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EXPLICIT EXACT TRAVELING WAVE SOLUTIONS AND BIFURCATIONS OF THE KUNDU-ECKHAUS EQUATION

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Abstract. The paper deals with the nonlinear complex Kundu-Eckhaus (KE) equation, a basic model in nonlinear optics which describes the propagation of solitons through the optical fiber. The bifurcation analysis is performed on the dynamic system associated to traveling wave solutions, showing the existence of periodic wave solutions, bright solitons, dark solitons, kink wave and anti-kink wave solutions, in different parametric domains. Explicit parametric representations of the traveling wave solutions are also obtained. Phase portraits and simulations are presented to illustrate the theoretical results.

Key words: Kundu-Eckhaus equation, exact solution, bifurcation, kink wave solution.

1. INTRODUCTION

In this paper, we study the exact traveling wave solutions of the nonlinear complex Kundu-Eckhaus (KE for short) equation in the form [12, 14, 16, 17, 24]:

$$iu_t + u_{xx} - 2\rho|u|^2u + \delta^2|u|^4u + 2i\delta(|u|^2)_x u = 0, \tag{1}$$

which describes the propagation of the ultra-short femtosecond pulses in an optical fiber. The complex KE equation (1) has great significant in the quantum theory, weakly nonlinear dispersive water waves and nonlinear optics. Optical soliton and rogue wave solutions of KE equation were obtained in [12, 14, 16, 17, 24]. Soliton collisions for the KE equation with variable coefficients were studied in Xie et al. [37, 38]. Different from their method in [7, 9–12, 14–16, 24, 26, 30, 34, 35], this paper employed the bifurcation theory of planar dynamic systems to find the exact traveling wave solutions to the KE equation (1). Motivated by the works of (the monograph [18], also papers e.g. [2–4, 8, 13, 19–23, 25, 27–29, 32, 33, 36, 39–46]), we give the sketch of the method as follows.

Step 1: Transfer the nonlinear PDE to an ODE by a suitable travelling wave transformation. Compute the first integrals for the obtained ODE.

Step 2: Perform the bifurcation analysis of ODE and make phase portrait.

Step 3: Dynamic behavior of ODE corresponds to wave solution of the PDE. We see that a periodic orbit of ODE corresponds to a periodic wave solution of a nonlinear wave equation (PDE); a homoclinic orbit of ODE corresponds to a solitary wave solution of a nonlinear wave equation (PDE); a heteroclinic orbit of system ODE corresponds to a kink (or anti-kink) wave solution of a nonlinear wave equation (PDE). Also, the exact expression of the solutions to nonlinear wave equation is given.

Assume that Eq.(1) has the traveling wave solutions in the form

$$u(x,t) = \phi(\xi)e^{i\eta(\xi)}, \quad \xi = x - ct, \tag{2}$$

where c is the wave speed. Substituting it into equation (1) and decomposing into real and imaginary parts, we get the following equations

$$-c\phi' + 2\phi'\eta' + \phi\eta'' + 4\delta\phi^2\phi' = 0, \tag{3}$$

$$c\phi\eta' + \phi'' - \phi(\eta')^2 - 2\rho\phi^3 + \delta^2\phi^5 = 0, \tag{4}$$

where ϕ' is the derivative with respect to ξ. Multiplying Eq.(3) by ϕ and integrating with respect to ξ, we have

$$\eta' = \frac{2g + c\phi^2 - 2\delta\phi^4}{2\phi^2}, \tag{5}$$

where g is an integral constant. Substituting Eq.(5) into (4) yields

$$\phi'' = \frac{8\rho\phi^6 - (c^2 + 8g\delta)\phi^4 + 4g^2}{4\phi^3}. \tag{6}$$

Equation (6) is equivalent to the following planar dynamical system

$$\frac{\mathrm{d}\phi}{\mathrm{d}\xi} = y, \quad \frac{\mathrm{d}y}{\mathrm{d}\xi} = \frac{8\rho\phi^6 - (c^2 + 8g\delta)\phi^4 + 4g^2}{4\phi^3}, \tag{7}$$

which has the first integral

$$H(\phi, y) = y^2 - \rho\phi^4 + \frac{(c^2 + 8g\delta)}{4}\phi^2 + \frac{g^2}{\phi^2} = h. \tag{8}$$

