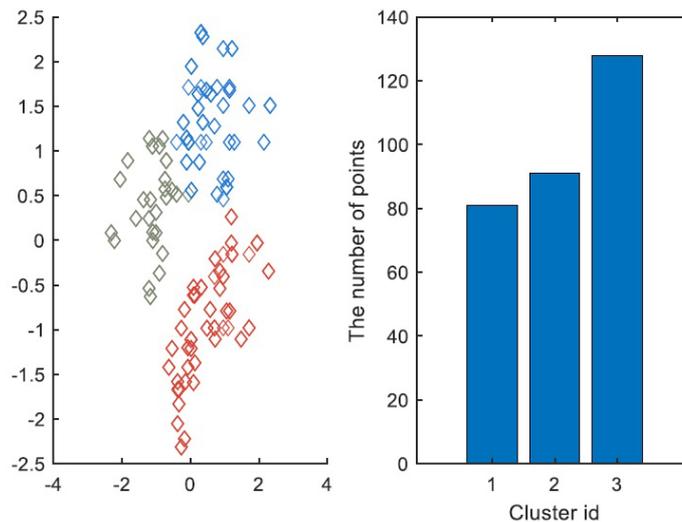


Fig. 1 – Clustering results of mixed signals with CLA algorithm.

As are shown in Fig. 1, we find 3 clusters. That is to say, the number of source signals is 3.

### 2.2. Fast-ICA with a novel non-linear function

In general, the number of the sources is equal to the number of the mixtures. The solution of ICA is to estimate a separation or demixing matrix  $B$  as the inverse of the mixing matrix  $A$ ,  $B = A^{-1}$ . According to the concept of mutual information, the differential entropy  $H(y)$  of a random vector  $y = R^N$  with the probability density  $p(y)$  is defined as follows:

$$H(y) = -\int p(y) \log p(y) dy. \quad (1)$$

Negentropy  $J(y)$  is defined as follows:

$$J(y) = H(y_G) - H(y), \quad (2)$$

where  $y_G$  is a Gaussian random vector of the same covariance matrix as  $y$  [9].

To use the definition of ICA given above, an approximation of the negentropy (the objective function of the Fast-ICA algorithm) based on the maximum-entropy principle is given by  $y_G$

$$J(y_i) \approx c \{E[G(y_i)] - E[G(v)]\}^2, \quad (3)$$

where  $c$  is an irrelevant constant,  $E(\cdot)$  is the mean operation,  $v$  and  $y_i$  are both Gaussian variables with zero mean and unit variance,  $G(\cdot)$  is practically any non-quadratic contrast function [12].

To find one independent component (source signal) as  $y_i = w^T x$ , we maximize the function  $J_G$  given by [6]

$$J_G(w) = \{E[G(w^T x)] - E[G(v)]\}^2, \quad (4)$$

where  $w$  is a vector constrained so that  $E[(w^T x)^2] = 1$ .  $w_i (i=1,2,\dots,N)$  gives one of the rows of the orthogonal matrix  $W$  that is derived from ICA [9]. The most frequently used contrast functions  $G(\cdot)$  and non-linear function (non-linearity)  $g(\cdot)$  are summarized as follows [6]:

$$\begin{cases} G_1(u) = \frac{1}{a_1} \log \cosh(a_1 u), & g_1(u) = \tanh(a_1 u), \\ G_2(u) = -\frac{1}{a_2} e^{-\frac{1}{2} a_2 u^2}, & g_2(u) = u e^{-\frac{1}{2} a_2 u^2}, \\ G_3(u) = \frac{1}{4} u^4, & g_3(u) = u^3, \end{cases} \quad (5)$$

where  $1 \leq a_1 \leq 2$ ,  $a_2 \approx 1$  are constants. The derivative function  $g = G'$  is called the non-linearity.  $g_1(\cdot)$ ,  $g_2(\cdot)$  and  $g_3(\cdot)$  are signed as 'tanh', 'gauss' and 'pow3', respectively. It is necessary to select the different non-linear functions according to the Gaussian characteristics of the signals.

