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# CONSENSUS TRACKING ITERATIVE LEARNING CONTROL OF SECOND-ORDER MULTI-AGENT SYSTEMS

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**Abstract**. In this paper, the problem of consensus tracking control for a class of second-order leader-following nonparametric uncertain multi-agent systems, which perform a given repetitive task over a finite interval with arbitrary initial error. By means of learning control and initial shift rectifying, a first-order attractor control algorithm is presented.In the tracking process, the proposed algorithm simultaneously rectifies all the initial state shifts, and after enough iterations, the all following multi-agents' states perfectly track the leader's state in the preset time interval. Finally, simulation results demonstrate the effectiveness of the learning control algorithm.

***Key words:*** initial rectifying, iterative learning control, multi-agent systems, nonparametric uncertainties.

## 1. INTRODUCTION

A multi-agent system is composed of multiple agents performing the same task. Each agent can only communicate with adjacent agents to obtain local state information. At runtime, each agent collaborates with each other to complete a complex task through this local communication mode. In recent years, due to the wide applications of multi-agent, more and more scholars have begun to study the interaction and cooperative control of multi-agent [1, 2]. For multi-agent systems, the fundamental problem is the consensus tracking control [3,4]. The so-called consensus problem refers to that each agent in the multi-agent system achieves the same output or the same state through mutual communication and cooperation. The research results is widely used in many fields, such as robot formation, distributed neural network and UAV control [5].

At the beginning of the 21st century, some scholars tried to use Lyapunov method to solve the consensus problem of multi-agent systems [6–14]. Reference [8] analyzed the consensus problem of multi-agent systems by using Laplacian matrix. In [10], a terminal sliding mode algorithm is proposed to solve the tracking problem of multi-robot systems. Compared with linear multi-agent systems and parameterized nonlinear multi-agent systems, the tracking control of nonparametric nonlinear multi-agent systems is more difficult. References [11] and [12] solve the consensus tracking control problem of nonparametric nonlinear multi-agent systems with lipschitz continuity, while [13] and [14] complete the consensus tracking control for such systems by utilizing neural network and fuzzy system.

In recent years, the iterative learning control(ILC) method has been applied to achieve the consensus tracking for multi-agent system [15, 16]. Repeatedly executing the same operation is an important feature of ILC, and when utilizing the ILC method [17, 18], it is generally required that the initial state offset of each iteration is equal to zero, which is impossible in practice due to positioning accuracy and measurement error, etc. In fact, the control system cannot achieve complete error-free tracking throughout the control process, and can only reach the practical complete tracking (PCT) in some specified interval. Obviously, in the PCT case, the system allows the initial state deviation of not zero. When the initial deviation is not zero, the system can achieve the complete tracking in the specified interval by initial shift rectifying [19, 20]. In [21], for a class of uncertain nonlinear systems with unknown virtual control coefficients, a fuzzy adaptive fixed time control scheme based on event triggered is proposed, which can guarantee that all signals of the controlled system are bounded and the tracking error can converge to a small neighborhood of the origin in a fixed time and the convergence time is independent of the initial state. Relying on the structural characteristics of the solution of linear differential equation, [22] designs initial state shifts rectifying schemes for high-order nonlinear system with arbitrary initial state shifts, which can ensure that the systems achieve complete tracking over the specified interval under the premise that the system input gain is unknown and the learning parameters strictly depend on the system state. Some scholars try to use fuzzy control strategy to rectify the initial state shifts [23]. In repeated environment, [24] and [25] propose consensus control algorithms for high-order and second-order nonlinear multi-agent systems (MAS) under alignment condition with both parametric and nonparametric system uncertainties, respectively. A distributed adaptive ILC method by using backstepping to design with the composite energy function (CEF) structure is proposed in [24]. [25] implements the consensus control problems of MAS with unknown control gain and uncertain disturbance by referencing the Nussbaum-type function and the neural networks.

This paper studies the solution of the consensus tracking problem for a class of second-order leader-following nonparametric uncertain multi-agent systems with arbitrary initial state errors. By using initial state rectifying, a consensus learning control algorithm is presented to realize consensus tracking of second-order multi-agent. Finally, a simulation example is given to verify the effectiveness of the proposed algorithm.

The main contributions of this work are as follows:

(1) This paper studies the nonparametric uncertain multi-agent system with arbitrary initial state shifts, which indicate that the systems do not have alignment conditions. Therefore, it is a meaningful work to design a tracking control scheme for an uncertain nonlinear multi-agent system.

(2) The idea of utilizing the attractor function to rectify the initial state shifts is realized for the multi-agent systems. After rectifying the state shifts, the proposed algorithm can enable the system to achieve complete tracking in the specified interval.

## 2. PROBLEM FORMULATION

Consider a class of second-order leader-following multi-agent systems, which contain one leader-agent and $n$ follower-agents. Dynamics of the $j^{\text{th}}$ $(j = 1, 2, \ldots, n)$ agent at the $k^{\text{th}}$ iteration is described as

$$\begin{cases} \dot{x}^1_{j,k}(t) = x^2_{j,k}(t), \\ \dot{x}^2_{j,k}(t) = f_{j,k}(t) + g_{j,k}(t) \cdot u_{j,k}(t) \end{cases} \tag{1}$$

where $t \in [0, T]$; $k = 1, 2, \ldots$ is the index of iteration; $x^i_{j,k} \in \mathbb{R}$ is the $i$th state variable of the agent $j$ at the $k$th iteration; $u_{j,k}(t) \in \mathbb{R}$ is the control input to be designed of the agent $j$ at the $k$th iteration; $f_{j,k}(t) = f_j(x^1_{j,k}(t), x^2_{j,k}(t), t)$, $g_{j,k}(t) = g_j(x^1_{j,k}(t), x^2_{j,k}(t), t)$ are unknown nonlinear functions that meet certain conditions. (The aforementioned variables can be abbreviated as $x^i_{j,k} (i = 1, 2)$, $u_{j,k}$, $f_{j,k}$, $g_{j,k}$).

*Remark* 1. The structure of the system model (1) is a widely discussed form referred to Brunovsky canonical form. For complex dynamic systems, nonparametric uncertainty is the main uncertainty factor. Nonparametric uncertain dynamic models are widely used in unmanned aerial vehicles, aircraft and spacecraft.

The leader-agent satisfies the following requirements

$$\begin{cases} \dot{x}^1_0(t) = x^2_0(t), \\ \dot{x}^2_0(t) = u_0(t) \end{cases} \tag{2}$$

where $u_0(t)$ (abbreviated as $u_0$) is unknown but certain, $x^i_0(t) (i = 1, 2)$ (abbreviated as $x^i_0 (i = 1, 2)$) maintains the same properties during each iteration.

This paper aims to design a novel controller $u_{j,k}$ for multi-agent system with arbitrary initial state $x^i_{j,k}(0)(i = 1,2)$, so that each follower-agent can achieve consensus tracking with the leader-agent in the specified interval $[t_p,T]$ $(0<t_p<T)$ after enough iterations, that is, $\lim\limits_{k\to\infty}(x^1_{j,k}(t)-x^1_0(t))=0$, $\lim\limits_{k\to\infty}(x^2_{j,k}(t)-x^2_0(t))=0$, $j=1,2,\ldots,n$, for $t\in[t_p,T]$.