Clearly, system (7) is a singular traveling wave system of the first class (see [18]) with one singular straight line $\phi = 0$ if $g \neq 0$. The existence of the singular straight line leads to a dynamical behavior of solutions with two scales. In particular, for $g = 0$, system (7) becomes a regular system as follows:

$$\frac{\mathrm{d}\phi}{\mathrm{d}\xi} = y, \quad \frac{\mathrm{d}y}{\mathrm{d}\xi} = 2\rho\phi^3 - \frac{c^2}{4}\phi. \tag{9}$$

2. BIFURCATIONS OF PHASE PORTRAITS OF SYSTEM (7)

We consider the associated regular system of (7) as follows

$$\frac{\mathrm{d}\phi}{\mathrm{d}\zeta} = 4\phi^3 y, \quad \frac{\mathrm{d}y}{\mathrm{d}\zeta} = 8\rho\phi^6 - (c^2 + 8g\delta)\phi^4 + 4g^2. \tag{10}$$

This system has the same first integral as (7), where $\mathrm{d}\xi = 4\phi^3\mathrm{d}\zeta$. The dynamics of system (10) and (7) are different in the neighborhood of the straight line $\phi = 0$. Specially, under some parameter conditions, the variable ζ is a fast variable while the variable ξ is a slow variable in the sense of the geometric singular perturbation theory.

To study the equilibrium points of (10), we write that $f(\phi) = 8\rho\phi^6 - (c^2 + 8g\delta)\phi^4 + 4g^2$. Let $t = \phi^2$, we obtain

$$f_1(t) = 8\rho t^3 - (c^2 + 8g\delta)t^2 + 4g^2, \quad f_1'(t) = 24\rho t\left(t - \frac{c^2 + 8g\delta}{12\rho}\right).$$

Apparently, $f_1'(t)$ has two zeros at $t = 0$ and $t = \frac{c^2+8g\delta}{12\rho} = t_1$. Also, we have $f_1(0) = 4g^2$ and $f_1(t_1) = 4g^2 - \frac{(c^2+8g\delta)^3}{432\rho^2}$.

Thus, we have the following conclusion:

1. If $g = 0$, (i) when $\rho < 0$, then system (10) has only one equilibrium point $E_0(0,0)$; (ii) when $\rho > 0$, then system (10) has three equilibrium points $E_0(0,0)$, $E_{r1}\left(\sqrt{\frac{c^2}{8\rho}}, 0\right)$ and $E_{r2}\left(-\sqrt{\frac{c^2}{8\rho}}, 0\right)$.

