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# DENOISING SOURCE SEPARATION BASED ON A NONLINEAR FUNCTION AND MAXIMUM OVERLAP DISCRETE WAVELET TRANSFORM DENOISING

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**Abstract**. To further improve the separation accuracy of the denoising source separation algorithm under the noisy model $X = AS + N$, this paper replaces the original nonlinear function with a new nonlinear function (sin) for non-Gaussian signals and the separated signal is denoised by the maximum overlap discrete wavelet transform (MODWT). The simulation results based on communication signals and image signals show that the improved algorithm achieves better performance in blind source separation (BSS) under the universal noisy model $X = A(S+n)+N$.

***Key words**:* denoising source separation, non-Gaussian signals, nonlinear function, wavelet denoising.

## 1. INTRODUCTION

Interference of noise seriously impedes signal propagation and recognition, and observation noise is often introduced by signal receiver. The conventional independent component analysis (ICA) algorithm can successfully separate the independent components in the noise-free environment [1, 2]. However, if the influence of noise, especially the noise with large amplitude, is added, conventional ICA algorithms does not work normally [3].

The noisy ICA model can be mathematically expressed in the vector form as follows:

$$X(t) = AS(t) + N(t) \tag{1}$$

where $A$ is an unknow $m\times n$ mixing matrix which order is $m\times n$, $S(t) = [s_1(t)\cdots s_n(t)]^T$ is an unknown source signal vector which order is $n\times 1$, $N(t) = [n_1(t)\cdots n_n(t)]^T$ is an unknown local noise vector which order is $n\times 1$, $X(t) = [x_1(t)\cdots x_m(t)]^T$ is a known mixed signal vector which order is $m\times 1$. The local noise $N(t)$ is superimposed in the transmission process, and then mixed with the source signal by the mixing matrix *A*. For convenience, equation (1) can be abbreviated as $X = AS + N$. This model can be separated by the ICA algorithm. But after separation, each component may be the mixture of the source signal and the noise, and the source signal and the noise are different from each other. In this way, the source signal can be recovered by proper filtering [4].

For the noisy model $X = AS + N$, *N* is the observation noise received by the sensor. The introduction of noise in this model has a great impact on ICA algorithm, which makes ICA algorithm fail to process. Denoising source separation (DSS) algorithm is a representative blind source separation (BSS) algorithm based on the above noisy model. DSS exploits a separation structure integrating non-Gaussian signal blind separation and signals with different time structures. This structure conducts source signal separation based on the noise reduction function (time filter). As a blind separation algorithm, DSS does not need any prior information about the channel and interference signal. DSS has been extensively applied to various fields, such as the de-noising of paroxysmal electroencephalogram (EEG) data [5], the source separation and recognition of mechanical vibration signal [6], the de-noising of magnetoencephalogram (MEG) record [7], the elimination of interictal spikes of electrocorticogram to improve the control of neural prosthesis [8], and the nonlinear mixing separation of combined images [9], etc. These applications have achieved a certain separation effect.

This paper takes the common non-Gaussian signals (communication signal and image signal) as the source signal, and proves the effectiveness of the algorithm in blind separation of these signals through experimental simulations. The DSS algorithm directly uses the nonlinear function $f(s)=\tanh(s)$ to improve the separation accuracy. The new ideas of the present study aims to find an alternative function sin($s$) to further improve the separation effect. In addition, in order to further obtain purer separated signals, we use the maximum overlap discrete wavelet transform to remove noise from the separated signals (See the description in Section 2.2 for specific reasons).

The remainder of this paper is organized as follows: Section 2 introduces the method, including the specific steps and block diagram of the improved DSS algorithm, and separated signal denoising with the improved algorithm. In Section 3, the effectiveness of the method is verified by simulation experiments. Section 4 summarizes the content of this paper.