To facilitate the discussion of the main result, we introduce the tracking error of the agent $j$ at the kth iteration as

$$\begin{cases} e_{j,k}(t)=x^1_{j,k}(t)-x^1_0(t), \\ \dot{e}_{j,k}(t)=x^2_{j,k}(t)-x^2_0(t). \end{cases} \tag{3}$$

And, the relative tracking error between the agent $j$ and the agent $l$ at the kth iteration is defined as

$$e_{jl,k}(t)=x^1_{j,k}(t)-x^1_{l,k}(t)=[x^1_{j,k}(t)-x^1_0(t)]-[x^1_{l,k(t)}-x^1_0(t)]=e_{j,k}(t)-e_{l,k}(t). \tag{4}$$

If two agents are adjacent, $e_{jl,k}$ plays an important role in achieving consensus tracking.

For simplicity, let $\boldsymbol{x}^1_k=(x^1_{1,k},x^1_{2,k},\ldots,x^1_{n,k})^T$, $\boldsymbol{x}^2_k=(x^2_{1,k},x^2_{2,k},\ldots,x^2_{n,k})^T$, $\boldsymbol{u}_k=(u_{1,k},u_{2,k},\ldots,u_{n,k})^T$, $\boldsymbol{x}^1_0=(x^1_0,x^1_0,\ldots,x^1_0)^T$, $\boldsymbol{x}^2_0=(x^2_0,x^2_0,\ldots,x^2_0)^T$, $\boldsymbol{u}_0=(u_0,u_0,\ldots,u_0)^T$, $\boldsymbol{e}_k=(e_{1,k},e_{2,k},\ldots,e_{n,k})^T$, then the overall models can be represented as

$$\begin{cases} \boldsymbol{e}_k=\boldsymbol{x}^1_k-\boldsymbol{x}^1_0, \\ \dot{\boldsymbol{e}}_k=\boldsymbol{x}^2_k-\boldsymbol{x}^2_0, \\ \ddot{\boldsymbol{e}}_k=\boldsymbol{f}_k+\boldsymbol{g}_k\boldsymbol{u}_k-\boldsymbol{u}_0 \end{cases} \tag{5}$$

where $\boldsymbol{f}_k=[f_1(x^1_0+e_{1,k},x^2_0+\dot{e}_{1,k},t),f_2(x^1_0+e_{2,k},x^2_0+\dot{e}_{2,k},t),\ldots,f_n(x^1_0+e_{n,k},x^2_0+\dot{e}_{n,k},t)]^T$, $\boldsymbol{g}_k=\mathrm{diag}\{g_1(x^1_0+e_{1,k},x^2_0+\dot{e}_{1,k},t),g_2(x^1_0+e_{2,k},x^2_0+\dot{e}_{2,k},t),\ldots,g_n(x^1_0+e_{n,k},x^2_0+\dot{e}_{n,k},t)\}$.

The topology connection between the $n$ follower-agents can be conveniently described by a simple and undirected graph $G(V,E,A)$, with $V=\{v_1,v_2,\ldots,v_n\}$ denotes the set of the $n$ agents, $E\subseteq V\times V$ is the set of edges, and $A=(a_{ij})\in\mathbb{R}^{n\times n}$ represents the weighted adjacency matrix. $(v_j,v_l)\in E$ indicates that information can flow between agent $j$ and agent $l$. The element $a_{jl}=a_{lj}>0$ if $(v_j,v_l)\in E$, otherwise $a_{jl}=0$, and $a_{jj}=0$ (because $G$ is simple which means there are no repeated edges and no self loops) . Then the Laplacian matrix of the weighted graph is defined as

$$L=D-A \tag{6}$$

where $D=\mathrm{diag}\{d_1,d_2,\ldots,d_n\}$, and $d_i=\sum_{j=1}^n a_{ij}$, $i=1,2,\ldots,n$. Meanwhile, there are some agents connected to the leader, and the connection weight between agent $i$ and the leader is denoted by $b_i$ ( $b_i>0$, if agent $i$ is connected to the leader, otherwise $b_i=0$), $i=1,2,\ldots,n$. Define $B=\mathrm{diag}\{b_1,b_2,\ldots,b_n\}$. Let $M=L+B$ for simplicity. Assume that the graph $G$ of the multi-agent systems is connected, and at least one agent is connected to the leader, that is, at least one $b_i>0$.

The next lemma [3] is given for $M$ .

LEMMA 1. *Let $M=L+B$, where $L$ is the Laplacian matrix associated with an undirected graph $G$, which contains $n$ vertices, $B=\mathrm{diag}\{b_1,b_2,\ldots,b_n\}(b_j\geq 0,j=1,2,\ldots,n)$. If there is at least one $b_i>0$, then $M$ is positive definite.*

*Assumption* 1. The initial state $x^1_{j,k}(0),x^2_{j,k}(0)$ are arbitrary but not infinite, in other words, the initial error of the system $\boldsymbol{e}_k(0),\dot{\boldsymbol{e}}_k(0)$ can be arbitrary value but bounded.

*Remark* 2. It only needs to satisfy the bounded condition, without the need for the exact boundary value. It is pointed out that most of existing work only considers the agents under the alignment conditions of second- or higher-order nonlinear dynamics with uncertainty. This paper deals with nonparametric uncertain multi-agent systems with arbitrary initial state errors, on which, to the best of authors' knowledge, very little research has been done. Therefore, this paper can be regarded as a useful supplement to current literature.

*Assumption* 2. $f_j(v,w,t)$ and $g_j(v,w,t)$ satisfy the Lipschitz condition with respect to $v$ and $w$, that is, for all $j(=1,2,\ldots,n)$, and $\forall v_1,v_2,w_1,w_2 \in \mathbb{R}$,

$$|f_j(v_1,w,t)-f_j(v_2,w,t)| \leq \alpha_j^1|v_1-v_2|, \tag{7}$$

$$|f_j(v,w_1,t)-f_j(v,w_2,t)| \leq \alpha_j^2|w_1-w_2|, \tag{8}$$

$$|g_j(v_1,w,t)-g_j(v_2,w,t)| \leq \beta_j^1|v_1-v_2|, \tag{9}$$

$$|g_j(v,w_1,t)-g_j(v,w_2,t)| \leq \beta_j^2|w_1-w_2|, \tag{10}$$

where $\alpha_j^1,\alpha_j^2,\beta_j^1,\beta_j^2$ are unknown but fixed nonnegative constants. Therefore,

$$|f_j(v_1,w_1,t)-f_j(v_2,w_2,t)| \leq \alpha_j^1|v_1-v_2|+\alpha_j^2|w_1-w_2|, \tag{11}$$

$$|g_j(v_1,w_1,t)-g_j(v_2,w_2,t)| \leq \beta_j^1|v_1-v_2|+\beta_j^2|w_1-w_2|. \tag{12}$$

*Assumption* 3. $g_{j,k}(t)>0$, and there is a continuous function $g_{Lj,k}(t)=g_{Lj}(x_{j,k}^1,x_{j,k}^2,t)$, satisfies

$$0<g_{Lj,k}(t)\leq g_{j,k}(t), \forall j, \forall k. \tag{13}$$

*Remark* 3. Due to the uncertainty of functions $f_{j,k}(t)$, $g_{j,k}(t)$, the control law and the parameter update laws should strictly depend on the state, and the above two assumptions are the only conditions that will be used for those related to $f_{j,k}(t)$, $g_{j,k}(t)$. The Lipschitz conditions are to limit the speed of functions change, so as to ensure that the functions will not grow indefinitely. The nonnegativity of $g_{Lj,k}(t)$ implies the control direction for all agents is certain.

