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STABILITY OF MATTER-WAVE SOLITONS IN QUASI-1D BOSE-EINSTEIN CONDENSATES TRAPPED IN PÖSCHL-TELLER POTENTIAL SUBJECT TO PERIODIC PERTURBATIONS THROUGH A NUMERICAL APPROACH

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Abstract: We presented for the first time a study regarding the stability of matter-wave solitons in a quasi one-dimensional Bose-Einstein condensate (cigar-shaped) consisting of atoms with attractive interatomic interactions trapped in a Pöschl-Teller potential subjected to periodic perturbations. The quasi-one-dimensional Bose-Einstein condensate was modeled by Gross-Pitaevskii equation. A numerical (split-step Crank-Nicolson method) approach has been proposed to investigate the dynamic properties of matter-wave solitons during the temporal evolution of this atomic system. The results obtained in this paper demonstrated the stability of the wave-matter solitons while they were perturbed by periodic oscillations of the Pöschl-Teller potential during temporal evolution. The results presented in this paper can open the way for several applications in atomic and molecular physics, condensed matter physics, solid state physics, nonlinear optics and material sciences.

Key words: Bose-Einstein condensates, solitons, Pöschl-Teller potential, split-step Crank-Nicolson method.

1. INTRODUCTION

The advent of Bose-Einstein Condensation opens the way for a better understanding of the physical phenomena related to ultracold atoms confined by external traps [1–3]. The dynamic behavior of Bose-Einstein condensates (BEC) at ultracold temperatures is very well described by the Gross-Pitaevskii equation (GPE) [4, 5]. The GPE is a nonlinear Schrödinger equation (NLSE) with the additional term related to the trapping potential [6].

The presence of nonlinearity, due to interatomic interactions, can result in nonlinear effects in BECs [7, 8]. Among these nonlinear effects, we can mention the solitons: packets of matter-waves that have their shape preserved during the system's temporal evolution [9]. Investigations of matter-wave solitons (MWS) in BECs became a hot area of research when they were observed experimentally [10]. Afterwards, theoretical [11–15] and experimental [10, 16–19] studies about MWSs in both BECs and superfluid Fermi gases (SFG) began to be studied [20, 21]. It is important to note that solitons have unique properties [22] and have applications in several areas of physics such as plasma physics [23], condensed matter physics [24], nonlinear optics [25], materials science [26], etc.

A problem of considerable interest in ultracold atom physics is the possibility of controlling the stability of MWSs in BECs trapped by external potentials subjects to perturbations. Among the possible disturbances,

we can assume a scenario in which the MWSs are subject to periodic perturbations derived from fluctuations in the external potential. Recently, Alotaibi et al [27] analyzed the dynamics of bright-bright solitons in two-component BECs subject to parametric perturbations to explore nonlinear resonances and splitting of the coupled bright solitons. Turmanov et al [28] studied the emergence of density waves in quasi-1D-BECs when dipole-dipole atomic interaction strength is periodically varied in time.

Although several studies have analyzed the stability of solitons in BECs trapped by both periodic [20, 29] and parabolic potentials [30, 31], there are still few studies that address soliton stability when atoms are trapped by a Poschl-Teller potential subject to periodic perturbations. Thus, motivated by the recent publication in [32] and applications of the MWSs in BECs, we propose in this paper an unprecedented investigation of the stability of MWSs in BECs with attractive interatomic interactions trapped in a PT potential subjected to periodic perturbations. For this purpose, we use a numerical approach (split-step Crank-Nicholson method) [33, 34] to solve the one-dimensional Gross-Pitaevskii equation (1D-GPE).

The paper is structured as follows: In Sec. 2, we formulate our dynamic model based on cigar-shaped BECs and obtain a quasi-1D-GPE. In Sec. 3, we present the numerical formulation based on the split-step Crank-Nicolson method to solve the quasi-1D-GPE. In Sec. 4, we present the numerical results related to the stability and chemical potential of MWSs. Finally, in Sec. 5, we present the conclusions and final considerations about this paper.