In industrial applications, the computing speed of an algorithm is one of the factors limiting its application. Based on this, we devote to finding new non-linearity. The basic principles of constructing non-linearities in the Fast-ICA algorithm are as follows: (1) The non-linearities should be simple and their calculation speed should be fast; (2) The separation performances should be better than that of the traditional non-linearities [9].

In this paper, we adopt contrast function  $G_4(u) = -\cos(u)$  and non-linearity  $g_4(u) = \sin(u)$  because our purpose is to find or design a new possible alternative to  $g_1$ ,  $g_2$  or  $g_3$  and achieve the same or better performance.  $g_4(\cdot)$  is signed as 'sin' here.

As stated previously, the improved Fast-ICA algorithm is listed in Table 1.

Table 1  
The improved Fast-ICA algorithm

Input: The mixed signals $X$ .
Output: The separation matrix $W$ .
1: Estimate the number of source signals $M$ by CLA clustering algorithm.
2: Whiten the mixed signals $X$ , namely $E(X_i) = 0$ and $E(X^T X) = I$ .
3: Choose the proposed non-linear function $g(u) = \sin(u)$ .
4: Initialize the number of estimated vectors, Let $p = 1$ .
5: Initialize $w_p$ , namely $w_p = w_p / \ w_p\ $ .
6: Calculate $w_p$ and let $w_p = E\{X g(w_p^T X)\} - E\{g'(w_p^T X)\} w$ .

Table 1 (continued)

$$7: w_{p+1} = w_{p+1} - \sum_{j=1}^p w_{p+1}^T w_j w_j.$$

8: If  $|w_{p+1} - w_p| < \varepsilon$  does not converge, return Step 6.

9: Let  $p = p + 1$ , if  $p < M$ , return Step 4.

### 3. EXPERIMENTS AND RESULTS

In this section, we compare the new non-linearity with the classical non-linearities ('tanh', 'gauss' and 'pow3') in the Fast-ICA algorithm by simulation. For each non-linearity, the experiments are carried out in the same software and hardware environments in order to make the comparison as fair as possible. All the simulations are carried out in MATLAB 2017b, system of the PC is Windows 7 (64-bit version), CPU is Intel® Core™ i7-2720QM Processor @2.20GHz, and RAM is 16 GB.

The Fast-ICA algorithm with the four non-linearities is compared from two aspects of statistical and computational.

(1) The statistical performance or separation accuracy is measured using a performance index  $C$  (correlation coefficient) defined as follows.

$$C(a,b) = \frac{\text{cov}(a,b)}{\sqrt{\text{cov}(a,a)}\sqrt{\text{cov}(b,b)}}. \quad (6)$$

$C(a,b)=0$  means that  $a$  and  $b$  are uncorrelated, and  $C(a,b)=1$  means that  $a$  and  $b$  are fully correlated. The signals correlation increases as  $C(a,b)$  approaches unity, the bigger the value  $C$  is, the better the separation performance of a BSS algorithm [13].

(2) The computational load is measured using running time needed for convergence [9]. To compare the computational speed, we record the average runtime of algorithm running ten times for different non-linearities.

The mixing square matrix  $A$  was randomly generated, such as  $A = \begin{pmatrix} 0.1946 & 0.8345 & 0.1477 \\ 0.2252 & 0.7008 & 0.2098 \\ 0.0967 & 0.8110 & 0.7473 \end{pmatrix}$ .

In the first experiment, the sources are three music tunes ('guitar.wav', 'piano.wav' and 'trumpet.wav'). For each source signal, the number of samples  $N=400000$ . The separation results by the improved Fast-ICA algorithm are shown in Fig. 2.

