THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A
Volume 23, Number 4/2022, pp. 361–368

COMMUTATION RELATIONS GENERALIZED, DUAL STATES AND HOW THEY SOLVE THE NATURALNESS PROBLEM

Renata JORA[1], Sudipa MITRA-KIRTLEY[2]

[1] National Institute of Physics and Nuclear Engineering PO Box MG-6, Bucharest-Magurele, Romania
[2] Rose Hulman Institute of Technology, Terre Haute IN 47803, United States, E-mail: `sudipa.kirtley@rose-hulman.edu`
Corresponding author: Renata JORA, E-mail: `rjora@theory.nipne.ro`

Abstract. In this work we generalize the commutation relations in quantum mechanics and quantum field theory to include the possibility that the lattice spacing for a certain discretization of space time is finite or equivalently that there is a maximum momentum allowed in the theory. We obtain a combination of commutators and anti-commutators for both bosons and fermions. In the two limits $a \to 0$ (a is the lattice spacing) and $a \to \infty$ the theory contains the initial particle respectively a ghost associated to it with opposite statistics. The two limits are dual. Our findings have important consequences and lead to a clear solution to the problem of quadratic divergences of the Higgs boson mass. Therefore our interpretation resolves the naturalness problem in particle physics.

Key words: canonical commutation relations, dual states, naturalness, Higgs boson.

1. INTRODUCTION

The standard model of elementary particles has received the final recognition with the discovery of the Higgs boson at the LHC experiment in 2012 [1, 2]. The average mass of the Higgs boson was established at $m_h = 125.25$ GeV [3].

In the subsequent years no hint of extra particles, extra dimensions, symmetry or phenomena was discovered at the large hadron collider. Therefore in the list of unsolved problem in physics the absence of naturalness of the theory [4] , mainly associated to the quadratic divergences of the Higgs boson mass remained unexplained. The Higgs boson mass in the standard model suffers from quadratic divergences given by [5]:

$$m_h^2 = m_0^2 + \frac{3}{16\pi^2}[m_Z^2 + 2m_W^2 + m_h^2 - 4m_t^2]\Lambda^2, \tag{1}$$

where m_h, m_Z, m_W, m_t are the masses of the Higgs, Z, W bosons and of the top quark. Here Λ is the cut-off scale of the theory. In a natural theory the corrections to the mass of the scalar should be of the order of the respective mass. However for the standard model if the cut off scale is of the order of the Planck scale the ratio of the Higgs mass to the Planck mass is $\frac{m_h}{m_P} \approx 10^{-17}$ so extremely small. This renders the Higgs theory and therefore the standard model highly unnatural. If another particle or symmetry were present in the theory then either the cut-off scale would be established at the mass of the extra particle or the quadratic divergences would simply vanish by the mere presence of these particles. Such would be the case if supersymmetry would be implemented in the standard model.

Early on in 1981 Veltman [6] proposed a solution for cancelling the quadratic divergences by solving for the Higgs mass at around $m_h = 314$ GeV. However in the LHC era we know that this cancellation cannot occur.

Despite the progresses in one direction or another the particle physics is at a crossroad. One may wonder if in the mere structure of quantum field theories is not an aspect or a property that we miss despite the remarkable advances.

In this work we start from the quantum partition function for a quantum mechanics theory to extrapolate to the path integral formalism for a quantum field theory to show that in adequate circumstances the theory admits generalization that do not contradict the present knowledge but offer new insights that offer immediate and natural explanations to important unsolved issues in particle physics. This includes the hierarchy problem. The gist of the new properties is that the theory presents an ad-hoc supersymmetry or a supersymmetric duality that acts selectively in terms of the regime of interest.

Section II contains a short preview of the commutation relations in the path integral formalism, section III introduces a generalization of the commutation relation for the quantum theory of the scalar field and section IV offers a solution to the naturalness problem in terms of the specific example of the Higgs boson mass. Section IV is dedicated to the Conclusions.