2. THE DYNAMICAL MODEL

2.1. The Gross-Pitaevski equation

Static and dynamical properties of a pure BEC made of N dilute and ultracold atoms are very well described by the three-dimensional Gross-Pitaevskii equation (3D-GPE) [6]:

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t)=\left[-\frac{\hbar^2}{2m}\nabla^2+V_{ext}(\mathbf{r})+g_{3D}\left|\Psi(\mathbf{r},t)\right|^2\right]\Psi(\mathbf{r},t), \tag{1}$$

where $\mathrm{i}=\sqrt{-1}$ is the imaginary unit, $\hbar=h/2\pi$ is the reduced Planck constant, m is the mass of each atom, ∇^2 is the Laplacian operator in 3D, $V_{ext}(\mathbf{r})$ is the external trapping potential and $g_{3D}=4\pi\hbar^2 a_s/m$ is the nonlinearity coefficient, being a_s the s-wave scattering length of atoms in the BEC; it is positive for repulsive interactions and negative for attractive interactions. The quantity $\Psi(\mathbf{r},t)$ denotes the macroscopic wave function of the BEC and, since $|\Psi(\mathbf{r},t)|^2$ is defined as the atomic density of the N condensed atoms [35], the Eq. (1) will be supplemented by the normalization condition:

$$\int_{-\infty}^{+\infty}\left|\Psi(\mathbf{r},t)\right|^2\mathrm{d}^3\mathbf{r}=N. \tag{2}$$

2.2. The external potential

We start by considering ultracold atoms in a BEC confined in a strongly elongated trap known as "cigar-shaped" trap, described by the following external potential:

$$V_{ext}(x,y,z)=-\frac{\hbar^2V_0}{mw^2}\mathrm{sech}^2\left(\frac{x}{w}\right)+\frac{1}{2}m\omega_{||}^2x^2+\frac{1}{2}m\omega_{\perp}^2\left(y^2+z^2\right). \tag{3}$$

In the above equation, the first term is the Pöchl-Teller (PT) potential and the other terms represent the parabolic potential. In Eq. (3), m is the mass of each atom, V_0 and w are, respectively, the amplitude and width of the PT potential and $\left\{\omega_{||},\omega_{\perp}\right\}$ are the angular frequencies of the parabolic potential. In this cigar-shaped configuration, the longitudinal and transverse confining frequencies (denoted by $\omega_{||}$ and $\omega_{\perp}$, respectively) are such that $\omega_{||}\ll\omega_{\perp}$, where $\omega_{||}\equiv\omega_x$ and $\omega_{\perp}\equiv\omega_y=\omega_z$. Thus, the external potential, $V_{ext}(x,y,z)$, can be

interpreted as a combination of a longitudinal, $V_{||}(x)$, and a transverse potential, $V_{\perp}(y,z)$, i.e., $V_{ext}(x,y,z) = V_{||}(x) + V_{\perp}(y,z)$ [36]. Hence, we can decompose the wave function of the BEC, $\Psi(\mathbf{r},t)$, in a longitudinal (along x) and a transverse (on the $y-z$ plane) component and the system can be governed by quasi-1D-GPE by assuming solutions on the form [37]:

$$\Psi(x,y,z,t) = \psi(x,t)\,\Phi(y,z)\,, \tag{4}$$

where the transversal wave function ,$\Phi(y,z)$, can be obtained analytically from the Schrödinger equation for the transverse harmonic oscillator given by the following equation:

$$\left[-\frac{\hbar^2}{2m}\nabla_{\perp}^2 + \frac{1}{2}m\omega_{\perp}^2\left(y^2+z^2\right)\right]\Phi(y,z) = E_{\perp}\Phi(y,z)\,. \tag{5}$$

Note that $\nabla_{\perp}^2$ is the Laplacian operator in 2D and $E_{\perp}$ is the energy associated to transverse confinement of the trap. Since the considered system is effectively 1D, it is natural to assume that the transverse component of the condensate wavefunction, $\Phi(y,z)$, remains in the ground state [38]. Thus, $\Phi(y,z)$ takes the form:

$$\Phi(y,z) = \sqrt{\frac{m\omega_{\perp}}{\pi\hbar}}\exp\left[-\frac{m\omega_{\perp}}{2\hbar}\left(y^2+z^2\right)\right], \tag{6}$$

with $E_{\perp} = \hbar\omega_{\perp}$ being the ground state energy corresponding to transverse confinement. Due to the weak parabolic confinement in the x direction, it becomes possible to omit the term $\omega_{||}$ from the external potential, thus, assuming that just the PT potential is uniform and is not affected by the presence of the additional confining parabolic potential [37]. Substituting the Eq. (6) into the Eq. (1) and integrating it over the transverse coordinate, we derive an quasi-1D-GPE:

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\psi(x,t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - \frac{\hbar^2 V_0}{mw^2}\mathrm{sech}^2\left(\frac{x}{w}\right) - g_{1D}\left|\psi(x,t)\right|^2 + \hbar\omega_{\perp}\right]\psi(x,t)\,, \tag{7}$$

where $g_{1D} = g_{3D}/2\pi a_{\perp}^2$ is the one-dimensional nonlinearity coefficient and $a_{\perp} = \sqrt{\hbar/(m\omega_{\perp})}$ is the transversal harmonic oscillator ground state width. The term $\hbar\omega_{\perp}$ was omitted because it does not affect the dynamics of the BEC [15].

Eq. (7) can be rewritten using the dimensionless variables $\{\tilde{x},\tilde{t},\tilde{\psi},\tilde{g}_{1D}\}$[1]:

$$x \equiv w\tilde{x}, \qquad t \equiv \frac{mw^2}{\hbar}\tilde{t}, \qquad \psi \equiv \sqrt{\frac{N}{w}}\tilde{\psi}, \qquad g_{1D} \equiv \frac{\hbar^2}{mw}\tilde{g}_{1D}, \tag{8}$$

and replacing the stationary variable of the PT potential with a dynamic variable that causes a periodic perturbation of amplitude Γ and angular frequency Ω around the global minimum point of the potential:

$$\mathrm{sech}^2(x) \to \mathrm{sech}^2(x - \Gamma\cos(\Omega t))\,, \tag{9}$$

we obtain, finally, the equation that governs the dynamic behavior of MWSs in a cigar-shaped BEC confined longitudinally by the PT potential subject to periodic perturbations:

$$\mathrm{i}\frac{\partial}{\partial t}\psi(x,t) = \left[-\frac{1}{2}\frac{\partial^2}{\partial x^2} - V_0\mathrm{sech}^2(x - \Gamma\cos(\Omega t)) - g\left|\psi(x,t)\right|^2\right]\psi(x,t)\,, \tag{10}$$

where $g \equiv g_{1D}N$ and the wave function is normalized to unity:

$$\int_{-\infty}^{+\infty}\left|\psi(x,t)\right|^2\mathrm{d}x = 1. \tag{11}$$

[1] In fact, dimensionless variables are represented by tilde. However, we omit it to simplify the notation.

3. METHODOLOGY

In order to introduce the split-step Crank-Nicolson (SSCN) method [33, 34, 39] to solve the quasi-1D-GPE, Eq. (10) can be expressed as follows:

$$\mathrm{i}\frac{\partial}{\partial t}\psi(x,t) = \left[-\frac{1}{2}\frac{\partial^2}{\partial x^2} - V_0\mathrm{sech}^2\left(x-\Gamma\cos(\Omega t)\right) - g\left|\psi(x,t)\right|^2\right]\psi(x,t) \equiv \hat{H}_{GP}\psi(x,t), \tag{12}$$

where the Hamiltonian $\hat{H}_{GP}$ is a operator that can be expressed by $\hat{H}_{GP} \equiv \hat{T} + \hat{V}$. Here, $\hat{T} \equiv -\frac{1}{2}\frac{\partial^2}{\partial x^2}$ is the kinetic energy operator and $\hat{V} \equiv \hat{V}_{ext} + \hat{V}_{NL}$ is the potential energy operator, with $\hat{V}_{ext}$ being the operator related to external trapping potential and $\hat{V}_{NL}$ the operator responsible for the non-linear term[2]. Eq. (12) is a non-linear partial differential equation defined in the spatial domain $x_\mathrm{i} \leq x \leq x_\mathrm{f}$ and in the temporal domain $t_\mathrm{i} \leq t \leq t_\mathrm{f}$. Here, the initial time is null ($t_\mathrm{i} = 0$). In addition, Eq. (12) is complemented by the initial condition, $\psi(x,0)$, and boundary conditions $\psi(x_\mathrm{i},t)$ and $\psi(x_\mathrm{f},t)$.

