THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A
Volume 22, Number 3/2021, pp. 205–212

REMARKS ON FRACTIONAL ID-[a,b]-FACTOR-CRITICAL COVERED NETWORK GRAPHS

Sufang WANG[1], Wei ZHANG[2]

[1] Jiangsu University of Science and Technology, School of Public Management, Zhenjiang, Jiangsu 212100, China

[2] Wenzhou University, Oujiang College, Wenzhou, Zhejiang 325035, China

Corresponding author: Sufang WANG, E-mail: wangsufangjust@163.com

Abstract. A graph G is called a fractional $[a,b]$-covered graph if for any $e \in E(G)$, G admits a fractional $[a,b]$-factor covering e. A graph G is called a fractional ID-$[a,b]$-factor-critical covered graph if for any independent set I of G, $G-I$ is a fractional $[a,b]$-covered graph. In this paper, we verify that if a graph G of order n with $n \geq \frac{(a+b-2)(a+2b-1)+a+b+1}{b}$ satisfies

$$\delta(G) \geq \frac{(a+b-1)n+a+b+1}{a+2b-1}$$

and

$$\delta(G) \geq \frac{(a+b-2)n+2\alpha(G)+2}{a+2b-2},$$

then G is a fractional ID-$[a,b]$-factor-critical covered graph, where a and b are two integers such that $2 \leq a \leq b$.

Key words: graph, fractional ID-$[a,b]$-factor-critical covered graph, minimum degree, independence number.

1. INTRODUCTION

We may use graphs to model real-world networks. The vertices of the graph correspond to the nodes of the network, and the edges of the graph represent the links between the nodes in the network. Next, we show an example: an online social network with nodes standing for persons and links acting for personal contacts of each user. Henceforth, we use the term “graph” instead of “network”.

The ruggedness and vulnerability of the network are the core issues of network security research, and it is also one of the key topics that researchers must consider during the network designing phase. In a communication network, the data transmission problem at the moment can be transformed to the existence problem of the fractional factor in the corresponding graph of the network. When some nonadjacent nodes with each other are damaged and a special channel is assigned, the possibility of data transmission in a communication network is equivalent to the existence of fractional ID-factor-critical covered graph [23]. Research on the existence of fractional ID-factor-critical covered graphs under specific network structures can help scientists design and construct networks with high data transmission rates. In this paper, we investigate the existence of fractional ID-$[a,b]$-factor-critical covered graphs which plays an important role in studying data transmissions of communication networks. We find that there is strong essential connection between independence number, minimum degree and the existence of fractional ID-$[a,b]$-factor-critical covered graphs, and hence investigations on independence number and minimum degree, which play an irreplaceable role in the vulnerability of the network and the feasibility of data transmission, can yield theoretical guidance to meet data transmission and network security requirements.

The graphs discussed here will be finite, undirected and simple. Let $G=(V(G),E(G))$ be a graph, where $V(G)$ denotes the vertex set of G and $E(G)$ denotes the edge set of G. For $x\in V(G)$, the degree of x in G, denoted by $d_G(x)$, is the number of vertices in G which are adjacent to x. For $x\in V(G)$, the neighborhood of x in G, denoted by $N_G(x)$, is the set of vertices in G which are adjacent to x. We use $\delta(G)$ to denote the minimum degree of G, that is, $\delta(G)=\min\{d_G(x):x\in V(G)\}$. For a vertex subset X of G, we write $N_G(X)=\bigcup_{x\in X}N_G(x)$, and we denote by $G[X]$ the subgraph of G induced by X and by $G-X$ the subgraph obtained from G by deleting all the vertices in X together with their incident edges. We say that X is an independent set of G if $G[X]$ has no edges. The independence number, denoted by $\alpha(G)$, of a graph G is the maximum cardinality of any subset $X\subseteq V(G)$ such that X is an independent set. For $F\subseteq E(G)$, we denote by $G[F]$ the subgraph of G induced by F. We denote by K_n the complete graph of order n.