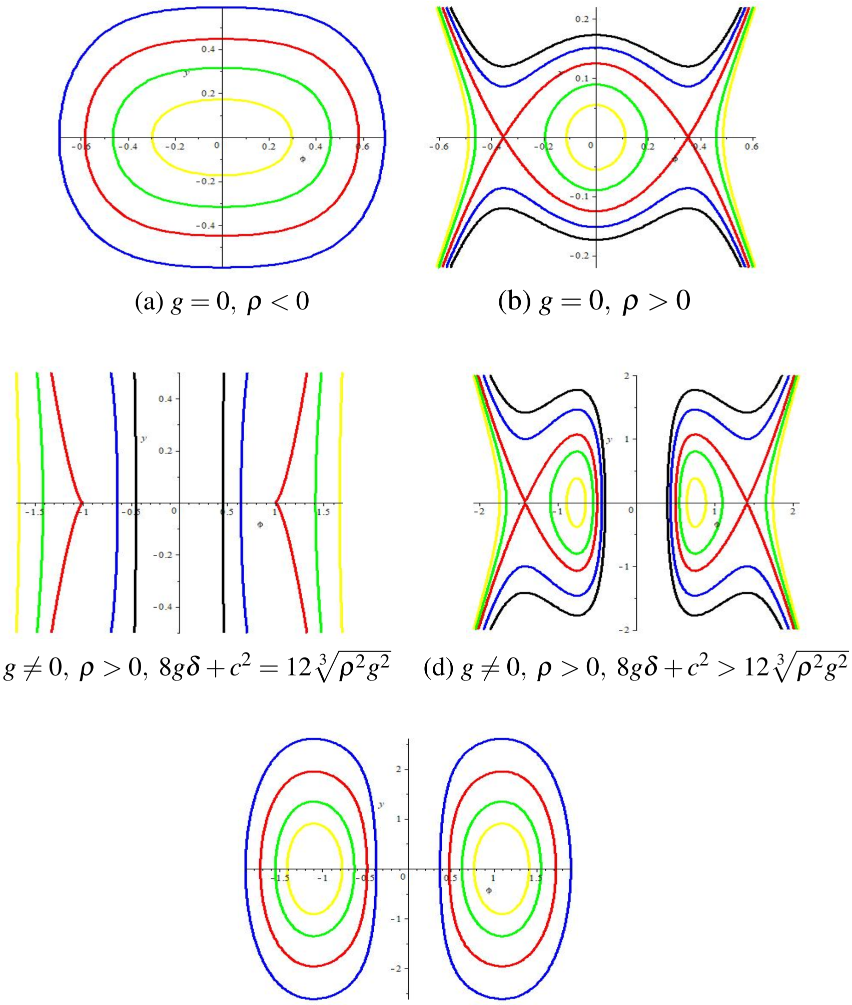
2. If $g \neq 0$, (i) when $\rho > 0,\ c^2 + 8g\delta - 12\sqrt[3]{\rho^2 g^2} = 0$ or when $\rho < 0, c^2 + 8g\delta \neq 0$, then system (10) has two equilibrium points $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$, where $\phi_1 > \phi_2$; (ii) when $\rho > 0, c^2 + 8g\delta > 0, f_1(t_1) < 0$, then system (10) has four equilibrium points $E_1(\phi_1, 0)$, $E_2(\phi_2, 0)$, $E_3(\phi_3, 0)$ and $E_4(\phi_4, 0)$, where $\phi_1 > \phi_2 > \phi_3 > \phi_4$.

Let $M(\phi_j, 0)$ be the coefficient matrix of the linearized system of (10) at an equilibrium point E_j and $J(\phi_j, 0) = \mathrm{detM}(\phi_j, 0)$. We have

$$J(0,0) = \frac{c^2}{4} > 0,\ J\left(\pm\sqrt{\frac{c^2}{8\rho}}, 0\right) = -\frac{c^2}{2} < 0,\ J(\phi_j, 0) = -192\rho\phi_j^6\left(\phi_j^2 - \frac{c^2 + 8g\delta}{12\rho}\right).$$

For an equilibrium point of the planar system (10), the following classification holds true: if $J < 0$, then the equilibrium point is a saddle; if $J > 0$, then it is a center; if $J = 0$ and the index of the equilibrium point is zero, then it is a cusp.

Let $h_0 = H(0,0) = 0$, $h_r = H\left(\pm\sqrt{\frac{c^2}{8\rho}}, 0\right) = \frac{c^4}{64\rho}$ and $h_j = H(\phi_j, 0)$, where H is given by (8). Without loss of generality, we discuss the case $g \geq 0$. By using the above information to do qualitative analysis, we have the following bifurcations of the phase portraits of system (10) shown in Fig.1.



(a) $g = 0,\ \rho < 0$

(b) $g = 0,\ \rho > 0$

(c) $g \neq 0,\ \rho > 0,\ 8g\delta + c^2 = 12\sqrt[3]{\rho^2 g^2}$

(d) $g \neq 0,\ \rho > 0,\ 8g\delta + c^2 > 12\sqrt[3]{\rho^2 g^2}$

(e) $g \neq 0,\ \rho < 0,\ 8g\delta + c^2 \neq 0$

Fig. 1 – Bifurcations of phase portraits of system (7) in the (ϕ, y)-phase plane: (a) $g = 0,\ \rho < 0$; (b) $g = 0,\ \rho > 0$; (c) $g \neq 0,\ \rho > 0,\ 8g\delta + c^2 = 12\sqrt[3]{\rho^2 g^2}$; (d) $g \neq 0,\ \rho > 0,\ 8g\delta + c^2 > 12\sqrt[3]{\rho^2 g^2}$; (e) $g \neq 0,\ \rho < 0,\ 8g\delta + c^2 \neq 0$.