2. COMMUTATION RELATIONS IN THE PATH INTEGRAL FORMALISM

The Lagrangian for the nonrelativistic particle motion in a potential in quantum mechanics has the expression:

$$\mathscr{L} = \frac{1}{2}m\dot{q}_i^2 - V(q), \tag{2}$$

where q_i are the three space cooordinates, $\dot{q}_i = \frac{dq_i}{dt}$ and $V(q)$ is the potential. The corresponding partition function is :

$$Z = \int dq(t)\exp[i\int dt\mathscr{L}]. \tag{3}$$

We discretize $0 \le t \le T$ in small equal intervals $T = N\varepsilon$ and denote $T_k = k\varepsilon$ and $q_{ik} = q_i(t_k)$. For simplicity we shall consider only one spatial dimension and drop the subscript i. Then one may write:

$$\mathscr{L} = \sum_{k=0}^{N}[\mathscr{T}_k - V_k]. \tag{4}$$

Here,

$$\mathscr{T}_k = \frac{1}{2}m[\frac{(q_{k+1}-q_k)^2}{\varepsilon^2} + \frac{(q_k - q_{k-1})^2}{\varepsilon^2}], \tag{5}$$

where $\mathscr{T}_k$ is the kinetic energy corresponding to the partition k. Moreover $q_0 = q_a$ and $q_N = q_b$ where q_a and q_b are the coordinates boundaries on the interval of time $[0,T]$. It is convenient the use the potential of an harmonic oscillator, therefore we shall take:

$$V = \frac{1}{2}k_s q^2. \tag{6}$$

Then,

$$V_k = \frac{1}{2}k_s\frac{(q_{k+1}+q_k)^2}{4} + \frac{1}{2}k_s\frac{(q_k+q_{k-1})^2}{4}. \tag{7}$$

We consider the quantity:

$$\int dq(t)q_k\frac{d}{dq_k}\exp[i\int dt\mathscr{L}] = -1, \tag{8}$$

to obtain:

$$\begin{aligned}&\langle -\frac{m}{\varepsilon}[(q_{k+1}-q_k)q_k - (q_k - q_{k-1})q_k] - \\ &\frac{k\varepsilon}{4}[(q_{k+1}+q_k)q_k + (q_k+q_{k-1})q_k] = i\rangle.\end{aligned} \tag{9}$$

One defined $\dot{q}_k = \frac{q_{k+1}-q_k}{\varepsilon}$ or $\dot{q}_k = \frac{q_k - q_{k-1}}{\varepsilon}$. There is an ambiguity of the definition from the mathematical point of view and also in the definition of external particle in physics. This is resolved by taking the Weyl ordering of the particles which is equivalent to time ordering of them. In this situation Eq. (9) becomes:

$$\begin{aligned}&\langle m[q_k \frac{q_k - q_{k-1}}{\varepsilon} - \frac{q_{k+1}-q_k}{\varepsilon} q_k] - \\ &\frac{\varepsilon k}{4}[(q_{k+1}+q_k)q_k + q_k(q_k+q_{k-1})] = i\rangle.\end{aligned} \tag{10}$$

By taking the continuum limit $\varepsilon \to 0$ one obtains:

$$[q,p] = qp - pq = i. \tag{11}$$

In the above we illustrated how one can obtain the standard commutation relations in quantum mechanics from the path integral formalism as it was derived early on by Dirac [7] and Feynman [8].

3. COMMUTATION RELATIONS GENERALIZED

First we will generalize the procedure outlined in section II to the Lagrangian of the free scalar in quantum field theory (consider Minkowski or euclideean space):

$$\mathscr{L} = \frac{1}{2}\partial_\mu \Phi(x) \partial^\mu \Phi(x) - \frac{1}{2} m^2 \Phi^2. \tag{12}$$

We consider a cubic lattice such that at each point $x_\mu = k_\mu \varepsilon$ with $0 \le k_\mu \le N$. The lattice spacing is $\varepsilon = \frac{\pi}{P_{max}}$ where P_{max} is the maximum momentum in the theory. Then one can write immediately:

$$\begin{aligned}S = \sum_{0 \le k_\mu \le N} \Bigg[&\frac{\varepsilon^2}{2}[(\Phi(x^\mu_{k+1}) - \Phi(x^\mu_k))^2 + (\Phi(x^\mu_k) - \Phi(x^\mu_{k-1})^2] - \\ &\frac{\varepsilon^4 m^2}{16}[(\Phi(x^\mu_{k+1}) + \Phi(x^\mu_k))^2 + (\Phi(x^\mu_k) + \Phi(x^\mu_{k-1})^2].\end{aligned} \tag{13}$$

Here we denoted generically by k a point on the lattice with components k_μ and summation over k involves summation over the four components k_μ.