## 2. METHODOLOGY

### 2.1. Improved DSS algorithm

The DSS algorithm framework was proposed by Jaakko et al. for the mixed noisy model $X = AS + N$, which integrates signal separation and denoising [10]. The algorithm is derived from the expectation maximization algorithm and exploits the ideas of maximum likelihood method and Bayesian probability estimation. The DSS algorithm first preprocesses (including centralization and whitening) the observation data $X$ to get $\tilde{X}$, and then separates the source signals one by one. The improved DSS algorithm has five steps as follows:

Step 1. The random initialization separation vector $w$, i.e., the inverse of the mixed matrix $A$, is used to estimate the source signal vector

$$s = w^T \tilde{X} . \tag{2}$$

Step 2. In the noise reduction step, the function taking $s$ as an independent variable is as follows:

$$s^+ = f(s), \tag{3}$$

where $f(s)$ is the de-noising function of the source signal $s$. $s^+$ is the expectation of the source signal vector obtained through $f(s)$ in the noise reduction step. All source separation methods with noise reduction are defined in DSS [10], thus DSS provides a basic computational framework for source separation. To deal with a specific problem, the best method can be obtained by selecting or constructing the corresponding noise reduction function $f(s)$.

The blind separation algorithm exploiting the maximum non-Gaussian property of signal is unified with the denoising blind separation algorithm in form. Specifically, the kurtosis of mixed signal is smaller than that of single non-Gaussian signal. When the kurtosis of each signal is maximized, the signal is separated. As for the denoising problem, kurtosis is equal to 0 for the Gaussian noise because it represents a higher-order cumulant. Let the separated signal be denoted as $s = s' + N$. $s'$ represents the source signal estimation without affecting by noise, which has the maximum non-Gaussian property and kurtosis. By maximizing the kurtosis of the noisy output $s$, the adverse effect of Gaussian noise can be eliminated. Therefore, the purpose and means of separation and denoising are consistent, and the whole process can be unified by the nonlinear function $f(\bullet)$. To make the algorithm more robust, a smoother nonlinear function should be used, and hyperbolic tangent function is a good alternative. According to different types of source signals, there are two options for the nonlinear function:

① When the source signal is sub-Gaussian: $f_1(s) = \tanh(s)$

② When the source signal is super-Gaussian: $f_2(s) = s - \tanh(s)$

The selection of denoising function is the key of DSS. The iterative method of DSS algorithm is consistent with Fast-ICA algorithm in form [11]. Based on this similarity, $f_1(s)$ and $f_1(s)$ is replaced with $f_3(s) = \sin(s)$ directly in this paper [12, 13].

Step 3. The estimated value of column separation vector $w^+$ is calculated for signal re-estimation:

$$w^+ = \tilde{X}s^{+T} = \tilde{X}(f(s))^T = \tilde{X}(f(w^T\tilde{X}))^T \tag{4}$$

Step 4. The EM algorithm is used to standardize the separation vector and update the separation vector $w_{new}$ at the same time to normalize the separation vector to avoid its divergence in the iterative process:

$$w_{new} = \frac{w^+}{\| w^+ \|} \tag{5}$$

Step 5. Step 3 and step 4 are repeated until all the source signals are separated and the separation vector $w$ is obtained. In this case, the estimation of the source signal vector can be expressed as $s = w^T\tilde{X}$.

The improved DSS algorithm is referred to as DSS (sin) in subsequent content.

### 2.2. Separated signal denoising

In the process of signal transmission and reception, all kinds of noise are accumulated. Therefore, the communication signal after blind separation often involves a lot of interference noise. To improve the performance of blind separation system, it is necessary to denoise the separated signal. Wavelet Transform (WT) provides a new approach for signal de-noising because of its time-frequency localization characteristics and the flexibility of wavelet basis selection. WT inherits and develops the idea of short-time Fourier transform localization. To overcome the shortcoming that window size does not change with frequency, WT provides a "time-frequency" window, which is the basis for signal time-frequency analysis and signal processing.

Discrete wavelet transform (DWT) is widely used to analyze the time and spectrum characteristics of non-stationary signals. Compared with continuous wavelet transform (CWT), DWT is easier to implement. By sampling the scale and translation parameters of CWT, DWT is mainly used in signal denoising or compression. The maximum overlap discrete wavelet transform (MODWT) is evolved from DWT, but it differentiates from traditional DWT. The threshold of MODWT is determined adaptively to avoid the error caused by the traditional method of setting the threshold in advance. At the same time, the MODWT can operation in real-time because of the small amount of computation, showing great potential to be applied in the field of signal processing.