3. EXPLICIT EXACT PARAMETRIC REPRESENTATIONS OF THE SOLUTIONS OF SYSTEM (7)

We now consider the explicit exact parametric representations of the solutions of system (7). We see from (8) and the first equation of (7) that

$$\xi = \int_{\phi_0}^{\phi} \frac{\mathrm{d}\phi}{y(\phi)} = \int_{\phi_0}^{\phi} \frac{\mathrm{d}\phi}{\sqrt{\rho\phi^4 - \frac{(c^2+8g\delta)}{4}\phi^2 + h - \frac{g^2}{\phi^2}}}. \tag{11}$$

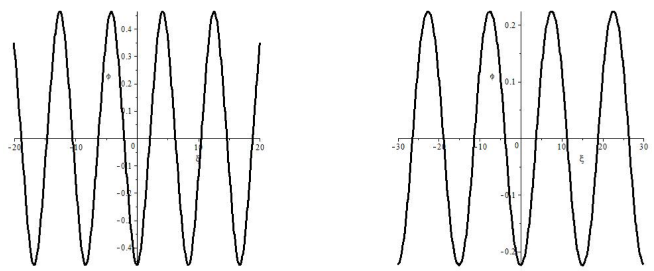
3.1. The case of $g = 0,\ \rho < 0$ (see Fig.1a)

Corresponding to the level curves defined by $H(\phi, y) = h, h \in (0, \infty)$, there exists a family of periodic orbits of system (7), enclosing the equilibrium point $E(0,0)$. Now, $y^2 = -\rho(r_1 - \phi)(\phi + r_1)(\phi - r_2)(\phi - \bar{r_2})$, where $r_1 > 0$, r_2 and $\bar{r_2}$ are complex.

Then, we have the parametric representation of the periodic solution as follows:

$$\phi(\xi) = -r_1 \mathrm{cn}(g_1\xi, k_1), \tag{12}$$

where $a_1^2 = -\frac{(r_2-\bar{r_2})^2}{4}$, $A^2 = r_1^2 + a_1^2$, $g_1 = A\sqrt{-\rho}$, $k_1^2 = \frac{r_1^2}{A^2}$, $\mathrm{cn}(\xi, k)$ is Jacobian elliptic function (see [1]).



(a) Periodic wave given by (12) (b) Periodic wave given by (13)

Fig. 2 – Periodic waves of system (7) given by (12) and (13).

3.2. The case of $g = 0,\ \rho > 0$ (see Fig.1b)

(i) Corresponding to the level curves defined by $H(\phi, y) = h, h \in (0, h_r)$, there exists a family of periodic orbits of system (7), enclosing the equilibrium point $E(0,0)$. Now, $y^2 = \rho(r_1 - \phi)(r_2 - \phi)(\phi + r_2)(\phi + r_1)$, where $r_1 > r_2 > 0$.

Then, we have the parametric representation of the periodic solution as follows:

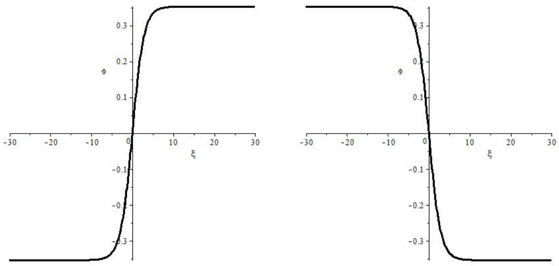
$$\phi(\xi) = \frac{2r_1r_2\mathrm{sn}^2(g_2\xi, k_2) - (r_1 + r_2)r_2}{(r_1 + r_2) - 2r_2\mathrm{sn}^2(g_2\xi, k_2)}, \tag{13}$$

where $g_2 = \frac{(r_1+r_2)\sqrt{\rho}}{2}$, $k_2^2 = \frac{4r_1r_2}{(r_1+r_2)^2}$.

(ii) Corresponding to the level curves defined by $H(\phi, y) = h_r$, there exist two heteroclinic orbits connecting the equilibrium points $E_{r1}\left(\sqrt{\frac{c^2}{8\rho}}, 0\right)$ and $E_{r2}\left(-\sqrt{\frac{c^2}{8\rho}}, 0\right)$, enclosing the equilibrium point $E_0(0,0)$. Now, $y^2 = \rho\left(\sqrt{\frac{c^2}{8\rho}} - \phi\right)^2\left(\phi + \sqrt{\frac{c^2}{8\rho}}\right)^2$.