In quantum field theories the commutation or anticommutation relations take place at equal time. We can proceed as such or we can consider the most general case where all four coordinates may vary. We shall apply the latter case (which justifies the factor in the front of the square bracket in the second line of Eq. (13)). In what follows the sum over the μ components is suppressed and will be retrieved at the end of the calculations. One alternatively may consider only one coordinate and generalize in the end to the full set.

On the lattice one has:

$$\int d\Phi(x_k)\Phi(x_k)\frac{d}{d\Phi(x_k)} \exp[iS_k] = -\int d\Phi(x_k) \exp[iS_k], \tag{14}$$

which leads to:

$$\begin{aligned}i\varepsilon^2 \Bigg[&[-\Phi(x_{k+1}) + \Phi(x_k)]\Phi(x_k) + [\Phi(x_k) - \Phi(x_{k-1})]\Phi(x_k)] - \\ &\frac{m^2\varepsilon^2}{8}[[\Phi(x_{k+1}) + \Phi(x_k)]\Phi(x_k) + [\Phi(x_k) + \Phi(x_{k-1})]\Phi(x_k)] \Bigg] = -1.\end{aligned} \tag{15}$$

Again we apply Weyl ordering for all coordinates. Since we considered time on the same footing with the space coordinates the Weyl ordering is applied to all of them. The criterion is to place all quantities with the index k larger at the beginning of the products. Then Eq. (15) becomes:

$$\varepsilon^2\Bigg[[-\Phi(x_{k+1})+\Phi(x_k)]\Phi(x_k)+\Phi(x_k)[\Phi(x_k)-\Phi(x_{k-1})]]-$$
$$\frac{m^2\varepsilon^2}{8}[[\Phi(x_{k+1})+\Phi(x_k)]\Phi(x_k)+\Phi(x_k)[\Phi(x_k)+\Phi(x_{k-1})]]\Bigg]=i. \tag{16}$$

This may be written as:

$$\Bigg[[-\Phi(x_{k+1})+\Phi(x_k)]\Phi(x_k)+\Phi(x_k)[\Phi(x_k)-\Phi(x_{k-1})]]-$$
$$\frac{m^2\varepsilon^2}{8}[[\Phi(x_{k+1})+\Phi(x_k)]\Phi(x_k)+\Phi(x_k)[\Phi(x_k)+\Phi(x_{k-1})]]\Bigg]=i\frac{1}{\varepsilon^2}. \tag{17}$$

We observe that if the term in the first line contains only to the time derivative and we take the limit $\varepsilon \to 0$ we obtain the standard commutation for a scalar field in quantum field theory corresponding to equal time, equal space coordinates:

$$[\Phi(x),\Pi(x)]=i\frac{1}{\varepsilon^3}, \tag{18}$$

where $\Pi(x)=\frac{\partial\Phi(x)}{\partial t}$. Here one may take $\delta(0)=\frac{1}{\varepsilon^3}$. The relation in Eq. (18) can be easily generalized to the complete commutation relation using exactly the same procedure if one considers the set-up in Eq. (14) slightly modified as in:

$$\int d\Phi(x)\Phi(x_k)\frac{d}{d\Phi(x_p)}\exp[iS]=-\int d\Phi(x)\exp[iS]\delta_k^p, \tag{19}$$

Note that in the product in the partition function the two quantities have different space time indices k and p.

In our set-up the lattice spacing is finite and related to the maximum momentum in the theory. Consequently Eq. (17) will contain both commutator and anticommutators. To understand what is happening let us assume that $\frac{m^2\varepsilon^2}{8}\gg 1$. Then the first term in the equation can be neglected and one obtains:

$$\left[\frac{m^2\varepsilon^2}{8}[\Phi(x_{k+1})+\Phi(x_k)]\Phi(x_k)+\Phi(x_k)[\Phi(x_k)+\Phi(x_{k-1})]\right]=-i\frac{1}{\varepsilon^2} \tag{20}$$

This case in QFT corresponds to the situation when the cut-off scale is smaller than the particle mass. Then in the standard quantum field theory one just simply assumes that the particle decouples from the theory apart for some suppressed terms. For a real scalar Eq. (20) might be interpreted in a trivial way. However if in the initial set-up one considered a complex scalar field the outcome would be evidently:

$$\{\Phi(x_k),\Phi(x_k)^*\}=-i\frac{8}{m^2\varepsilon^4}, \tag{21}$$

where the curl bracket refers to anticommutators. Therefore one has two distinct limits: the limit $\frac{m^2\varepsilon^2}{8}\ll 1$ in which the particle has the usual statistics (the scalar behaves like a scalar) and the limit $\frac{m^2\varepsilon^2}{8}\gg 1$ in which the particle has the statistics reversed (the scalar behaves like an anticommuting ghost). There is more to it which will be discussed in the next section.