*Definition* 1 [22]. Consider the following differential equation

$$\dot{\psi}(t)+c\psi(t)+r(t)=0, \tag{14}$$

where $\psi(t),r(t)\in\mathbb{R}$, and $c$ is a constant. If there is $\psi(t)=0, \dot{\psi}(t)=0$ for $t\geq t_1$, then $r(t)$ is called the first-order attractor rectifying function. If there is only $\dot{\psi}(t)=0$, then $r(t)$ is called the zero-order attractor rectifying function. We can choose $r(t)$ as follows

$$r(t)=\phi(t)\exp(-c(t-t_0)+\psi(t_0)). \tag{15}$$

Solve the differential equation Eqn. (14), and obtain

$$\psi(t)=(\psi(t_0)-\exp(\psi(t_0))\int_{t_0}^{t}\phi(\tau)\mathrm{d}\tau)\cdot\exp(-c(t-t_0)). \tag{16}$$

The derivative of $\psi(t)$ is given by

$$\begin{aligned}\dot{\psi}(t)=&-(\psi(t_0)-\exp(\psi(t_0))\int_{t_0}^{t}\phi(\tau)\mathrm{d}\tau)\cdot c\exp(-c(t-t_0))\\&-\exp(\psi(t_0))\cdot\phi(t)\exp(-c(t-t_0)), \quad t\in[t_0,t_1].\end{aligned} \tag{17}$$

When $t\geq t_1$, in order to ensure that $\dot{\psi}(t)=0$ and $\psi(t)=0$, the function $\phi(t)$ is required to satisfy the following conditions

$$\begin{cases}\phi(t)=0, \quad t<t_0,t\geq t_1,\\ \phi(t_0)=-(\dot{\psi}(t_0)+c\psi(t_0))\exp(-\psi(t_0)),\\ \int_{t_0}^{t_1}\phi(\tau)\mathrm{d}\tau=\psi(t_0)\exp(-\psi(t_0)),\\ \lim_{t\to t_1^-}\phi(t)=0.\end{cases} \tag{18}$$

*Remark* 4. The second equation of Eqn. (18) is to ensure that when $t = t_0$, the left and right sides of Eqn. (16) are equal to $\psi(t_0)$, and the ones of Eqn. (17) are equal to $\dot{\psi}(t_0)$, so as to be consistent with the initial conditions. The third equation of Eqn. (18) is to ensure that $\psi(t) = 0$ and $\dot{\psi}(t) = 0$, when $t = t_1$, in order to make $r(t)$ rectify the shifts of $\psi(t)$ and $\dot{\psi}(t)$ at the same time.

*Remark* 5. In particular, when $c = 0$, Eqn. (14) can be simplified to $\dot{\psi}(t) = -r(t)$. In this case, it is easy to find a zero-order attractor rectifying function such that $\dot{\psi}(t) = 0$ ( Just let $\dot{\psi}(t) = 0$, and do not care about the value of $\psi(t)$ , that is, $\psi(t)$ may or may not be equal to zero). For instance, we can choose $r(t)$ as $r(t) = -\dot{\psi}(t_0)$ for $t = t_0$, $r(t) = 0$ for $t \geq t_1$, $r(t)$ transits smoothly from $-\dot{\psi}(t_0)$ to 0 for $t \in [t_0, t_1]$.

*Remark* 6. According to the above definition and solution procedure, the continuity and differentiability of $\psi(t)$ are strictly dependent on $\phi(t)$, so $\phi(t)$ should not only be continuous but also smooth.

The next definition and lemma are given for convenience of subsequent convergence analysis.

*Definition* 2 [22]. The saturation function $\mathrm{sat}(p, \bar{p}^1, \bar{p}^2)$ is defined by

$$\mathrm{sat}(p, \bar{p}^1, \bar{p}^2) = \begin{cases} \bar{p}^1, & p > \bar{p}^1, \\ p, & \bar{p}^2 \leq p \leq \bar{p}^1, \\ \bar{p}^2, & p < \bar{p}^2, \end{cases} \tag{19}$$

where $\bar{p}^1$ and $\bar{p}^2$ are corresponding upper and lower bounds. For $\boldsymbol{p} = (p_1, p_2, \ldots, p_n)^T \in \mathbb{R}^n$, $\mathrm{sat}(\boldsymbol{p}, \bar{p}^1, \bar{p}^2)$ represents the above operation is performed on each component $p_i (i = 1, 2, \ldots)$ of $\boldsymbol{p}$, that is,

$$\mathrm{sat}(\boldsymbol{p}, \bar{p}^1, \bar{p}^2) = (\mathrm{sat}(p_1, \bar{p}^1, \bar{p}^2), \mathrm{sat}(p_2, \bar{p}^1, \bar{p}^2), \cdots, \mathrm{sat}(p_n, \bar{p}^1, \bar{p}^2))^T \tag{20}$$

LEMMA 2 [22]. *Let a be a real number, and $a = sat(a, \bar{p}^1, \bar{p}^2)$, then*

$$(a - sat(p, \bar{p}^1, \bar{p}^2))(p - sat(p, \bar{p}^1, \bar{p}^2)) \leq 0. \tag{21}$$

*Notation* 1. The notations used throughout the paper are defined as follows. The symbol $|\cdot|$ is defined by $|\boldsymbol{a}| = (|a_1|, |a_2|, \ldots, |a_n|)^T$ for any $\boldsymbol{a} = (a_1, a_2, \ldots, a_n)^T$.

## 3. CONTROLLER DESIGN

At the $k$th iteration, the consensus error among the agent $j$ and other agents is defined as follows

$$\xi_{j,k}(t) = \sum_{i=1}^{n} a_{ji} \cdot e_{ji,k}(t) + b_j \cdot e_{j,k}(t). \tag{22}$$

Let $\boldsymbol{\xi}_k(t) = [\xi_{1,k}(t), \xi_{2,k}(t), \ldots, \xi_{n,k}(t)]^T$(abbreviated as $\boldsymbol{\xi}_k$), then $\boldsymbol{\xi}_k = (L + B)\boldsymbol{e}_k = M\boldsymbol{e}_k$. It follows that the consensus error satisfies

$$\begin{cases} \dot{\boldsymbol{\xi}}_k = M\dot{\boldsymbol{e}}_k = M(\boldsymbol{x}_k^2 - \boldsymbol{x}_0^2), \\ \ddot{\boldsymbol{\xi}}_k = M\ddot{\boldsymbol{e}}_k = M(\boldsymbol{f}_k + \boldsymbol{g}_k \boldsymbol{u}_k - \boldsymbol{u}_0). \end{cases} \tag{23}$$

*Remark* 7. By Assumption 1, the initial errors $\boldsymbol{e}_k(0), \dot{\boldsymbol{e}}_k(0)$ of the system are arbitrary but bounded, hence the initial values of the consensus errors $\boldsymbol{\xi}_k(0)$, $\dot{\boldsymbol{\xi}}_k(0)$ are bounded.