Let $b\geq a$ be two nonnegative integers, and let $h:E(G)\to[0,1]$ be a function with $a\leq d_G^h(x)\leq b$ for each $x\in V(G)$, where $d_G^h(x)=\sum_{e\in E(x)}h(e)$ and $E(x)$ denotes the set of edges incident to vertex x. We write $F_h=\{e:e\in E(G),h(e)>0\}$. Then we call $G[F_h]$ a fractional $[a,b]$-factor [1] of G with indicator function h. A fractional $[k,k]$-factor is simply called a fractional k-factor, where k is a nonnegative integer. In particular, a fractional $[a,b]$-factor is just an $[a,b]$-factor and a fractional k-factor is just a k-factor if $h(e)\in\{0,1\}$ for any $e\in E(G)$.

Anstee [1] gave a characterization of a graph with a fractional $[a,b]$-factor, which is a special case of Anstee's fractional (g,f)-factor theorem. Liu and Zhang [9] later put forward a simple proof.

THEOREM 1 (Anstee [1], Liu and Zhang [9]). *Let G be a graph and let a,b be two nonnegative integers with $a\leq b$. Then G admits a fractional $[a,b]$-factor if and only if for any $S\subseteq V(G)$ and $T=\{x:x\in V(G)\setminus S,d_{G-S}(x)\leq a\}$,*

$$\lambda_G(S,T)=b|S|+d_{G-S}(T)-a|T|\geq 0,$$

where $d_{G-S}(T)=\sum_{x\in T}d_{G-S}(x)$.

A graph G is called a fractional $[a,b]$-covered graph if for any $e\in E(G)$, G has a fractional $[a,b]$-factor $G[F_h]$ with $h(e)=1$. A fractional $[k,k]$-covered graph is simply called a fractional k-covered graph.

Li, Yan and Zhang [7] derived a criterion for a fractional $[a,b]$-covered graph, which is a special case of Li, Yan and Zhang's fractional (g,f)-covered graph theorem.

THEOREM 2 (Li, Yan and Zhang [7]). *Let G be a graph and let a,b be two nonnegative integers with $a\leq b$. Then G is a fractional $[a,b]$-covered graph if and only if for any $S\subseteq V(G)$ and $T=\{x:x\in V(G)\setminus S,d_{G-S}(x)\leq a\}$,*

$$\lambda_G(S,T)=b|S|+d_{G-S}(T)-a|T|\geq\varepsilon(S),$$

where $\varepsilon(S)$ is defined by

$$\varepsilon(S)=\begin{cases}2, & if\ S\ is\ not\ independent,\\ 1, & if\ S\ is\ independent,\ and\ there\ is\ an\ edge\ joining\ S\ and\ V(G)\setminus(S\cup T)\ or\\ & there\ is\ an\ edge\ e=uv\ joining\ S\ and\ T\ with\ d_{G-S}(v)=a\ for\ v\in T,\\ 0, & otherwise.\end{cases}$$

A graph G is called a fractional ID-$[a,b]$-factor-critical graph if for any independent set I of G, $G-I$ has a fractional $[a,b]$-factor. If $a=b=k$, then a fractional ID-$[a,b]$-factor-critical graph is called a fractional ID-k-factor-critical graph. A graph G is called a fractional ID-$[a,b]$-factor-critical covered graph if for any independent set I of G, $G-I$ is a fractional $[a,b]$-covered graph. If $a=b=k$, then a fractional ID-$[a,b]$-factor-critical covered graph is called a fractional ID-k-factor-critical covered graph.