4. INTERPRETATION AND APPLICATIONS

In the previous section we showed that for a lattice with finite size there are two distinct limits for the commutation relations, one with $\frac{m^2\varepsilon^2}{8} \ll 1$ in which the particle has the normal statistics, one with $\frac{m^2\varepsilon^2}{8} \gg 1$ in which the particle behaves as a ghost with opposite statistics. In this case we will deal with the situation in which the quantity $\frac{m^2\varepsilon^2}{8}$ has an arbitrary value and therefore the actual commutation relations are mixed.

We start by rewriting the commutation relations in Eq. (16) in the partition function:

$$\int d\Phi(x_k)\Bigg[i\varepsilon^2\Bigg[[-\Phi(x_{k+1})+\Phi(x_k)]\Phi(x_k)+[\Phi(x_k)-\Phi(x_{k-1})]\Phi(x_k)]-$$
$$\frac{m^2\varepsilon^2}{8}[\Phi(x_{k+1})+\Phi(x_k)]\Phi(x_k)+[\Phi(x_k)+\Phi(x_{k-1})]\Phi(x_k)]\Bigg]\exp[i(K-aU)]=$$
$$-\int d\Phi(x_k)\exp[i(K-aU)], \tag{22}$$

where:

$$K=\sum_{0\le n_\mu\le N}\left[\frac{\varepsilon^2}{2}[(\Phi(x^\mu_{k+1})-\Phi(x^\mu_k))^2+(\Phi(x^\mu_k)-\Phi(x^\mu_{k-1}))^2]\right]$$
$$U=\sum_{0\le n_\mu\le N}\left[\frac{\varepsilon^2}{2}[(\Phi(x^\mu_{k+1})+\Phi(x^\mu_k))^2+(\Phi(x^\mu_k)+\Phi(x^\mu_{k-1}))^2]\right]$$
$$a=\frac{\varepsilon^2m^2}{8}. \tag{23}$$

We consider the second line in Eq. (22) and make a change of variables $\Phi(x_k)=(-1)^{|k|^2}\Phi'(x_k)$ where $|k|=\sqrt{k_\mu k^\mu}$. Note that each time $k_\mu\to k_\mu\pm1$ the field $\Phi(x^\mu_{|k|\pm1})$ becomes negative with respect to $\Phi(x^\mu_k)$. Then we obtain:

$$\int d\Phi(x_k)\frac{m^2\varepsilon^2}{8}[\Phi(x_{k+1})+\Phi(x_k)]\Phi(x_k)+[\Phi(x_k)+\Phi(x_{k-1})]\Phi(x_k)]\exp[i(K-aU)]=$$
$$\int d\Phi(x_k)'(-1)^{|k|^2}\frac{m^2\varepsilon^2}{8}[-\Phi(x_{k+1})'+\Phi(x_k)']\Phi(x_k)'+[\Phi(x_k)'-\Phi(x_{k-1})']\Phi(x_k)']\times$$
$$\exp[i(-aK+U)]. \tag{24}$$

Notice the change in the action in the exponential.

Applying Eq. (24) to Eq. (22) upon a Weyl ordering one obtains:

$$\int d\Phi(x)\varepsilon^2\Bigg[[\Phi(x_k),\Pi(x_k)]\exp[i(K-aU)]-$$
$$\int d\Phi(x)'\exp[i\pi\sum|k|^2]a[\Phi(x_k)',\Pi(x_k)']\exp[i(-aK+U)]=$$
$$-i\int d\Phi(x_k)\exp[i(K-aU)]. \tag{25}$$

Here $\exp[i\pi\sum|k|^2]=\exp[i\pi N_0]$ is just an angle depending on N_0 which is a function of N and can be easily calculated for Minkowski and euclideean space. Here we will assume that one picks N such that this contribution is one. Therefore we will ignore this factor in what follows.