The SSCN method is based on the finite difference (FD) method. The basic idea of the FD method is to transform the problem of solving a partial differential equation, linear or non-linear, into a problem of solving a systems of algebraic equations [33], using the discretization of spatial and temporal domains of the wave function and the substitution of derivatives present in the differential equation by approximations involving only numerical values of the wave function [40]. The numerical discretization of the spatial domain consists of dividing the interval $x_\mathrm{i} \leq x \leq x_\mathrm{f}$ in $m+1$ points separated by a space step Δx and, with respect to temporal domain, numerical discretization is based on dividing the interval $t_\mathrm{i} \leq t \leq t_\mathrm{f}$ into $n+1$ points separated by a time step Δt. Both steps are defined by $\Delta x = (x_\mathrm{f} - x_\mathrm{i})/m$ and $\Delta t = (t_\mathrm{f} - t_\mathrm{i})/n$, and the grid of spatial and temporal points can be represented by the following equations:

$$x_j = x_\mathrm{i} + j\Delta x, \quad j = 0,1,\ldots m; \qquad\qquad t_k = t_\mathrm{i} + k\Delta t, \quad k = 0,1,\ldots,n. \tag{13}$$

The initial idea of the SSCN method is based on dividing the Hamiltonian $\hat{H}_{GP}$ into different derivative and non-derivative parts. In this paper, we split $\hat{H}_{GP}$ into three parts: $\hat{H}_{GP} = \hat{H}_1 + \hat{H}_2 + \hat{H}_3$, where

$$\hat{H}_1 = \frac{\hat{V}}{2} = -\frac{1}{2}\left[V_0\mathrm{sech}^2\left(x-\Gamma\cos(\Omega t)\right) + g\left|\psi(x,t)\right|^2\right], \tag{14}$$

$$\hat{H}_2 = \hat{T} = -\frac{1}{2}\frac{\partial^2}{\partial x^2}, \tag{15}$$

$$\hat{H}_3 = \hat{H}_1. \tag{16}$$

On the other hand, the solution of Eq. (12) can be obtained through the following expression:

$$\psi(x_j, t_k + \Delta t) = \mathrm{e}^{-\mathrm{i}\hat{H}_{GP}\Delta t}\psi(x_j,t_k), \tag{17}$$

being $\psi(x_j, t_k + \Delta t)$ the solution after a temporal evolution of an instant of time Δt. For simplicity of notation, we take $\psi_j^k \equiv \psi(x_j,t_k)$. Thus, operator $\mathrm{e}^{-\mathrm{i}\hat{H}_{GP}\Delta t}$ can be rewritten using Eqs. (14)-(16) and the Eq. (17) becomes:

$$\psi_j^{k+1} = \mathrm{e}^{-\mathrm{i}\hat{H}_3\Delta t}\mathrm{e}^{-\mathrm{i}\hat{H}_2\Delta t}\mathrm{e}^{-\mathrm{i}\hat{H}_1\Delta t}\psi_j^k. \tag{18}$$

The solution ψ_j^{k+1} can be obtained by means of the temporal evolution of the wave function at a given time t_k, ψ_j^k, advancing temporally from t_k to t_{k+1} through iterations [33]. For this, the operator $\hat{H}_1$, initially, produces an intermediate solution $\psi_j^{k+1/3}$ from ψ_j^k. As there is no derivative in $\hat{H}_1$ this propagation is performed exactly through the operation:

$$\psi_j^{k+1/3} = \mathscr{O}_\mathrm{nd}(\hat{H}_1)\psi_j^k \equiv \mathrm{e}^{-\mathrm{i}\hat{H}_1\Delta t}\psi_j^k = \mathrm{e}^{\mathrm{i}\left[V_0\mathrm{sech}^2\left(x_j-\Gamma\cos(\Omega t_k)\right)+g\left|\psi_j^k\right|^2\right]\frac{\Delta t}{2}}\psi_j^k, \tag{19}$$

[2]Note that the nonlinearity term will be considered as a potential.