Define the sliding mode error function [23] as follows

$$\begin{cases} \sigma_{j,k}(t) = \dot{\xi}_{j,k}(t) + c\xi_{j,k}(t) + r_{j,k}(t), \\ \sigma_{j,k}(0) = 0, \end{cases} \tag{24}$$

for all $j = 1,2,\dots,n$, $t \in [0,T]$, and

$$r_{j,k}(t) = \phi_{j,k}(t)\exp(-ct + \xi_{j,k}(0)), \tag{25}$$

where $\phi_{j,k}(t)$ is given by

$$\phi_{j,k}(t) = h_{j,k}t^2 + p_{j,k}t + q_{j,k}. \tag{26}$$

The parameters of $\phi_{j,k}(t)$ need to satisfy Eqn. (18), hence,

$$\frac{1}{3}h_{j,k}t_p^3 + \frac{1}{2}p_{j,k}t_p^2 + q_{j,k}t_p = \xi_{j,k}(0)\exp(-\xi_{j,k}(0)), \tag{27}$$

$$q_{j,k} = -(\dot{\xi}_{j,k}(0) + c\xi_{j,k}(0))\exp(-\xi_{j,k}(0)), \tag{28}$$

$$h_{j,k}t_p^2 + p_{j,k}t_p + q_{j,k} = 0. \tag{29}$$

Solve the above equations to obtain

$$\begin{pmatrix} h_{j,k} \\ p_{j,k} \\ q_{j,k} \end{pmatrix} = \begin{pmatrix} \frac{1}{3}t_p^3 & \frac{1}{2}t_p^2 & t_p \\ t_p^2 & t_p & 1 \\ 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} \varepsilon_{j,k}^1 \\ 0 \\ \varepsilon_{j,k}^2 \end{pmatrix}, \tag{30}$$

where $\varepsilon_{j,k}^1 = \xi_{j,k}(0)\exp(-\xi_{j,k}(0))$, $\varepsilon_{j,k}^2 = -(\dot{\xi}_{j,k}(0) + c\xi_{j,k}(0))\exp(-\xi_{j,k}(0))$. Let

$$\boldsymbol{\sigma}_k(t) = [\sigma_{1,k}(t), \sigma_{2,k}(t), \dots, \sigma_{n,k}(t)]^T, \tag{31}$$

$$\boldsymbol{r}_k(t) = [r_{1,k}(t), r_{2,k}(t), \dots, r_{n,k}(t)]^T, \tag{32}$$

which are abbreviated as $\boldsymbol{\sigma}_k$,$\boldsymbol{r}_k$, then

$$\begin{cases} \boldsymbol{\sigma}_k = \dot{\boldsymbol{\xi}}_k + c\boldsymbol{\xi}_k + \boldsymbol{r}_k, \\ \boldsymbol{\sigma}_k|_{t=0} = \boldsymbol{0}. \end{cases} \tag{33}$$

Therefore, the derivative of $\boldsymbol{\sigma}_k$ is

$$\dot{\boldsymbol{\sigma}}_k = \ddot{\boldsymbol{\xi}}_k + c\dot{\boldsymbol{\xi}}_k + \dot{\boldsymbol{r}}_k = M(\boldsymbol{f}_k + \boldsymbol{g}_k\boldsymbol{u}_k - \boldsymbol{u}_0 + c\dot{\boldsymbol{e}}_k + M^{-1}\dot{\boldsymbol{r}}_k). \tag{34}$$

*Remark* 8. In order to achieve the control objectives, we introduce the sliding mode error function $\boldsymbol{\sigma}_k$. According to the definition of the attractor rectifying function and the characteristics of its parameters, when $\boldsymbol{\sigma}_k$ converges to zero, it can ensure that the consensus error converges to zero in the specified interval $[t_p, T]$ by utilizing the initial state shifts rectifying.

Define a continuously differentiable, positive definite functional $V_k(t)$ as

$$V_k(t) = \frac{1}{2}\boldsymbol{\sigma}_k^T(t)M^{-1}\boldsymbol{\sigma}_k(t). \tag{35}$$

*Remark* 9. Introducing the error rectifying function $r_k(t)$ into $\sigma_k(t)$ has two main purposes. The first is to make $V_k(t)$ satisfy the condition in Babalat Lemma, i.e. $V_k(0) = 0$, and then prove $\lim_{k\to\infty} V_k(t) = 0$ to ensure the system convergence. Secondly, the introduction of the function $r_k(t)$ allows the system constantly rectify state shifts in the control process, thus facilitating rapid convergence of the system.

Taking the derivative of $V_k$, we obtain

$$\begin{aligned}\dot{V}_k =&\boldsymbol{\sigma}_k^T[\boldsymbol{f}_k+\boldsymbol{g}_k\boldsymbol{u}_k-\boldsymbol{u}_0+c\dot{\boldsymbol{e}}_k+M^{-1}\dot{\boldsymbol{r}}_k]\\ =&\boldsymbol{\sigma}_k^T[(\boldsymbol{f}_k-\boldsymbol{f}_0)+(\boldsymbol{g}_k\boldsymbol{u}_k-\boldsymbol{g}_k\boldsymbol{u}_{0k})+(\boldsymbol{g}_0\boldsymbol{u}_{0k}-\boldsymbol{g}_0\boldsymbol{u}_0)+(\boldsymbol{g}_k\boldsymbol{u}_{0k}-\boldsymbol{g}_0\boldsymbol{u}_{0k})\\ &+\boldsymbol{f}_0+\boldsymbol{g}_0\boldsymbol{u}_0-\boldsymbol{u}_0+c\dot{\boldsymbol{e}}_k+M^{-1}\dot{\boldsymbol{r}}_k],\end{aligned}\tag{36}$$

where $\boldsymbol{u}_{0k}$ is the estimate of $\boldsymbol{u}_0$, and

$$\boldsymbol{f}_0=[f_1(x_0^1,x_0^2,t),f_2(x_0^1,x_0^2,t),\dots,f_n(x_0^1,x_0^2,t)]^T,\tag{37}$$

$$\boldsymbol{g}_0=diag\{g_1(x_0^1,x_0^2,t),g_2(x_0^1,x_0^2,t),\dots,g_n(x_0^1,x_0^2,t)\}.\tag{38}$$