Egawa and Kotani [2] showed that a graph G admits a 4-factor if G is a 2-connected $K_{1,t}$-free graph with $\delta(G)\geq\frac{3t+1}{2}$, where $t\geq 3$ is an integer. Zhou et. al [19–22, 24, 28] derived some results on the existence of [1,2]-factors in graphs. Kano, Lee and Suzuki [4] investigated the problem of [1,2]-factors in graphs. Kouider and Lonc [6] verified a sufficient condition for the existence of an $[a,b]$-factor in a graph. Liu and Long [8], Zhou, Zhang and Xu [29], Wang and Zhang [11] got some results on the existence of edge-disjoint factors in graphs. Katerinis [5] presented two sufficient conditions for regular graphs to have fractional k-factors. Zhou, Xu and Sun [26], Zhou [17], Yuan and Hao [16], Wang and Zhang [10] justified some theorems on the existence of fractional $[a,b]$-factors in graphs. Wang and Zhang [12], Zhou [18], Zhou, Sun and Pan [25] discussed the existence of fractional factors in graphs. Yuan and Hao [14] put forward a degree condition for a graph to be a fractional $[a,b]$-covered graph. Zhou, Xu and Xu [27] derived a sufficient condition for a graph to be a fractional ID-k-factor-critical graph. Yuan and Hao [15] posed a neighborhood union condition for a graph to be a fractional ID-$[a,b]$-factor-critical graph. Jiang [3], Zhou, Liu and Xu [23] verified some results on fractional ID-$[a,b]$-factor-critical covered graphs. In this article, we investigate fractional ID-$[a,b]$-factor-critical covered graphs, and establish a relationship between independence number, minimum degree and fractional ID-$[a,b]$-factor-critical covered graphs. In what follows, we give our main result.

THEOREM 3. *Let a and b be two integers such that $2 \le a \le b$, and let G be a graph of order n with $n \ge \frac{(a+b-2)(a+2b-1)+a+b+1}{b}$. Then G is a fractional ID-$[a,b]$-factor-critical covered graph if*

$$\delta(G) \ge \frac{(a+b-1)n+a+b+1}{a+2b-1}$$

and

$$\delta(G) \ge \frac{(a+b-2)n+2\alpha(G)+2}{a+2b-2}.$$

The following corollary is derived if $a=b=k$ in Theorem 3.

COROLLARY 1. *Let k be an integer such that $k \ge 2$, and let G be a graph of order n with $n \ge 6k-6+\frac{3}{k}$. Then G is a fractional ID-k-factor-critical covered graph if*

$$\delta(G) \ge \frac{(2k-1)n+2k+1}{3k-1}$$

and

$$\delta(G) \ge \frac{(2k-2)n+2\alpha(G)+2}{3k-2}.$$

2. PROOF OF THEOREM 3

Proof of Theorem 3. It suffices to prove that H is a fractional $[a,b]$-covered graph, where $H = G - I$ for any independent set I of G. On the contrary, we assume that H is not a fractional $[a,b]$-covered graph. Then by Theorem 2, we have

$$\lambda_H(S,T) = b|S| + d_{H-S}(T) - a|T| \le \varepsilon(S) - 1 \qquad (1)$$

for some $S \subseteq V(H)$, where $T = \{x : x \in V(H) \setminus S, d_{H-S}(x) \le a\}$.

It follows from the definition of $\varepsilon(S)$ that $\varepsilon(S) \le |S|$. If $T = \emptyset$, then we get

$$\lambda_H(S,T) = b|S| \ge |S| \ge \varepsilon(S),$$

which contradicts (1). Hence, $T \ne \emptyset$. In what follows, we define

$$\beta = \min\{d_{H-S}(x) : x \in T\}.$$

According to the definition of T, we admit $0 \le \beta \le a$.

CLAIM 1. $|S| \geq \delta(G) - |I| - \beta$.

Proof. We choose $t \in T$ such that $d_{H-S}(t) = \beta$. Thus, we have

$$\delta(H) \leq d_{H-S}(t) + |S| = \beta + |S|. \tag{2}$$

Note that $H = G - I$. Hence, we get $\delta(H) = \delta(G-I) \geq \delta(G) - |I|$. Combining this with (2), we derive

$$\delta(G) - |I| \leq \delta(H) \leq \beta + |S|,$$

that is,

$$|S| \geq \delta(G) - |I| - \beta.$$

This completes the proof of Claim 1. □

Note that I is an independent set of G. Thus, we obtain

$$|I| \leq n - \delta(G). \tag{3}$$

Case 1. $1 \leq \beta \leq a$.