Here, a comparative experiment of MODWT and DWT denoising is conducted. Considering the original signal $x(t)$, the sampling frequency is set to $f$ is set to 1000 Hz; the number of samples $N$ is set to 1000, and the transient time is set to 0.3 seconds and 0.72 seconds, respectively. The noisy signal is constructed by superposition of 15 dB Gaussian white noise, and then the noise is respectively denoised by DWT and MODWT. Based on the specific equations of DWT and MODWT in references [14, 15, 16], when the original signal $x(t)$ with superimposed Gaussian white noise is input, we get the comparison results in Fig. 1.

It can be seen from Fig. 1 that MODWT performs better than DWT in terms of detail. The correlation coefficient between the DWT de-noising signal and the source signal is 0.9958; the correlation coefficient between the MODWT de-noising signal and the source signal is 0.9975, indicating that the MODWT de-noising signal is closer to the original signal. Therefore, MODWT is selected by this paper to handle with the post-processing of DSS, to denoise the separated signal, and further improves the performance of blind separation system.

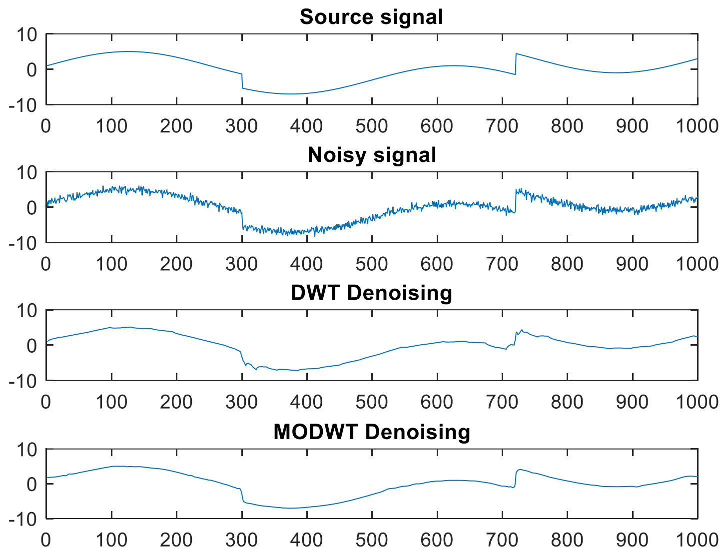


Fig. 1 – Comparison of DWT and MODWT denoising.

### 2.3. Steps and block diagram of proposed algorithm

The improved algorithm proposed in this paper introduces a new nonlinear function. Then, the separated signal is denoised to improve the separation effect of the source signal. Here, the number of source signals is set to $N$, and the number of observation signals is set to $P$, where $P \geq N$. The source signal is assumed to be independent and identically distributed, and of non-Gaussian propoerty. Combined with the new nonlinear function $f(u)=\sin(u)$ and the MODWT de-noising, the DSS(sin) algorithm algorithm is conducted as described in Table 1.