By Assumption 2, we have

$$\boldsymbol{\sigma}_k^T(\boldsymbol{f}_k-\boldsymbol{f}_0)\le|\boldsymbol{\sigma}_k^T|(\alpha_{max}^1|\boldsymbol{e}_k|+\alpha_{max}^2|\dot{\boldsymbol{e}}_k|),\tag{39}$$

and

$$\boldsymbol{\sigma}_k^T(\boldsymbol{g}_k\boldsymbol{u}_{0k}-\boldsymbol{g}_0\boldsymbol{u}_{0k})\le|\boldsymbol{\sigma}_k^T|(\hat{\beta}_{max}^1|\boldsymbol{e}_k|+\hat{\beta}_{max}^2|\dot{\boldsymbol{e}}_k|),\tag{40}$$

where

$$\begin{aligned}&\alpha_{max}^1=\max\{\alpha_1^1,\alpha_2^1,\dots,\alpha_n^1\}, &&\alpha_{max}^2=\max\{\alpha_1^2,\alpha_2^2,\dots,\alpha_n^2\},\\ &\hat{\beta}_{max}^1=\max\{\beta_1^1\bar{u}_{0,1},\beta_2^1\bar{u}_{0,2},\dots,\beta_n^1\bar{u}_{0,n}\}, &&\hat{\beta}_{max}^2=\max\{\beta_1^2\bar{u}_{0,1},\beta_2^2\bar{u}_{0,2},\dots,\beta_n^2\bar{u}_{0,n}\},\end{aligned}$$

with $\bar{u}_{0,1},\bar{u}_{0,2},\dots,\bar{u}_{0,n}$ is the corresponding saturation limit ( see Eqn. (43) ). The rectifying function $r_k(t)$ defined as Eqn. (25) is continuously differentiable on $[0,T]$, hence each component of the vector $|M^{-1}\dot{\boldsymbol{r}}_k|$ has an upper bound $\omega_j\ge 0(j=1,2,\dots,n)$. Define $\boldsymbol{\omega}=(\omega_1,\omega_2,\dots,\omega_n)^T$. By utilizing Eqns. (39) and (40), we have

$$\dot{V}_k\le|\boldsymbol{\sigma}_k^T|(\boldsymbol{e}_k^*+\boldsymbol{\omega})+\boldsymbol{\sigma}_k^T[(\boldsymbol{g}_k\boldsymbol{u}_k-\boldsymbol{g}_k\boldsymbol{u}_{0k})+(\boldsymbol{g}_0\boldsymbol{u}_{0k}-\boldsymbol{g}_0\boldsymbol{u}_0)+\boldsymbol{\eta}],\tag{41}$$

where

$$\begin{aligned}\boldsymbol{\eta}=&\boldsymbol{f}_0+\boldsymbol{g}_0\boldsymbol{u}_0-\boldsymbol{u}_0,\\ \boldsymbol{e}_k^*=&[e_{1,k}^*,e_{2,k}^*,\dots,e_{n,k}^*]^T=(\alpha_{max}^1+\hat{\beta}_{max}^1)|\boldsymbol{e}_k|+(\alpha_{max}^2+\hat{\beta}_{max}^2+|c|)|\dot{\boldsymbol{e}}_k|.\end{aligned}$$

For the purposes of achieving consensus tracking, we now construct the iterative learning control algorithm for the multi-agent system described by Eqn. (1)-(2) as follows

$$\boldsymbol{u}_k=\boldsymbol{u}_{0k}-\boldsymbol{u}_{ek}-\boldsymbol{u}_{\omega k}-\boldsymbol{u}_{\eta k}-b\boldsymbol{\sigma}_k,\tag{42}$$

$$\boldsymbol{u}_{0k}=\mathrm{sat}_{\bar{u}_0}(\hat{\boldsymbol{u}}_{0k}),\hat{\boldsymbol{u}}_{0k}=\mathrm{sat}_{\bar{u}_0}(\hat{\boldsymbol{u}}_{0k-1})-\gamma_1\boldsymbol{\sigma}_k,\tag{43}$$

$$\boldsymbol{u}_{ek}=(g_{L1,k}^{-1}e_{1,k}^*\arctan(k^2\sigma_{1,k}e_{1,k}^*),g_{L2,k}^{-1}e_{2,k}^*\arctan(k^2\sigma_{2,k}e_{2,k}^*),\cdots,g_{Ln,k}^{-1}e_{n,k}^*\arctan(k^2\sigma_{n,k}e_{n,k}^*))^T\tag{44}$$

$$\boldsymbol{\omega}_k=\mathrm{sat}_{\bar{\omega}}(\hat{\boldsymbol{\omega}}_k),\hat{\boldsymbol{\omega}}_k=\mathrm{sat}_{\bar{\omega}}(\hat{\boldsymbol{\omega}}_{k-1})+\gamma_2|\boldsymbol{\sigma}_k|,\tag{45}$$

$$\boldsymbol{u}_{\omega k}=(g_{L1,k}^{-1}\omega_{1,k}\arctan(k^2\sigma_{1,k}\omega_{1,k}),g_{L2,k}^{-1}\omega_{2,k}\arctan(k^2\sigma_{2,k}\omega_{2,k}),\cdots,g_{Ln,k}^{-1}\omega_{n,k}\arctan(k^2\sigma_{n,k}\omega_{n,k}))^T\tag{46}$$

$$\boldsymbol{\eta}_k = \mathrm{sat}_{\bar{\eta}}(\hat{\boldsymbol{\eta}}_k), \hat{\boldsymbol{\eta}}_k = \mathrm{sat}_{\bar{\eta}}(\hat{\boldsymbol{\eta}}_{k-1}) + \gamma_3 \boldsymbol{\sigma}_k, \tag{47}$$

$$\boldsymbol{u}_{\eta k} = (g_{L1,k}^{-1}\eta_{1,k}\arctan(k^2\sigma_{1,k}\eta_{1,k}), g_{L2,k}^{-1}\eta_{2,k}\arctan(k^2\sigma_{2,k}\eta_{2,k}), \cdots, g_{Ln,k}^{-1}\eta_{n,k}\arctan(k^2\sigma_{n,k}\eta_{n,k}))^T \tag{48}$$

where $b$ is a positive constant, $\gamma_1 > 0$, $\gamma_2 > 0$ and $\gamma_3 > 0$ are the ILC gains. $\boldsymbol{u}_{0k} = (u_{0k,1}, u_{0k,2}, \ldots, u_{0k,n})^T$, $\boldsymbol{\omega}_k = (\omega_{1,k}, \omega_{2,k}, \ldots, \omega_{n,k})^T$, $\boldsymbol{\eta}_k = (\eta_{1,k}, \eta_{2,k}, \ldots, \eta_{n,k})^T$ are the ILC updating laws for $\boldsymbol{u}_0$, $\boldsymbol{\omega}$, $\boldsymbol{\eta}$, respectively. $\bar{u}_0$, $\bar{\omega}$ and $\bar{\eta}$ are the saturation bounds of $\boldsymbol{u}_{0k}$, $\boldsymbol{\omega}_k$ and $\boldsymbol{\eta}_k$, which are defined such that $|u_{0k,j}| \le \bar{u}_0$, $|\omega_{j,k}| \le \bar{\omega}$, $|\eta_{j,k}| \le \bar{\eta}$, $j = 1, 2, \ldots, n$. $\hat{\boldsymbol{u}}_{0k} = \boldsymbol{0}$, $\hat{\boldsymbol{\omega}}_k = \boldsymbol{0}$, $\hat{\boldsymbol{\eta}}_k = \boldsymbol{0}$, when $k = -1$.