In terms of (3), $|S| + |T| + |I| \leq n$, $\delta(G) \geq \frac{(a+b-1)n+a+b+1}{a+2b-1}$ and Claim 1, we derive

$$\begin{aligned}
\lambda_H(S,T) &= b|S| + d_{H-S}(T) - a|T| \\
&\geq b|S| + \beta|T| - a|T| \\
&= b|S| - (a-\beta)|T| \\
&\geq b|S| - (a-\beta)(n - |S| - |I|) \\
&= (a+b-\beta)|S| + (a-\beta)|I| - (a-\beta)n \\
&\geq (a+b-\beta)(\delta(G) - |I| - \beta) + (a-\beta)|I| - (a-\beta)n \\
&= (a+b-\beta)\delta(G) - b|I| - (a+b-\beta)\beta - (a-\beta)n \\
&\geq (a+b-\beta)\delta(G) - b(n - \delta(G)) - (a+b-\beta)\beta - (a-\beta)n \\
&= (a+2b-\beta)\delta(G) - (a+b-\beta)\beta - (a+b-\beta)n \\
&\geq (a+2b-\beta) \cdot \frac{(a+b-1)n+a+b+1}{a+2b-1} - (a+b-\beta)\beta - (a+b-\beta)n.
\end{aligned}$$

Let $D(\beta) = (a+2b-\beta) \cdot \frac{(a+b-1)n+a+b+1}{a+2b-1} - (a+b-\beta)\beta - (a+b-\beta)n$. Then it follows from $1 \leq \beta \leq a$ and $n \geq \frac{(a+b-2)(a+2b-1)+a+b+1}{b}$ that

$$\begin{aligned}
D'(\beta) &= -\frac{(a+b-1)n+a+b+1}{a+2b-1} - (a+b) + 2\beta + n \\
&= \frac{bn-a-b-1}{a+2b-1} - (a+b) + 2\beta \\
&\geq \frac{bn-a-b-1}{a+2b-1} - (a+b-2) \\
&\geq 0,
\end{aligned}$$

which implies that $D(\beta)$ is an increasing function at $1 \leq \beta \leq a$, and so

$$\lambda_H(S,T) \geq D(\beta) \geq D(1). \tag{4}$$

In light of (1), (4), $\varepsilon(S) \leq 2$

$$\begin{aligned}
1 &\geq \varepsilon(S)-1\geq \lambda_H(S,T)\geq D(1)\\
&= (a+2b-1)\cdot\frac{(a+b-1)n+a+b+1}{a+2b-1}-(a+b-1)-(a+b-1)n\\
&= 2,
\end{aligned}$$

which is a contradiction.

Case 2. $\beta=0$.

Write $X=\{x: x\in T, d_{H-S}(x)=0\}$, $Y=\{x: x\in T, d_{H-S}(x)=1\}$, $Y_1=\{x: x\in Y, N_{H-S}(x)\subseteq T\}$ and $Y_2=Y\setminus Y_1$. We easily see that the graph induced by Y_1 in $H-S$ admits maximum degree at most 1. We use Q to denote a maximum independent set of this graph. It is obvious that $|Q|\geq\frac{1}{2}|Y_1|$. Obviously, $X\cup Q\cup Y_2$ is an independent set of H by our definitions. Therefore, we infer

$$\alpha(H)\geq|X|+|Q|+|Y_2|\geq|X|+\frac{1}{2}|Y_1|+\frac{1}{2}|Y_2|=|X|+\frac{1}{2}|Y|. \tag{5}$$