*Table 1*

Steps of DSS(sin) algorithm

| |
|---|
| **Input:** mixed signal $x(t)$. |
| **Output:** separated signal $s_p(t)$. |
| Step 1: The mixed signal $x(t)$ is centralized and whitened to obtain the observed signal $z(t)$. |
| Step 2: The number of independent components to be estimated is set to *n*; the sequence number *p* of the separated signal source is initialized to 1, and the separation precision $\delta$ of the algorithm can be any small positive real number. |
| Step 3: When the initialization separation vector $w_p$ with unit norm is selected, an estimation of the source signal is performed $s_p(t)=w_p{}^T z(t)$. |
| Step 4: A new nonlinear function $f(u)=\sin(u)$ is selected to estimate $s_p(t)$, i.e., $s_p{}^+(t)=f(s_p(t))$. |
| Step 5: Update the value of separation vector, i.e., $w_p{}^+ = z(t)s_p{}^{+T}(t)$. |
| Step 6: The vector $w_p{}^+$ is orthogonalized by $w_p{}^+ = w_p{}^+ - \sum_{j=1}^{p-1}(w_p{}^{+T} w_j)w_j$ and normalized by $w_p = \dfrac{w_p{}^+}{\lVert w_p{}^+ \rVert}$. |
| Step 7: If $\lVert w_p - w_p{}^{old} \rVert \leq \delta$, the algorithm converges and goes to step 8 with $w_p{}^{old}$ as the separation vectors of the last iteration; otherwise, it returns to step 4. |
| Step 8: Let $p=p+1$. If $p<n$, the algorithm returns to step 3. |
| Step 9: The separated signal is denoised by MODWT. |

The new system model of BSS based on the above algorithm is illustrated in Fig. 2.

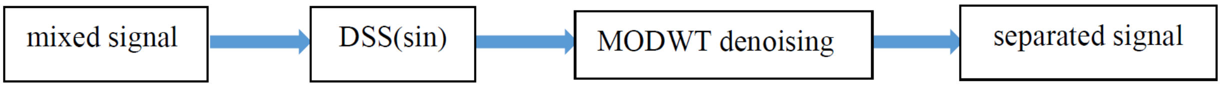


Fig. 2 – BSS system model based on the DSS(sin) algorithm.

## 3. EXPERIMENTS AND RESULTS

In this section, according to the algorithm listed in Table 1, the simulation experiment of mixed blind separation for communication signal and image signal is conducted, which is based on the noise mixed model $X = AS + N$ under low signal-to-noise ratio. To prove the feasibility of the improved algorithm, two popular ICA algorithms are chosen for comparison. Then, the feasibility of post de-noising for separated signals is verified by the universal noisy model $X = A(S+n)+N$ . The performance measure adopted in this paper is correlation coefficient. It is defined as follows.

$$C(s,y) = \frac{\mathrm{cov}(s,y)}{\sqrt{\mathrm{cov}(s,s)}\sqrt{\mathrm{cov}(y,y)}}, \tag{6}$$

where: $s$ denote the source signal vector, $y$ denote the separated signal vector, $C(s,y)=0$ means that $s$ and $y$ are uncorrelated, and $C(s,y)=1$ means that $s$ and $y$ are fully correlated. The bigger the value $C(s,y)$ is, the better the separation performance of a BSS algorithm [4]. The computational load is measured using running time needed for convergence [13].

The download link of the original DSS program can be obtained from the URL: http://www.cis.hut.fi/projects/dss/package/download.shtml, we only need to modify the nonlinear function in the main program to the proposed function, which is the named DSS(sin) algorithm in Table 1. There are three nonlinear functions involved in the comparison: two commonly used $f_1(\cdot)$ and $f_2(\cdot)$ in DSS, and $f_3(\cdot)$ proposed in this paper. To make the comparison as fair as possible, the experiments involving the algorithms for comparison are conducted in the same hardware and software environment (Experimental Platform: MATLAB 2017b, Operating System: 64-bit Windows 7, CPU: Intel® Core™ i7-2720QM Processor @2.20GHz, and RAM: 16 GB). Based on the same source signal and randomly generated mixed matrix, different DSS algorithms are used for separation. Each simulations are repeated for many times, and the number of iterations is set to 50. Finally, the obtained correlation coefficient and running time are averaged to reduce the randomness and improve the reliability of the results. The average correlation coefficient is the average value of the correlation coefficient between the separated signal and the source signal, and the average running time is the average value of the running time of each program (in seconds).

**Simulation 1.** Three kinds of common communication signals, i.e., “square wave”, “sine wave” and “cosine wave”, are taken as source signals. The three kinds of signals have kurtosis values of –1.9358, –1.5092 and –1.5078, respectively. The mixed matrix is generated randomly. For each source signal, the number of samples is set to 100, and the signal-to-noise ratio is –4 dB. The result of separation with the improved DSS (sin) algorithm is shown in Fig. 3.