*Remark* 10. Due to the topology connection between the $n$ follower-agents and the definition of the graph Laplacian matrix, the proposed iterative learning protocol for agent $j$ only uses the information of itself and its neighboring agents, which is very different from the control law proposed in [22].

## 4. CONVERGENCE ANALYSIS

For the control law designed in the previous section, there is the following convergence theorem. For simplicity of presentation, variables are replaced with corresponding abbreviations during the proof process.

THEOREM 1. *If the multi-agent system described by Eqns.* (1)–(2) *satisfies Assumptions1–3, then the designed controller Eqns.* (42)–(48) *can make the sliding mode error function $\sigma_k$ and energy function $V_k$ converge to zero uniformly on* $[0, T]$*, as $k \to \infty$.*

*Proof.* Consider the following Lyapunov-like function

$$L_k = V_k + \frac{1}{2\gamma_1}\int_0^t \tilde{\boldsymbol{u}}_{0k}^T g_0 \tilde{\boldsymbol{u}}_{0k} \mathrm{d}\tau + \frac{1}{2\gamma_2}\int_0^t \tilde{\boldsymbol{\omega}}_k^T \tilde{\boldsymbol{\omega}}_k \mathrm{d}\tau + \frac{1}{2\gamma_3}\int_0^t \tilde{\boldsymbol{\eta}}_k^T \tilde{\boldsymbol{\eta}}_k \mathrm{d}\tau, \tag{49}$$

where $\tilde{\boldsymbol{u}}_{0k} = \boldsymbol{u}_0 - \boldsymbol{u}_{0k}$, $\tilde{\boldsymbol{\omega}}_k = \boldsymbol{\omega} - \boldsymbol{\omega}_k$, $\tilde{\boldsymbol{\eta}}_k = \boldsymbol{\eta} - \boldsymbol{\eta}_k$.

Define $\Delta L_k = L_k - L_{k-1}$, and obtain

$$\begin{aligned}
\Delta L_k = & \int_0^t \dot{V}_k \mathrm{d}\tau + \frac{1}{2\gamma_1}\int_0^t (\tilde{\boldsymbol{u}}_{0k}^T \boldsymbol{g}_0 \tilde{\boldsymbol{u}}_{0k} - \tilde{\boldsymbol{u}}_{0k-1}^T \boldsymbol{g}_0 \tilde{\boldsymbol{u}}_{0k-1})\mathrm{d}\tau \\
& + \frac{1}{2\gamma_2}\int_0^t (\tilde{\boldsymbol{\omega}}_k^T \tilde{\boldsymbol{\omega}}_k - \tilde{\boldsymbol{\omega}}_{k-1}^T \tilde{\boldsymbol{\omega}}_{k-1})\mathrm{d}\tau + \frac{1}{2\gamma_3}\int_0^t (\tilde{\boldsymbol{\eta}}_k^T \tilde{\boldsymbol{\eta}}_k - \tilde{\boldsymbol{\eta}}_{k-1}^T \tilde{\boldsymbol{\eta}}_{k-1})\mathrm{d}\tau - V_{k-1} \\
\le & \int_0^t |\boldsymbol{\sigma}_k^T|(\boldsymbol{e}_k^* + \boldsymbol{\omega})\mathrm{d}\tau + \int_0^t \boldsymbol{\sigma}_k^T [\boldsymbol{g}_k(-\boldsymbol{u}_{ek} - \boldsymbol{u}_{\omega k} - \boldsymbol{u}_{\eta k} - b\boldsymbol{\sigma}_k) + \boldsymbol{g}_0(\boldsymbol{u}_{0k} - \boldsymbol{u}_0) + \boldsymbol{\eta}]\mathrm{d}\tau \\
& + \frac{1}{2\gamma_1}\int_0^t (\tilde{\boldsymbol{u}}_{0k}^T \boldsymbol{g}_0 \tilde{\boldsymbol{u}}_{0k} - \tilde{\boldsymbol{u}}_{0k-1}^T \boldsymbol{g}_0 \tilde{\boldsymbol{u}}_{0k-1})\mathrm{d}\tau \\
& + \frac{1}{2\gamma_2}\int_0^t (\tilde{\boldsymbol{\omega}}_k^T \tilde{\boldsymbol{\omega}}_k - \tilde{\boldsymbol{\omega}}_{k-1}^T \tilde{\boldsymbol{\omega}}_{k-1})\mathrm{d}\tau + \frac{1}{2\gamma_3}\int_0^t (\tilde{\boldsymbol{\eta}}_k^T \tilde{\boldsymbol{\eta}}_k - \tilde{\boldsymbol{\eta}}_{k-1}^T \tilde{\boldsymbol{\eta}}_{k-1})\mathrm{d}\tau - V_{k-1}.
\end{aligned} \tag{50}$$

By Eqns. (44), (46) and (48), we have

$$\begin{aligned}
|\boldsymbol{\sigma}_k^T|\boldsymbol{e}_k^* - \boldsymbol{\sigma}_k^T \boldsymbol{g}_k \boldsymbol{u}_{ek} &\le \sum_{i=1}^n |\sigma_{i,k} e_{i,k}^*| - \sum_{i=1}^n \sigma_{i,k} e_{i,k}^* \arctan(k^2 \sigma_{i,k} e_{i,k}^*), \\
|\boldsymbol{\sigma}_k^T|\boldsymbol{\omega}_k - \boldsymbol{\sigma}_k^T \boldsymbol{g}_k \boldsymbol{u}_{\omega k} &\le \sum_{i=1}^n |\sigma_{i,k} \omega_{i,k}| - \sum_{i=1}^n \sigma_{i,k} \omega_{i,k} \arctan(k^2 \sigma_{i,k} \omega_{i,k}), \\
\boldsymbol{\sigma}_k^T \boldsymbol{\eta}_k - \boldsymbol{\sigma}_k^T \boldsymbol{g}_k \boldsymbol{u}_{\eta k} &\le \sum_{i=1}^n |\sigma_{i,k} \eta_{i,k}| - \sum_{i=1}^n \sigma_{i,k} \eta_{i,k} \arctan(k^2 \sigma_{i,k} \eta_{i,k}).
\end{aligned}$$

Notice that the function arctan$(z)$ satisfies the inequality $\mid z \mid -z\arctan(\theta z) \leq \dfrac{0.2759}{\theta}$ for any $\theta > 0$ [22]. Therefore

$$|\boldsymbol{\sigma}_k^T|\boldsymbol{e}_k^* - \boldsymbol{\sigma}_k^T \boldsymbol{g}_k \boldsymbol{u}_{ek} \leq \frac{0.2759n}{k^2}, \tag{51}$$

$$|\boldsymbol{\sigma}_k^T|\boldsymbol{\omega}_k - \boldsymbol{\sigma}_k^T \boldsymbol{g}_k \boldsymbol{u}_{\omega k} \leq \frac{0.2759n}{k^2}, \tag{52}$$

$$\boldsymbol{\sigma}_k^T \boldsymbol{\eta}_k - \boldsymbol{\sigma}_k^T \boldsymbol{g}_k \boldsymbol{u}_{\eta k} \leq \frac{0.2759n}{k^2}. \tag{53}$$