Then, we have the parametric representations of the kink wave solution and anti-kink wave solutions as follows:

$$\phi(\xi) = \pm\sqrt{\frac{c^2}{8\rho}}\tanh\left(\sqrt{\frac{c^2}{8}}\xi\right). \tag{14}$$



(a) Kink wave (b) Anti-kink wave

Fig. 3 – Kink and anti-kink waves of system (7) given by (14).

3.3. The case of $g \neq 0,\ \rho > 0,\ 8g\delta + c^2 > 12\sqrt[3]{\rho^2 g^2}$ (see Fig. 1d)

(i) Corresponding to the level curves defined by $H(\phi, y) = h,\ h \in (h_2, h_1)$, there exist two families of periodic orbits of system (7), enclosing the equilibrium points $E_2(\phi_2, 0)$ and $E_3(\phi_3, 0)$, respectively. Now, $y^2 = \frac{\rho(r_1-\phi^2)(r_2-\phi^2)(\phi^2-r_3)}{\phi^2}$, where $0 < r_3 < r_2 < r_1$.

Then, we have the parametric representations of the two families of periodic wave solutions as follows:

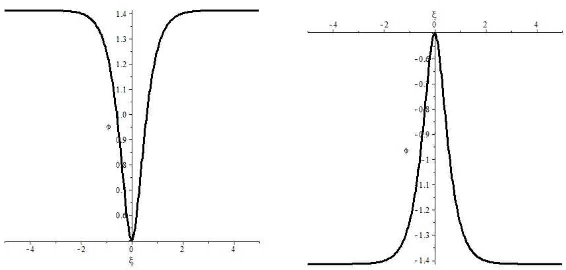
$$\phi(\xi) = \pm\sqrt{r_3 + (r_2 - r_3)\mathrm{sn}^2(g_3\xi, k_3)}, \tag{15}$$

where $g_3 = \sqrt{\rho(r_1 - r_3)}$, $k_3^2 = \frac{r_2 - r_3}{r_1 - r_3}$.

(ii) Corresponding to the level curves defined by $H(\phi, y) = h_1$, there exist two homoclinic orbits enclosing the equilibrium points $E_2(\phi_2, 0)$ and $E_3(\phi_3, 0)$, respectively. Now, $y^2 = \frac{\rho(\phi_1^2-\phi^2)^2(\phi^2-r_1)}{\phi^2}$, where $r_1 > 0$.

Then, we have the parametric representations of the two solitary wave solutions as follows:

$$\phi(\xi) = \pm\sqrt{r_1 + \left(\phi_1^2 - r_1\right)\tanh^2\left(\sqrt{\rho(\phi_1^2 - r_1)}\xi\right)}. \tag{16}$$



(a) Dark solitary wave (b) Bright solitary wave

Fig. 4 – Solitary waves of system (7) given by (16).

3.4. The case of $g \neq 0,\ \rho < 0,\ 8g\delta + c^2 \neq 0$ (see Fig. 1e)

Corresponding to the level curves defined by $H(\phi, y) = h,\ h \in (h_1, \infty)$, there exist two families of periodic orbits of system (7), enclosing the equilibrium points $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$, respectively. Now, $y^2 = \frac{-\rho(r_1-\phi^2)(\phi^2-r_2)(\phi^2-r_3)}{\phi^2}$, where $r_3 < 0 < r_2 < r_1$.

Then, we have the parametric representations of the two families of periodic wave solutions as follows:

$$\phi(\xi) = \pm\sqrt{\frac{(r_1-r_3)r_2-(r_1-r_2)r_3\mathrm{sn}^2(g_4\xi,k_4)}{(r_1-r_3)-(r_1-r_2)\mathrm{sn}^2(g_4\xi,k_4)}}, \tag{17}$$

where $g_4 = \sqrt{\rho(r_3-r_1)}$, $k_4^2 = \frac{r_1-r_2}{r_1-r_3}$.

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