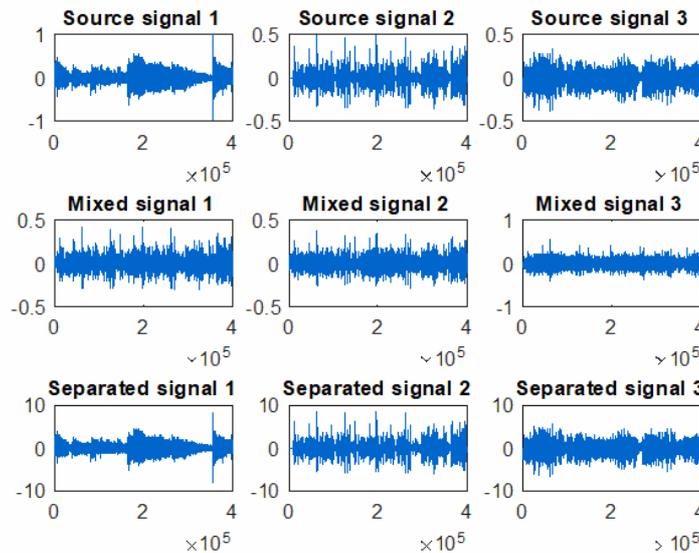


Fig. 2 – Displaying source signals, mixed signals and separated signals for music – 1, 2 and 3.

In the second experiment, the sources are three commonly used test images ('lenna.bmp', 'pepper.bmp' and 'sailboat.bmp'). For each source signal, image size is 512 by 512 and bit depth of image is 24. The separation results by the improved Fast-ICA algorithm are shown in Fig. 3.



Fig. 3 – Displaying source images, mixed images and separated images for image – 1, 2 and 3.

In the third experiment, the sources are three commonly used communication signals ('square wave', 'sine wave' and 'cosine wave'). For each source signal, the number of samples  $N=100$ . The separation results by the improved Fast-ICA algorithm are shown in Fig. 4.

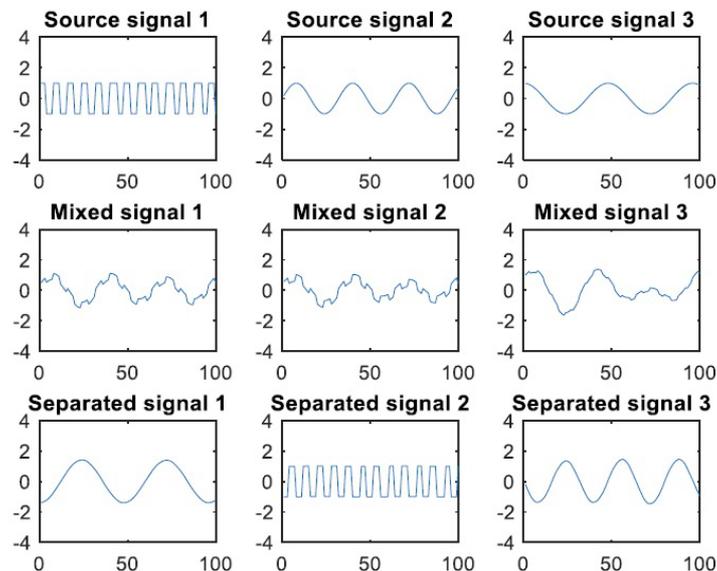


Fig. 4 – Displaying source signals, mixed signals and separated signals – 1, 2 and 3.

In the fourth experiment, the improved Fast-ICA algorithm is used for signal de-noising. It is assumed that the energy (amplitude) of the noise is higher than that of the source signal. Therefore, the source signal completely “submerged” in the noise [14]. The sources are signals of UWB-LFM (Ultra-Wideband Linear Frequency Modulation), QPSK (Quadrature Phase Shift Keying) and WGN (White Gaussian Noise). For each source signal, the number of samples  $N=1000$ . The separation results by the improved Fast-ICA algorithm are shown in Fig. 5.