Substituting Eqns. (51), (52) and (53) into Eqn. (50), yields

$$\begin{aligned}\Delta L_k \leq & \int_0^t -b\boldsymbol{\sigma}_k^T \boldsymbol{g}_k \boldsymbol{\sigma}_k \mathrm{d}\tau + \int_0^t \frac{0.8277n}{k^2}\mathrm{d}\tau + \int_0^t \boldsymbol{\sigma}_k^T \boldsymbol{g}_0(\boldsymbol{u}_{0k} - \boldsymbol{u}_0)\mathrm{d}\tau \\ & + \int_0^t |\boldsymbol{\sigma}_k^T|\tilde{\boldsymbol{\omega}}_k \mathrm{d}\tau + \int_0^t \boldsymbol{\sigma}_k^T \tilde{\boldsymbol{\eta}}_k \mathrm{d}\tau + \frac{1}{2\gamma_1}\int_0^t (\tilde{\boldsymbol{u}}_{0k}^T \boldsymbol{g}_0 \tilde{\boldsymbol{u}}_{0k} - \tilde{\boldsymbol{u}}_{0k-1}^T \boldsymbol{g}_0 \tilde{\boldsymbol{u}}_{0k-1})\mathrm{d}\tau \\ & + \frac{1}{2\gamma_2}\int_0^t (\tilde{\boldsymbol{\omega}}_k^T \tilde{\boldsymbol{\omega}}_k - \tilde{\boldsymbol{\eta}}_{k-1}^T \tilde{\boldsymbol{\eta}}_{k-1})\mathrm{d}\tau + \frac{1}{2\gamma_3}\int_0^t (\tilde{\boldsymbol{\eta}}_k^T \tilde{\boldsymbol{\eta}}_k - \tilde{\boldsymbol{\eta}}_{k-1}^T \tilde{\boldsymbol{\eta}}_{k-1})\mathrm{d}\tau - V_{k-1}.\end{aligned} \tag{54}$$

Considering $\tilde{\boldsymbol{u}}_{0k}^T \boldsymbol{g}_0 \tilde{\boldsymbol{u}}_{0k} - \tilde{\boldsymbol{u}}_{0k-1}^T \boldsymbol{g}_0 \tilde{\boldsymbol{u}}_{0k-1}$, we can obtain

$$\begin{aligned}&\tilde{\boldsymbol{u}}_{0k}^T \boldsymbol{g}_0 \tilde{\boldsymbol{u}}_{0k} - \tilde{\boldsymbol{u}}_{0k-1}^T \boldsymbol{g}_0 \tilde{\boldsymbol{u}}_{0k-1} \\ &\quad = -2\tilde{\boldsymbol{u}}_{0k}^T \boldsymbol{g}_0(\boldsymbol{u}_{0k} - \boldsymbol{u}_{0k-1}) - (\tilde{\boldsymbol{u}}_{0k} - \tilde{\boldsymbol{u}}_{0k-1})^T \boldsymbol{g}_0(\tilde{\boldsymbol{u}}_{0k} - \tilde{\boldsymbol{u}}_{0k-1}) \\ &\quad = -2\tilde{\boldsymbol{u}}_{0k}^T \boldsymbol{g}_0(\hat{\boldsymbol{u}}_{0k} - \boldsymbol{u}_{0k-1}) + 2\tilde{\boldsymbol{u}}_{0k}^T \boldsymbol{g}_0(\hat{\boldsymbol{u}}_{0k} - \boldsymbol{u}_{0k}) - (\tilde{\boldsymbol{u}}_{0k} - \tilde{\boldsymbol{u}}_{0k-1})^T \boldsymbol{g}_0(\tilde{\boldsymbol{u}}_{0k} - \tilde{\boldsymbol{u}}_{0k-1}).\end{aligned} \tag{55}$$

According to Lemma 2 and Eqn. (43), we have

$$\begin{aligned}\tilde{\boldsymbol{u}}_{0k}^T \boldsymbol{g}_0(\hat{\boldsymbol{u}}_{0k} - \boldsymbol{u}_{0k}) &= (\boldsymbol{u}_0 - \boldsymbol{u}_{0k})^T \boldsymbol{g}_0(\hat{\boldsymbol{u}}_{0k} - \boldsymbol{u}_{0k}) \\ &= (\boldsymbol{u}_0 - \mathrm{sat}_{\bar{u}_0}(\hat{\boldsymbol{u}}_{0k}))^T \boldsymbol{g}_0(\hat{\boldsymbol{u}}_{0k} - \mathrm{sat}_{\bar{u}_0}(\hat{\boldsymbol{u}}_{0k})) \leq 0.\end{aligned} \tag{56}$$

Using Eqns. (43) and (56), Eqn. (55) can be reduced to

$$\begin{aligned}\tilde{\boldsymbol{u}}_{0k}^T \boldsymbol{g}_0 \tilde{\boldsymbol{u}}_{0k} - \tilde{\boldsymbol{u}}_{0k-1}^T \boldsymbol{g}_0 \tilde{\boldsymbol{u}}_{0k-1} &\leq -2\tilde{\boldsymbol{u}}_{0k}^T \boldsymbol{g}_0(\hat{\boldsymbol{u}}_{0k} - \boldsymbol{u}_{0k-1}) \\ &= -2\tilde{\boldsymbol{u}}_{0k}^T \boldsymbol{g}_0(-\gamma_1 \boldsymbol{\sigma}_k) = -2\gamma_1 \boldsymbol{\sigma}_k^T \boldsymbol{g}_0(\boldsymbol{u}_{0k} - \boldsymbol{u}_0).\end{aligned} \tag{57}$$

Similarly, we derive

$$\begin{aligned}\tilde{\boldsymbol{\omega}}_k^T \tilde{\boldsymbol{\omega}}_k - \tilde{\boldsymbol{\omega}}_{k-1}^T \tilde{\boldsymbol{\omega}}_{k-1} &\leq -2\tilde{\boldsymbol{\omega}}_k^T (\hat{\boldsymbol{\omega}}_k - \boldsymbol{\omega}_{k-1}) \\ &= -2\gamma_2 |\boldsymbol{\sigma}_k^T| \tilde{\boldsymbol{\omega}}_k,\end{aligned} \tag{58}$$

$$\begin{aligned}\tilde{\boldsymbol{\eta}}_k^T \tilde{\boldsymbol{\eta}}_k - \tilde{\boldsymbol{\eta}}_{k-1}^T \tilde{\boldsymbol{\eta}}_{k-1} &\leq -2\tilde{\boldsymbol{\eta}}_k^T (\hat{\boldsymbol{\eta}}_k - \boldsymbol{\eta}_{k-1}) \\ &= -2\gamma_3 \boldsymbol{\sigma}_k^T \tilde{\boldsymbol{\eta}}_k.\end{aligned} \tag{59}$$