Note that $H=G-I$. Then we have $\alpha(G)\geq\alpha(H)$. Combining this with (5), we obtain

$$\alpha(G)\geq|X|+\frac{1}{2}|Y|. \tag{6}$$

It follows from (1), (3), (6), $\varepsilon(S)\leq 2$, $|S|+|T|+|I|\leq n$, $2\leq a\leq b$, $\beta=0$ and Claim 1 that

$$\begin{aligned}
1 &\geq \varepsilon(S)-1\geq\lambda_H(S,T)=b|S|+d_{H-S}(T)-a|T|\\
&= b|S|+d_{H-S}(T\setminus(X\cup Y))+|Y|-a|T|\\
&\geq b|S|+2|T\setminus(X\cup Y)|+|Y|-a|T|\\
&= b|S|-(a-2)|T|-2\left(|X|+\frac{1}{2}|Y|\right)\\
&\geq b|S|-(a-2)(n-|S|-|I|)-2\alpha(G)\\
&= (a+b-2)|S|+(a-2)|I|-2\alpha(G)-(a-2)n\\
&\geq (a+b-2)(\delta(G)-|I|-\beta)+(a-2)|I|-2\alpha(G)-(a-2)n\\
&= (a+b-2)\delta(G)-b|I|-2\alpha(G)-(a-2)n\\
&\geq (a+b-2)\delta(G)-b(n-\delta(G))-2\alpha(G)-(a-2)n\\
&= (a+2b-2)\delta(G)-2\alpha(G)-(a+b-2)n,
\end{aligned}$$

namely,

$$\delta(G)\leq\frac{(a+b-2)n+2\alpha(G)+1}{a+2b-2},$$

which contradicts $\delta(G)\geq\frac{(a+b-2)n+2\alpha(G)+2}{a+2b-2}$. This completes the proof of Theorem 3. □

3. REMARK

For two graphs G_1 and G_2, $G_1\vee G_2$ denotes the join of G_1 and G_2. In what follows, we shall show that

$$\delta(G)\geq\frac{(a+b-1)n+a+b+1}{a+2b-1}$$

and

$$\delta(G)\geq\frac{(a+b-2)n+2\alpha(G)+2}{a+2b-2}$$

in Theorem 3 cannot be replaced by

$$\delta(G) \geq \frac{(a+b-1)n+a+b+1}{a+2b-1} - 1$$

and

$$\delta(G) \geq \frac{(a+b-2)n+2\alpha(G)+2}{a+2b-2} - 1.$$

Let a,b and ρ be three integers such that $2 \leq a \leq b$, $a+b-1 < 2\rho < 2a+3b-3$ and $\frac{2\rho(a-1)+1}{b}$ is an integer. We consider a graph $G = ((2\rho-1)K_1 \vee K_{\frac{2\rho(a-1)+1}{b}}) \vee (\rho K_2)$. Then $\alpha(G) = 2\rho - 1$ (since $V((2\rho-1)K_1)$ is a maximum independent set of G) and

$$\begin{aligned} n &= |V(G)| = 4\rho - 1 + \frac{2\rho(a-1)+1}{b} \\ &= \frac{2\rho(a+2b-1)-b+1}{b} \\ &\geq \frac{(a+b)(a+2b-1)-b+1}{b} \\ &= \frac{(a+b-2)(a+2b-1)+2a+3b-1}{b} \\ &> \frac{(a+b-2)(a+2b-1)+a+b+1}{b}. \end{aligned}$$

And we easily calculate that

$$\begin{aligned} \delta(G) &= n - 2\rho + 1 \\ &= \frac{(a+2b-2)(n-2\rho+1)}{a+2b-2} \\ &= \frac{(a+b-2)n+bn-(a+2b-2)(2\rho-1)}{a+2b-2} \\ &= \frac{(a+b-2)n+2\rho(a+2b-1)-b+1-(a+2b-2)(2\rho-1)}{a+2b-2} \\ &= \frac{(a+b-2)n+2\rho+a+b-1}{a+2b-2} \\ &< \frac{(a+b-2)n+2\rho+2\rho}{a+2b-2} \\ &= \frac{(a+b-2)n+2\alpha(G)+2}{a+2b-2}, \end{aligned}$$