By renaming Φ' by Φ in the second line of Eq. (25) one obtains a clear commutator:

$$\int d\Phi(x)\varepsilon^2[\Phi(x_k),\Pi(x_k)][\exp[i(K-aU)]-a\exp[-aK+U]]=$$
$$-i\int d\Phi(x_k)\exp[i(K-aU)]. \tag{26}$$

Notice that at this point the commutations relation are correct for a scalar field. Nevertheless the action and the partition function changes according to:

$$\int d\Phi[\exp[i(K-aU)]-a\exp[-aK+U]]=(1-a)\int d\Phi\exp[i(K-aU)], \tag{27}$$

where in the second term we made the change back in the variable $\Phi(x)$.

The significance of our approach is evident if one considers the propagator in the Fourier space:

$$\begin{aligned}&\langle\Phi(p)\Phi(-p)\rangle=\\&\delta(0)_p\left[i\frac{1}{p^2-m^2}-ia\frac{1}{ap^2-\frac{1}{\varepsilon^2}}\right]=\\&\delta(0)_p\left[i\frac{1}{p^2-m^2}-i\frac{1}{p^2-\frac{1}{a\varepsilon^2}}\right].\end{aligned} \tag{28}$$

Since $a\varepsilon^2=\frac{m^2\varepsilon^4}{8}=\frac{m^2\pi^4}{8\Lambda^4}$ Eq. (28) clearly corresponds to the introduction of a Pauli Villars regulator. Note that this is a consequence of the introduction of a correct commutator in the theory and arrives naturally in this approach without any artifice.

5. APPLICATION TO THE HIGGS QUADRATIC DIVERGENCES

The problem of the quadratic divergences of the Higgs boson was briefly outlined in the Introduction.

Here we will show how this problem can be solved easily in the framework introduced in this paper.

One can apply the same generalizations of the commutation relations to the fermions with the amend that the derivatives and mass terms are linear. Therefore in the case of fermions the separation is made as follows. Both K and U will contain as factors in front ε^3 and $a_f=\frac{m\varepsilon}{8}$.

The quadratic divergences come from the Lagrangian,

$$\mathscr{L}=-y_f h\bar{f}f, \tag{29}$$

and the contribution to the Higgs boson mass is:

$$\Pi^f_{hh}(0)=-4y_f^2\int\frac{d^4k}{2\pi)^4}[\frac{1}{k^2-m_f^2}+\frac{2m_f^2}{(k^2-m_f^2)^2}]. \tag{30}$$

Here the first term is the quadratic contribution. In our set-up the mass term will become the kinetic term for the associated ghost and therefore the momentum in $\bar{f}\gamma^\mu p_\mu f$ will be multiplied by a_f. Therefore the quadratic divergence will be multiplied by a factor $\frac{1}{a_f^2}$.

The coupling constant will be exactly the known one. For the contribution to the Higgs mass we need those particles that appear as ghosts in the higher theory. Since mass of the W boson functions as a cut-off the next particle with mass of interest will be the bottom quark. In this set-up then $\varepsilon=\frac{\pi}{m_W}$. The bottom quark will thus act in the higher theory as a scalar ghost with the opposite statistics to that of a fermion. We know how to calculate the associate quadratic divergence:

$$\delta_B = \frac{3}{16\pi^2}\Lambda^2 \frac{1}{a_f^2}\frac{4}{2}m_b^2 =$$
$$\frac{3}{16\pi^2}\Lambda^2 (\frac{8m_W}{\pi m_b})^2 \frac{4}{2} m_b^2. \tag{31}$$

The factor $\frac{1}{2}$ comes form the conversion to the scalar statistics.

We will consider the contribution above and all the other contributions to the Higgs boson quadratic corrections known except for the contribution from the top quark which is taken as unknown. We require the cancellation of the quadratic divergences,

$$\frac{3}{16\pi^2}[m_Z^2 + 2m_W^2 + m_h^2 - 4m_t^2]\Lambda^2 + \delta_b = 0, \tag{32}$$

to determine the mass of the top quark $m_t = 173.72$ GeV which is well within the experimental range. Here we used the central values of the masses as taken from [3]: $m_W = 80.377 \pm 0.012$ GeV, $m_Z = 91.1876 \pm 0.0021$ GeV, $m_h = 125.25 \pm 0.17$ GeV, $m_t = 172.69 \pm 0.30$ GeV and $m_b = 4.18^{+0.03}_{-0.02}$ GeV.