where $\mathscr{O}_{\rm nd}(\hat{H}_1)$ denotes time-evolution operation with $\hat{H}_1$ and the suffix 'nd' denotes non derivative [33]. Then, we perform the time propagation corresponding to the operator $\hat{H}_2$ numerically by the following Crank-Nicholson scheme [39]:

$$\mathrm{i}\frac{\psi_j^{k+2/3}-\psi_j^{k+1/3}}{\Delta t}=-\frac{1}{2}\left[\left(\frac{\psi_{j+1}^{k+2/3}-2\psi_j^{k+2/3}+\psi_{j-1}^{k+2/3}}{2\left(\Delta x\right)^2}\right)+\left(\frac{\psi_{j+1}^{k+1/3}-2\psi_j^{k+1/3}+\psi_{j-1}^{k+1/3}}{2\left(\Delta x\right)^2}\right)\right]. \tag{20}$$

The formal solution of Eq. (20) can be obtained by the Cayley form [41] given by the following expression:

$$\psi_j^{k+2/3}=\mathscr{O}_{\rm CN}(\hat{H}_2)\psi_j^{k+1/3}\equiv \mathrm{e}^{-\mathrm{i}\hat{H}_2\Delta t}\psi_j^{k+1/3}\equiv\frac{1-\mathrm{i}(\hat{T}/2)\Delta t}{1+\mathrm{i}(\hat{T}/2)\Delta t}\psi_j^{k+1/3}, \tag{21}$$

where $\mathscr{O}_{\rm CN}$ denotes the time-evolution operation with $\hat{H}_2$ and the suffix 'CN' refers to the Crank-Nicholson algorithm. Operation $\mathscr{O}_{\rm CN}$ is used to propagate the intermediate solution $\psi_j^{k+1/3}$ by time step Δt to generate the second intermediate solution $\psi_j^{k+2/3}$. Finally, the final solution ψ_j^{k+1} is obtained by acting on the operator $\hat{H}_3$ in $\psi_j^{k+2/3}$:

$$\psi_j^{k+1}=\mathscr{O}_{\rm nd}(\hat{H}_3)\psi_j^{k+2/3}\equiv \mathrm{e}^{-\mathrm{i}\hat{H}_3\Delta t}\psi_j^{k+2/3}\equiv \mathrm{e}^{\mathrm{i}\left[V_0\mathrm{sech}^2\left(x_j-\Gamma\cos(\Omega t_{k+1})\right)+g\left|\psi_j^{k+2/3}\right|^2\right]\frac{\Delta t}{2}}\psi_j^{k+2/3}. \tag{22}$$

According to Adhikari et al [33], splitting the potential energy operator symmetrically around the kinetic energy operator increases enormously the stability of the method and reduces numerical error. The choice of the initial condition is fundamental for the temporal evolution of the wave function. In this paper, a natural choice is the hyperbolic secant function (normalized to unity) [42]:

$$\psi(x,0)=\frac{1}{\sqrt{2\sigma_0}}\mathrm{sech}\left(\frac{x}{\sigma_0}\right), \tag{23}$$

because at the linear limit of the quasi-1D-GPE it is possible to obtain stationary solutions for the 1D linear Schrödinger equation for the PT potential. Another plausible reason for choosing this initial conditional is the fact that the hyperbolic secant function is an exact soliton solution (nonlinear solution) of the 1D nonlinear Schrödinger equation. Here, σ_0 is the initial width of the MWS and we use boundary conditions $\psi_0^k=\psi_m^k=0$ to satisfy $\lim\limits_{x\to\pm\infty}\psi(x,t)=0$.