$$\begin{aligned} \delta(G) &= \frac{(a+b-2)n+2\rho+a+b-1}{a+2b-2} \\ &= \frac{(a+b-2)n+2\rho+2a+3b-3-(a+2b-2)}{a+2b-2} \\ &> \frac{(a+b-2)n+2\rho+2\rho-(a+2b-2)}{a+2b-2} \\ &= \frac{(a+b-2)n+2\alpha(G)+2}{a+2b-2} - 1 \end{aligned}$$

and

$$\begin{aligned}\delta(G) &= n-2\rho+1\\ &= \frac{(a+2b-1)(n-2\rho+1)}{a+2b-1}\\ &= \frac{(a+b-1)n+bn-(a+2b-1)(2\rho-1)}{a+2b-1}\\ &= \frac{(a+b-1)n+2\rho(a+2b-1)-b+1-(a+2b-1)(2\rho-1)}{a+2b-1}\\ &= \frac{(a+b-1)n+a+b}{a+2b-1}.\end{aligned}$$

Thus, we obtain

$$\frac{(a+b-1)n+a+b+1}{a+2b-1}-1<\delta(G)<\frac{(a+b-1)n+a+b+1}{a+2b-1}$$

and

$$\frac{(a+b-2)n+2\alpha(G)+2}{a+2b-2}-1<\delta(G)<\frac{(a+b-2)n+2\alpha(G)+2}{a+2b-2}.$$

Consider $I=V((2\rho-1)K_1)$, and so I is an independent set of G. Set $H=G-I$, $S=V(K_{\frac{2\rho(a-1)+1}{b}})$ and $T=V(\rho K_2)$. Thus, we admit $|S|=\frac{2\rho(a-1)+1}{b}$, $|T|=2\rho$, $d_{H-S}(T)=2\rho$ and $\varepsilon(S)=2$. Hence, we derive

$$\begin{aligned}\lambda_H(S,T) &= b|S|+d_{H-S}(T)-a|T|\\ &= 2\rho(a-1)+1+2\rho-a\cdot 2\rho\\ &= 1<2=\varepsilon(S).\end{aligned}$$

According to Theorem 2, H is not a fractional $[a,b]$-covered graph, and so G is not a fractional ID-$[a,b]$-factor-critical covered graph.

REFERENCES

1. R. P. ANSTEE, *An algorithmic proof of Tutte's f-factor theorem*, Journal of Algorithms, **6**, pp. 112–131, 1985.
2. Y. EGAWA, K. KOTANI, *4-factors in 2-connected star-free graphs*, Discrete Mathematics, **309**, pp. 6265–6270, 2009.
3. J. JIANG, *A sufficient condition for fractional ID-$[a,b]$-factor-critical covered graphs*, Utilitas Mathematica, **114**, pp. 173–179, 2020.
4. M. KANO, C. LEE, K. SUZUKI, *Path and cycle factors of cubic bipartite graphs*, Discussiones Mathematicae Graph Theory, **28**, pp. 551–556, 2008.
5. P. KATERINIS, *Fractional l-factors in regular graphs*, Australasian Journal of Combinatorics, **73**, pp. 432–439, 2019.
6. M. KOUIDER, Z. LONC, *Stability number and $[a,b]$-factors in graphs*, Journal of graph Theory, **46**, pp. 254–264, 2004.
7. Z. LI, G. YAN, X. ZHANG, *On fractional (g,f)-covered graphs*, OR Transactions (China), **6**(4), pp. 65–68, 2002.
8. G. LIU, H. LONG, *Randomly orthogonal (g,f)-factorizations in graphs*, Acta Mathematicae Applicatae Sinica, English Series, **18**, pp. 489–494, 2002.
9. G. LIU, L. ZHANG, *Fractional (g,f)-factors of graphs*, Acta Mathematica Scientia, Series B, **21**, pp. 541–545, 2001.
10. S. WANG, W. ZHANG, *Discussion on fractional (a,b,k)-critical covered graphs*, Acta Mathematicae Applicatae Sinica, English Series (accepted).
11. S. WANG, W. ZHANG, *On k-orthogonal factorizations in networks*, RAIRO-Operations Research, **55**, pp. 969–977, 2021.
12. S. WANG, W. ZHANG, *Research on fractional critical covered graphs*, Problems of Information Transmission, **56**, pp. 270–277, 2020.
13. S. WANG, W. ZHANG, *Toughness for fractional $(2,b,k)$-critical covered graphs*, Journal of the Operations Research Society of China, 2021, DOI: 10.1007/s40305-021-00359-4.
14. Y. YUAN, R. HAO, *A degree condition for fractional $[a,b]$-covered graphs*, Information Processing Letters, **143**, pp. 20–23, 2019.
15. Y. YUAN, R. HAO, *A neighborhood union condition for fractional ID-$[a,b]$-factor-critical graphs*, Acta Mathematicae Applicatae Sinica, English Series, **34**, pp. 775–781, 2018.