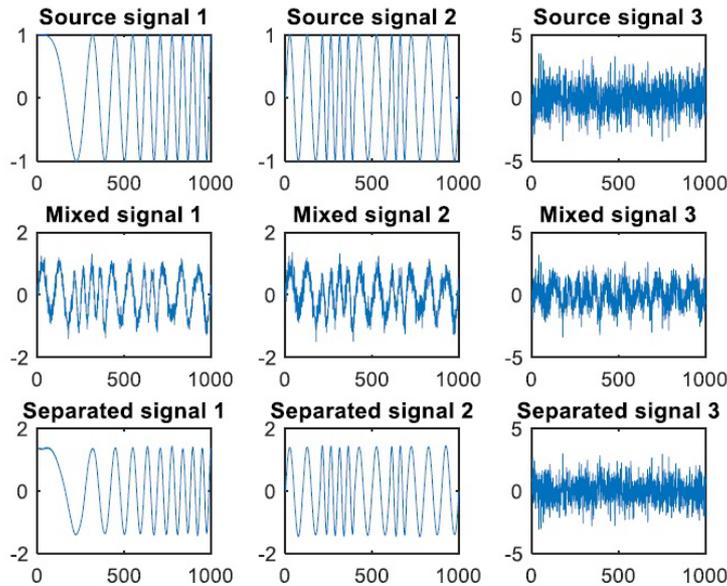


Fig. 5 – Displaying source signals, mixed signals and separated signals – 1, 2 and 3.

The Fast-ICA algorithms with different non-linearities are compared with the separation accuracy and the computational speed, the average correlation coefficient (C-ave) and the average runtime (T-ave) after ten executions of the algorithm are listed in Table 2, Table 3, Table 4, and Table 5.

As are shown in Fig. 2, Fig. 3, Fig. 4 and Fig. 5, the sources are well recovered after separation. From Table 2, Table 3, Table 4, and Table 5, we can see that new non-linearity ‘sin’ achieves the same or better separation accuracy and the computational speed (in bold text).

Table 2

C-ave and T-ave after ten executions of the algorithm based on Experiment 1

non-linearities	C-ave	T-ave (s)
‘tanh’	1.0000	0.6576
‘gauss’	1.0000	0.6210
‘pow3’	0.9999	0.7931
‘sin’	<b>1.0000</b>	<b>0.6108</b>

Table 3

C-ave and T-ave after ten executions of the algorithm based on Experiment 2

non-linearities	C-ave	T-ave (s)
‘tanh’	1.0000	0.8142
‘gauss’	1.0000	0.8346
‘pow3’	0.9999	1.0158
‘sin’	<b>1.0000</b>	<b>0.5588</b>

Table 4

C-ave and T-ave after ten executions of the algorithm based on Experiment 3

non-linearities	C-ave	T-ave (s)
‘tanh’	0.9621	1.6135
‘gauss’	0.9727	1.5860
‘pow3’	0.9696	2.1706
‘sin’	<b>0.9797</b>	<b>1.4326</b>

Table 5

C-ave and T-ave after ten executions of the algorithm based on Experiment 4

non-linearities	C-ave	T-ave (s)
'tanh'	0.9945	0.1635
'gauss'	0.9939	0.1457
'pow3'	0.9967	0.1763
'sin'	<b>0.9969</b>	<b>0.1440</b>

#### 4. CONCLUSION

There are three original contrast functions in the Fast-ICA algorithm to separate super-Gaussian and sub-Gaussian sources, different non-linearities are selected according to the Gaussian characteristics of the signal. Considering this shortcoming, the present study intends to find a new general non-linearity, which can be used as another choice to perform BSS tasks. In this paper, the improved Fast-ICA algorithm based on source number estimation and a novel non-linear function is proposed. The simulation results show that the proposed non-linearity achieves the same or better separation accuracy and the computational speed when comparing with the classical non-linearities.

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