4. RESULTS AND DISCUSSION

The numerical results presented in this paper were obtained through typical experimental parameters [6] in order to predict realistic results. Thus, we consider the scenario where a BEC consisting of $N=1000$ ^{7}Li atoms weakly interacting. According to Ref. [6], the mass and scattering length of lithium are $m=1.16\times10^{-26}\,\mathrm{kg}$ and $|a_s|=1.23\,\mathrm{nm}$, respectively. We propose the frequency of transverse confinement and the width of the PT potential as being $\omega_\perp=2\pi\times600\,\mathrm{Hz}$ and $w=1.0\,\mu\mathrm{m}$, respectively. Thus, through the aforementioned experimental parameters and provided that $\hbar=1.05\times10^{-34}\,\mathrm{J\cdot s}$, the dimensionless one-dimensional nonlinearity coefficient equation described in Sec. 2 corresponds to $g_{1D}=0.001$. In addition, the temporal evolution proposed in this paper simulates the dynamic behavior of the BEC with a duration of approximately 100 ms, which corresponds to a dimensionless time interval of 1000. In particular, for $V_0=0.5$, $g=1.0$ and $\Gamma=0$, Eq. (10) has an exact stationary solution, $\psi(x,t)=2^{-1/2}\mathrm{sech}\,(x)$, corresponding to the chemical potential $\mu=-0.5$ [32], which also justifies the proposed initial condition given by Eq. (23).

In order to appreciate the dynamic behavior of the temporal evolution of MWSs and to predict their stability, we numerically calculate the behavior of the center-of-mass position ξ and width σ using the following equations [42]:

$$\xi = \frac{1}{\aleph}\int_{-\infty}^{+\infty} x\,|\psi(x,t)|^2\,\mathrm{d}x, \tag{24}$$

$$\sigma = \left[\frac{1}{\aleph}\int_{-\infty}^{+\infty} \frac{12}{\pi^2}\,(x-\xi)^2\,|\psi(x,t)|^2\,\mathrm{d}x\right]^{1/2}, \tag{25}$$

where quantity $\aleph \equiv \int_{-\infty}^{+\infty} |\psi(x,t)|^2\,dx$ represents the normalization of the wave function at each iteration. Moreover, the chemical potential μ, which is also of interest in this paper, can be calculated from the following expression [34]:

$$\mu = \int_{-\infty}^{+\infty} \frac{1}{2}\left|\frac{\partial\psi}{\partial x}\right|^2 - V_0\mathrm{sech}^2\left(x-\Gamma\cos(\Omega t)\right)|\psi|^2 - g\,|\psi|^4\,\mathrm{d}x. \tag{26}$$