16. Y. YUAN, R. HAO, *Independence number, connectivity and all fractional* (a,b,k)*-critical graphs*, Discussiones Mathematicae Graph Theory, **39**, pp. 183–190, 2019.
17. S. ZHOU, *A result on fractional* (a,b,k)*-critical covered graphs*, Acta Mathematicae Applicatae Sinica, English Series, **37**, pp. 657–664, 2021, DOI: 10.1007/s10255-021-1034-8.
18. S. ZHOU, *Binding numbers and restricted fractional* (g,f)*-factors in graphs*, Discrete Applied Mathematics, **305**, pp. 350–356, 2021, DOI: 10.1016/j.dam.2020.10.017.
19. S. ZHOU, *Remarks on path factors in graphs*, RAIRO-Operations Research, **54**, pp. 1827–1834, 2020.
20. S. ZHOU, *Some results on path-factor critical avoidable graphs*, Discussiones Mathematicae Graph Theory, 2020, DOI: 10.7151/dmgt.2364.
21. S. ZHOU, Q. BIAN, Q. PAN, *Path factors in subgraphs*, Discrete Applied Mathematics, 2021, DOI: 10.1016/j.dam.2021.04.012.
22. S. ZHOU, Q. BIAN, Z. SUN, *Two sufficient conditions for component factors in graphs*, Discussiones Mathematicae Graph Theory, 2021, DOI: 10.7151/dmgt.2401.
23. S. ZHOU, H. LIU, Y. XU, *A note on fractional ID-*$[a,b]$*-factor-critical covered graphs*, Discrete Applied Mathematics, 2021, DOI: 10.1016/j.dam.2021.03.004.
24. S. ZHOU, Z. SUN, H. LIU, *Isolated toughness and path-factor uniform graphs*, RAIRO-Operations Research, **55**, pp. 1279–1290, 2021.
25. S. ZHOU, Z. SUN, Q. PAN, *A sufficient condition for the existence of restricted fractional* (g,f)*-factors in graphs*, Problems of Information Transmission, **56**, pp. 332–344, 2020.
26. S. ZHOU, Y. XU, Z. SUN, *Degree conditions for fractional* (a,b,k)*-critical covered graphs*, Information Processing Letters, **152**, art. no. 105838, 2019, DOI: 10.1016/j.ipl.2019.105838.
27. S. ZHOU, L. XU, Z. XU, *Remarks on fractional ID-k-factor-critical graphs*, Acta Mathematicae Applicatae Sinica, English Series, **35**, pp. 458–464, 2019.
28. S. ZHOU, F. YANG, L. XU, *Two sufficient conditions for the existence of path factors in graphs*, Scientia Iranica, **26**, pp. 3510–3514, 2019.
29. S. ZHOU, T. ZHANG, Z. XU, *Subgraphs with orthogonal factorizations in graphs*, Discrete Applied Mathematics, **286**, pp. 29–34, 2020.

Received May 7, 2021