The separation accuracy and calculation speed of the improved DSS algorithm under nonlinear functions of tanh and sin are compared. The average correlation coefficient (C-ave) and average running time (T-ave) of the algorithm after 50 times of execution are listed in Table 2.

It can be seen from Fig. 3 that the improved DSS algorithm achieves a good separation of the source signal. Meanwhile, the results in Table 2 indicate that using sin as the nonlinear function contributes to a higher separation performance and lower calculation time. Therefore, sin can be selected to replace tanh for separating of non-Gaussian source signal.

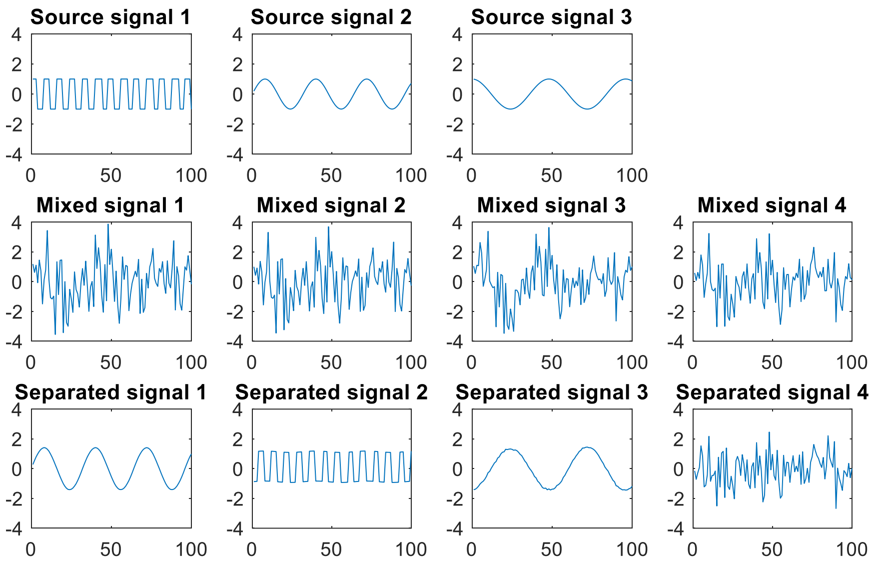


Fig. 3 – Separation result of improved DSS algorithm taking communication signal as source signal.

*Table 2*

Performance of the improved DSS algorithm with different nonlinear functions

| nonlinear functions | C-ave | T-ave (s) |
|---|---|---|
| $f_1(s) = \tanh(s)$ | 0.9973 | 0.2293 |
| $f_2(s) = s - \tanh(s)$ | 0.9964 | 0.2387 |
| $f_3(s) = \sin(s)$ | **0.9981** | **0.2229** |

**Simulation 2.** Blind signal separation can be used in image processing. For example, the main task of image restoration and reconstruction is to restore the original face from the contaminated image. There are various possible causes of image pollution, such as camera jitter, lens transformation, transmission noise superposition, etc. Three commonly used test images are selected as source signals in this experiment, including lenna.bmp, pepper.bmp, and sailboat.bmp. For each source signal, the image size is 512×512 and the bit depth is 24. The kurtosis values of the three source signals are –0.9225, –1.3135 and –1.4017, respectively, and these signals belong to non-Gaussian signal. The mixed matrix is randomly generated, and the signal-to-noise ratio is –4 dB. The results of separation by the improved DSS(sin) algorithm are shown in Fig. 4.


Fig. 4 – Improved DSS separation effect (–4dB) with source signal as image signal.

The separation accuracy and calculation speed of the improved DSS algorithm under nonlinear functions of tanh and sin are compared. The average correlation coefficient (C-ave) and average running time (T-ave) of the algorithm after 50 times of execution are listed in Table 3.