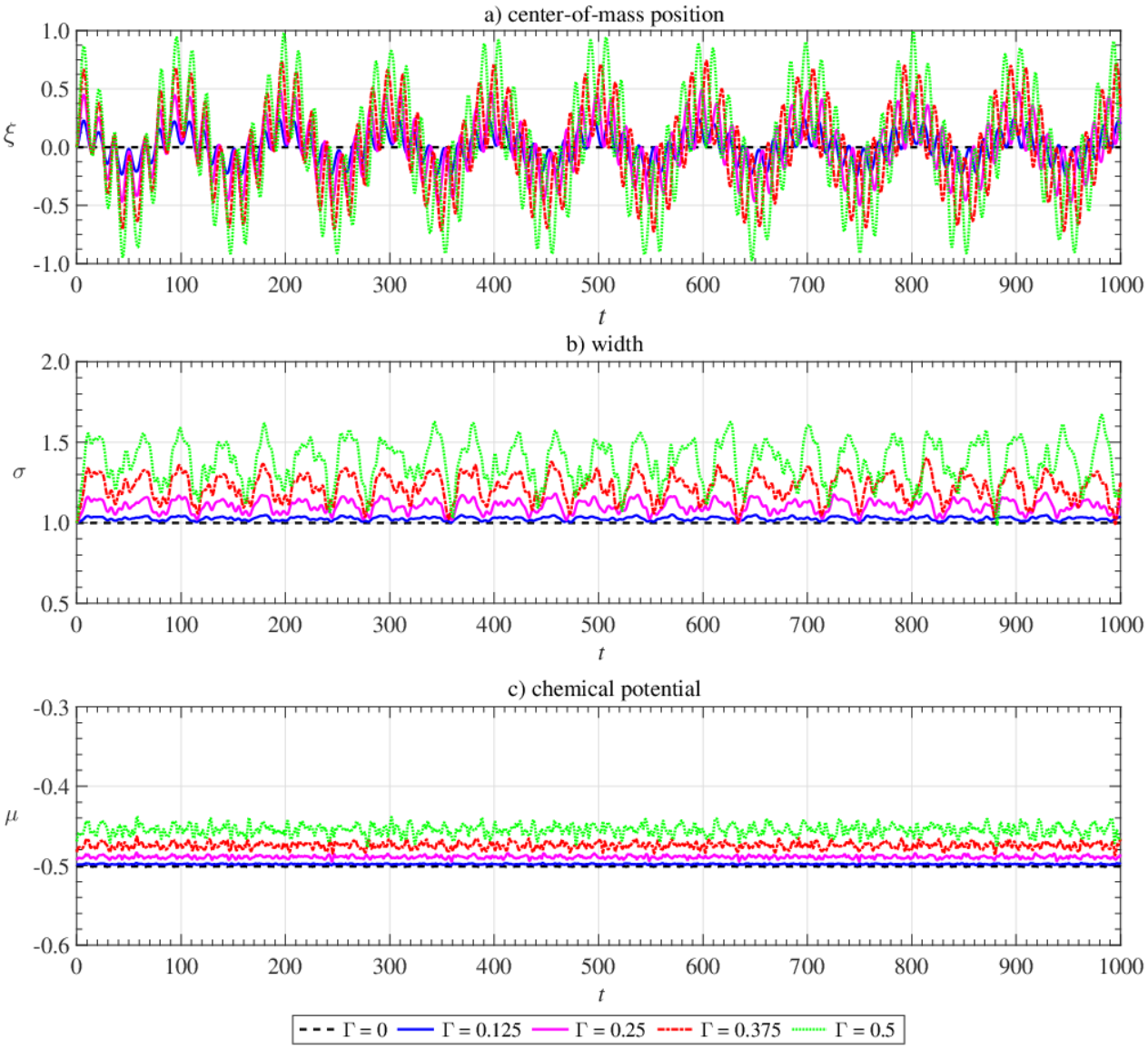

Fig. 1 – Temporal evolution of the center-of-mass position, width and chemical potential corresponding to matter-wave solitons for different oscillation amplitude values of the periodic disturbance. Parameters: $V_0 = 0.5$, $g = 1.0$, $\Omega = 2\pi \times 10^{-2}$, $\sigma_0 = 1.0$, $x_\mathrm{i} = -10$, $x_\mathrm{f} = 10$, $t_\mathrm{i} = 0$, $t_\mathrm{f} = 1000$, $\Delta x = 0.1$, $\Delta t = 0.005$.

Fig. 1 illustrates the numerical results of the dynamic behavior of the center-of-mass position, width and chemical potential of the solitons. As shown in Fig. 1a, the MWSs remained trapped during the time evolution for all the values of oscillation amplitude Γ used in the simulations, despite observing a strong deformation in the format (width) of the MWSs for values from $\Gamma > 0.4$ as seen in Fig. 1b. For large values of oscillation amplitude ($\Gamma > 0.5$) we contemplate the destruction of the BEC. Regarding the chemical potential, Fig. 1c illustrates a practically constant behavior with small fluctuations during the oscillatory perturbation of the potential. It is important to note that as the oscillation amplitude reaches ever greater values, we observe an increase in the chemical potential demonstrating a rate of loss of atoms in the BEC. For experimental purposes, the results obtained in this paper can be useful in controlling the stability of BECs trapped both by the Pöschl-Teller potential and by other confinement potentials, for example in optical lattices and magnetic traps. The temporal evolution of solitos trapped by longitudinal potentials has been confirmed from the experimental and theoretical results through the observation of the soliton's width [15]. In fact, our results demonstraed that MWSs can be stable in the presence of periodic perturbations.

5. CONCLUSIONS

In this paper, we consider the scenario where a quasi-1D BEC made up of diluted ultracold atoms with attractive interatomic interactions is confined by the PT potential. The results obtained demonstrated that MWSs can be stable in the presence of periodic perturbations around the minimum point of PT potential. In addition, we emphasize that the numerical results presented in this paper predicted a good correspondence with the exact analytical results, thus demonstrating the efficiency of the SSCN method. The results obtained in this paper can open the way for a better understanding of the dynamic properties of MWSs both in BECs and SFGs confined through the PT potential and motivate new studies that may have applications in other areas of physics such as solid state physics, condensed matter physics, nonlinear optics and materials sciences.

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