In essence we used the Veltman condition with the additional converted ghost coming from the next light particle in the spectrum. Here there are other options for selecting the scale of interest and the associated light particle. First we notice that the masses that we included in the quadratic divergence of the Higgs boson are all in the same range. Assume that the lower scale would be much higher than the mass of the top quark. Then these particles will become ghosts with opposite statistics. Their masses will be different. However their contribution to the quadratic divergence will be almost equal in magnitude (as in Eq. (31)) and will cancel by the simply counting of fermion and boson states: $\delta_z + 2\delta_W + \delta_h - 4\delta_t = 0$.

6. CONCLUSIONS

In this work we start from the path integral formalism for quantum mechanics and quantum field theories and the implementation of quantum commutation relations in this context stemming from the work of Dirac [7] and Feynman [8] in this direction based on time ordering or Weyl ordering. We notice that this formulation is adequate for a lattice field theory (or discrete one) only in the continuum limit or when the lattice spacing a $a \to 0$. However in real life theories there is always a maximum momentum of the theory which would correspond to a finite yet small lattice size $a = \frac{\pi}{P_{max}}$. If we allow this quantity to be finite then any commutation relation in the theory will also contain an anti-commutator showing that even for a free particles the statistics is mixed. This is exemplified in the paper for scalars but works as well for fermions.

A suitable change of variables in the partition function leads to to the diagonalization of the commutation relations such that they contain either only a commutator with the mass of the particle m or an ghost anticommutator with mass $m' \propto \frac{1}{m}$. These states are dual with one another and occur in two different limits $ma \to 0$ and $ma \to \infty$, where a is as before the lattice spacing. This properties are illustrated for a scalar particle. For a fermions there are also two dual states but the statistics is reversed.

First we show that in quantum field theories the method applied here is similar to the implementation of the Pauli Villars procedure in a natural way in the theory. Then we show that in the framework presented here the standard model is completely natural as the presence of dual particles solves at least the quadratic divergence problem of the Higgs boson.

The findings in this paper may lead to further breakthroughs. In quantum field theory whenever the mass of a particle is higher than the cut-off scale of the theory this particle decouples and some corrections appear in the effective theory. In our approach these corrections do not need to be determined. It would be sufficient to introduce an associated ghost with opposite statistics and a different mass and the same couplings as the massive particle and to compute the quantum corrections in this theory. This method would have important consequences on all processes including the behavior of a non abelian or abelian gauge theory.

REFERENCES

1. G. AAD, T. ABAJYAN, B. ABBOTT, J. ABDALLAH, S.A. KHALEK, A.A. ABDELALIM, R. ABEN, B. ABI, M. ABOLINS, O.S. ABOUZEID, H. ABRAMOWICZ, *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*, Phys. Lett. B, **716**, pp. 1–29, 2012.
2. S. CHATRICHYAN, V. KHACHATRYAN, A.M. SIRUNYAN, A. TUMASYAN, W. ADAM, E. AGUILO, T. BERGAUER, M. DRAGICEVIC, J. ERÖ, C. FABJAN, M. FRIEDL, *Observation of a new boson aa mass of 125 GeV with the CMS experiment at the LHC*, Phys. Lett. B. **716**, pp. 30–61, 2012.
3. P.A. ZYLA et al. (Particle Data Group), *The Review of Particle Physics*, Progr. Theor. Exp. Phys., 2020, 083 C01, 2020 and 2021 update.
4. M. DINE, *Naturalness under stress*, Annual Review of Nuclear and Particle Science, **65**, pp. 43–62, 2015.
5. J. MASINA, M. QUIROS, *On the Veltman condition, the hierarchy problem and high-scale supersymmetry*, Phys. Rev. D, **88**, art. 093003, 2013.
6. M.J.G. VELTMAN, *The infrared-ultraviolet connection*, Acta Phys. Polon. B, **12**, p. 437, 1981.
7. P. DIRAC, *The Lagrangian in quantum mechanics*, Physikalische Zeitschrift of Sovjet Union, **3**, pp. 64–72, 1933.
8. R. FEYNMAN, *Space-time approach to non relativistic quantum mechanics*, Rev. Mod. Phys., **20**, art. 367, 1948.

Received November 10, 2022