*Table 3*

Performance of the improved DSS algorithm with different nonlinear functions

| nonlinear functions | C-ave | T-ave (s) |
|---|---|---|
| $f_1(s) = \tanh(s)$ | 0.9785 | 1.0546 |
| $f_2(s) = s - \tanh(s)$ | 0.9736 | 1.0752 |
| $f_3(s) = \sin(s)$ | **0.9812** | **1.3240** |

As can be seen from Fig. 4, the improved DSS algorithm achieves good performance for separating source signals. Meanwhile, the results in Table 3 indicate that the nonlinear function of sin contributes to a higher separation performance at the cost of more average calculation time. However, with the emerge of high-speed computing platform, the separation effect should be highlighted more. Therefore, sin is also more appropriate for a source signal like super-Gaussian image.

**Simulation 3.** The advantages of the improved DSS algorithm for non-Gaussian communication signals with the nonlinear function of sin is verified by the results of the above two simulations. This experiment is to verify the separation performance of DSS(sin) algorithm and Fast-ICA algorithm in the noise mixed model $X = AS + N$ and 3×4 (Multi Input Multi Output, MIMO) model [17]. The source signal in this experiment is the same as the one used in simulation 1, and the signal-to-noise ratio is 2 dB. The result of Fast-ICA algorithm is shown in Fig. 5.

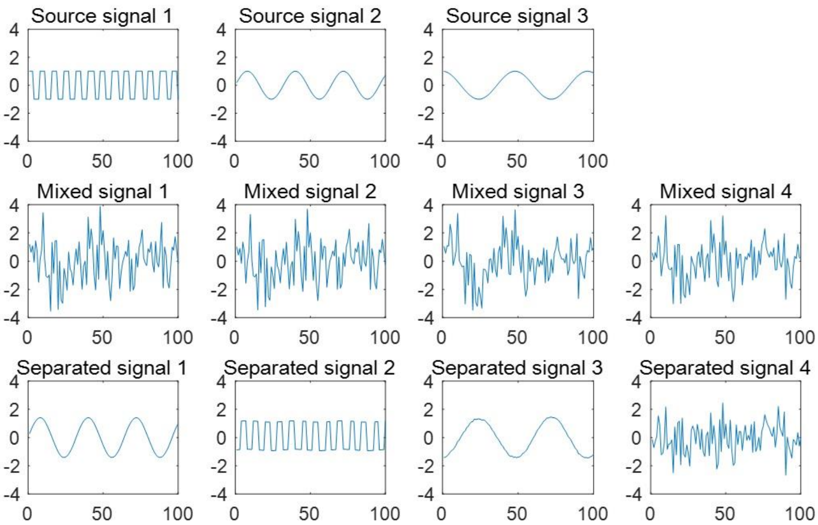


Fig. 5 – Separation performance Fast-ICA taking communication signal as source signal (2 dB).

Fig. 3 shows the separation performance of DSS (sin) under the condition of –4 dB, and Fig. 5 shows the separation performance of Fast-ICA under the condition of 2 dB. It can be seen intuitively from these two figures that the separation performance of DSS (sin) under low SNR is better than that of Fast-ICA under high SNR.

To further prove the feasibility of DSS (sin) algorithm, another ICA algorithm, i.e., radial (robust, accurate, direct ICA algorithm), is chosen for comparison [18]. Radial is a new ICA algorithm based on effective entropy estimation, which directly estimates the independent components by minimizing the Kullback Leibler divergence between the product of joint distribution and marginal distribution. The direct estimation of entropy does not require the density estimation as an intermediate step. In outlier test, the estimator is relatively insensitive to outliers.

To make the comparison as fair as possible, for each algorithm, the experiment is conducted in the same hardware and software environment. The following simulations are repeated for many times with the same signal-to-noise ratio of 2 dB and the same number of iterations of 50. Finally, the obtained correlation coefficient and running time are averaged, which can reduce the randomness and improve the reliability of the results. The average correlation coefficient is the average value of the correlation coefficient between the separated signal and the source signal; the average running time is the average value of the running time of each program (in seconds). The average correlation coefficient (C-ave) and average running time (T-ave) of the algorithm after 50 times of execution are listed in Table 4.

*Table 4*

Comparison of separation performance of improved DSS (sin), Fast-ICA and RADICAL-ICA

| Algorithm | C-ave | T-ave (s) |
|---|---|---|
| DSS(sin) | **0.9978** | 0.0497 |
| Fast-ICA [17] | 0.9500 | **0.0071** |
| RADICAL-ICA [18] | 0.9967 | 0.3128 |

It can be observed from Table 4 that DSS (sin) achieves higher separation performance, followed by RADICAL-ICA, which takes the most time; although Fast-ICA takes the least time, it achieves the worst separation performance. With the increasing speed of computing platform and considering separation performance, it is appropriate to choose DSS(sin).

**Simulation 4.** Considering that there are many kinds of additive noise $n$ in the transmission channel, there is a universal noisy model $X = A(S+n)+N$, as shown in Fig. 6. $n$ represents an unknown channel noise vector which order is $n\times 1$. The separation performance of the following three different schemes under the 3×4 MIMO model is verified. The SNR of the channel is 15 dB and that of the local sensor is 10 dB. The source signal in this simulation is the same as the one in simulation 1.

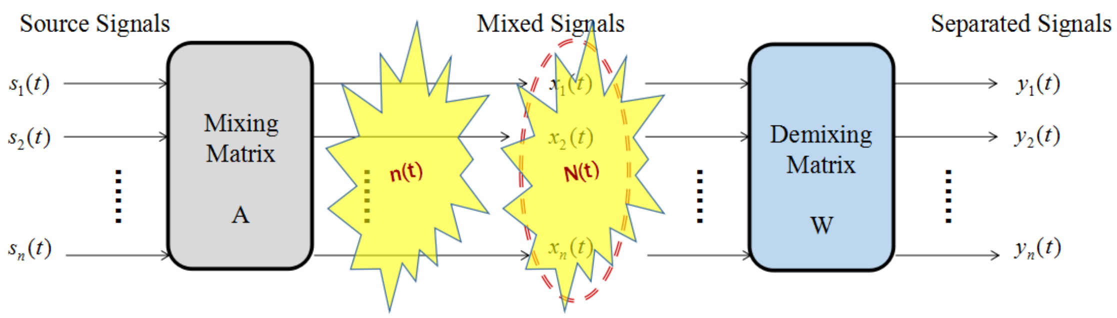


Fig. 6 – Hybrid superimposed noise blind source separation.

Scheme 0. Applying DSS on mixed signal $X(t)$ directly without wavelet de-noising.

Scheme 1. The mixed signal $X(t)$ is denoised by MODWT first, and then by DSS.

Scheme 2. The mixed signal $X(t)$ is denoised by DSS first, and then by MODWT.

The separation accuracy and calculation speed of the three schemes are compared. The average correlation coefficient (C-ave) and average running time (T-ave) of these schemes after 50 times of execution are listed in Table 5.

It can be seen from Table 5 that Scheme 2 achieves better performance than scheme 0 at the cost of a slightly higher calculation time. Meanwhile, it is obvious that the operation of wavelet de-noising in Scheme 1 affects the accuracy of the subsequent DSS separation. Based on the above results, it can be concluded that Scheme 2 is feasible, and it is reasonable to put the denoising module in the last position in Fig. 2.

*Table 5*

Comparison of separation performance of three schemes based on wavelet denoising

| Scheme | C-ave | T-ave (s) |
|---|---|---|
| Scheme 0 | 0.9824 | 0.0465 |
| Scheme 1 | 0.7103 | 0.1269 |
| Scheme 2 | **0.9826** | **0.0481** |

## 4. CONCLUSION

To further improve the separation accuracy of DSS algorithm, an improved DSS algorithm for non-Gaussian signal is proposed in this paper. Firstly, the original nonlinear function is replaced with ‘sin’, and then the wavelet de-noising (MODWT) is used for post-processing of the separated signal. The simulation results indicate that the proposed DSS algorithm is superior in blind separation of non-Gaussian signals in the mixed model of transmission channel and sensor array containing additive noise. However, the improved algorithm is suitable for the well-posed or overdetermined case, and its application in underdetermined case needs further research.

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