Substituting the above formulas Eqns. (57)-(59) into(54), it is true that

$$\Delta L_k \leq \frac{0.8277nt}{k^2} - V_{k-1}.$$

By $\sum_{i=1}^{+\infty}\dfrac{1}{i^2}=\dfrac{\pi^2}{6}$, the summation of $\Delta L_k$ satisfies

$$\sum_{i=1}^{k}\Delta L_i = L_k - L_0 \le \frac{0.2759\pi^2}{2}nt - \sum_{i=1}^{k}V_{i-1},$$

furthermore, it can be written as

$$0 \le L_k \le L_0 + \frac{0.2759\pi^2}{2}nt - \sum_{i=1}^{k}V_{i-1},$$

where $L_0$ is nonnegative bounded. Let $S_k=\sum_{i=0}^{k}V_i$. It is obvious that $\{S_k\}$ is a nonnegative monotone increasing sequence and satisfies $S_{k-1}\le L_0+\dfrac{0.2759\pi^2}{3}nt$. According to the Monotone Convergence Theorem, $\lim\limits_{k\to\infty}S_k$ exists, and $\lim\limits_{k\to\infty}(S_k-S_{k-1})=0$, i.e. $\lim\limits_{k\to\infty}V_k=0$, hence $\lim\limits_{k\to\infty}\boldsymbol{\sigma}_k=\mathbf{0}$. This completes the proof. □

*Remark* 11. For agent $j(j=1,2,\ldots,n)$, when $\lim\limits_{k\to\infty}\sigma_{j,k}=0$, from Eqns. (16) and (17), it can be seen that

$$\xi_{j,k}(t)=(\xi_{j,k}(0)-\exp(\xi_{j,k}(0)))\int_0^t\phi_{j,k}(\tau)\mathrm{d}\tau\cdot\exp(-ct), \tag{60}$$

$$\dot{\xi}_{j,k}(t)=-(\xi_{j,k}(0)-\exp(\xi_{j,k}(0)))\int_0^t\phi_{j,k}(\tau)\mathrm{d}\tau\cdot c\exp(-ct)-\exp(\xi_{j,k}(0))\phi_{j,k}(t)\cdot\exp(-ct). \tag{61}$$

Based on the definition of $\phi_{j,k}(t)$, the designed controller Eqns. (42)-(48) can guarantee that

$$\lim_{k\to\infty}\boldsymbol{\xi}_k(t)=0,\quad \lim_{k\to\infty}\dot{\boldsymbol{\xi}}_k(t)=0,\quad t\in[t_p,T]. \tag{62}$$

Utilizing $\boldsymbol{e}_k=M^{-1}\boldsymbol{\xi}_k$, we can obtain that $\lim\limits_{k\to\infty}\boldsymbol{e}_k(t)=0$, $\lim\limits_{k\to\infty}\dot{\boldsymbol{e}}_k(t)=0$, $t\in[t_p,T]$ .

*Remark* 12. According to Eqns. (60) and (61), if the value of $c$ is increased, not only can the convergence speed be accelerated, but the tracking error can also be reduced. From the definition of $\phi_k(t)$, it can be seen that when $\xi_k(t)$ and $\dot{\xi}_k(t)$ tend to zero, the rectification effect is not obvious.

## 5. SIMULATION RESULTS

In this section, we will verify the effectiveness of the proposed control strategy through a simulation example. Consider a multi-agent system running repeatedly on the interval $[0,T]$, which contains a leader-agent and 4 follower-agents. The topology of signal transmission between the agents is shown in Fig. 1.

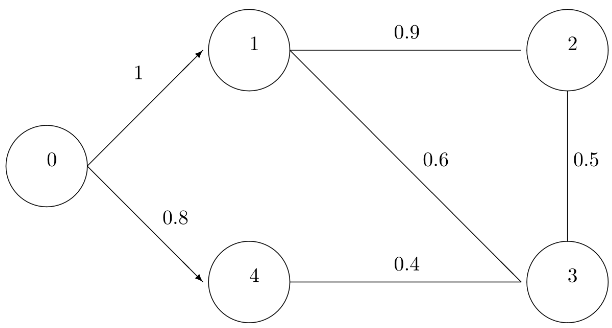


Fig. 1 – Topology of multi-agent system.

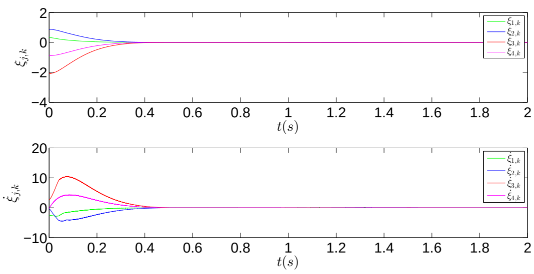


Fig. 2 – Consensus errors $\xi_{j,k}(t)$ and $\dot{\xi}_{j,k}(t)$, $k=30$.

The dynamics of the four follower-agents are given in structure of Eqn. (1) with

$$\begin{cases} \dot{x}^1_{i,k}(t) = x^2_{i,k}(t), i = 1,2,3,4 \\ \dot{x}^2_{1,k}(t) = -[1+\cos(x^2_{1,k}(t))]\cos(x^1_{1,k}(t)) + (2+t^2+\arctan(0.5x^1_{1,k}(t)))\cdot u_{1,k}(t), \\ \dot{x}^2_{2,k}(t) = -[1+\cos(x^1_{2,k}(t))]\sin(x^2_{2,k}(t)) + [1+0.1\cdot\sin(x^1_{2,k}(t))]\cdot u_{2,k}(t), \\ \dot{x}^2_{3,k}(t) = \dfrac{0.2x^2_{3,k}(t)}{1+(x^2_{3,k}(t))^2} + \sin t + [0.1\sin(x^1_{3,k}(t)) + 0.05\cos(x^2_{3,k}(t)) + 1.3]\cdot u_{3,k}(t), \\ \dot{x}^2_{4,k}(t) = 3\tanh(x^1_{4,k}(t)) + 0.1\sin^2(x^2_{4,k}(t)) + [2+0.1\cos^2(x^2_{4,k}(t))]\cdot u_{4,k}(t). \end{cases} \tag{63}$$

And the dynamic of the leader are $x^1_0(t) = \cos(\pi t)$, $x^2_0(t) = -\pi\sin(\pi t)$, $\dot{x}^2_0(t) = -\pi^2\cos(\pi t)$. Assume that the operation time interval of the multi-agent system is $[0,2]$, while the interval for rectifying the state shifts is $[0,0.5]$, i.e. $T = 2, t_p = 0.5$. The initial states of the follower-agents are described by

$$\begin{cases} x^1_{1,k}(0) = 1.5\,\text{rand}, & x^2_{1,k}(0) = -1\,\text{rand}, \\ x^1_{2,k}(0) = 0.8\,\text{rand}, & x^2_{2,k}(0) = -3\,\text{rand}, \\ x^1_{3,k}(0) = -2\,\text{rand}, & x^2_{3,k}(0) = 2\,\text{rand}, \\ x^1_{4,k}(0) = -0.5\,\text{rand}, & x^2_{4,k}(0) = 5\,\text{rand}, \end{cases} \tag{64}$$

where *rand* generates the random number between 0 and 1.

We choose $c = 8$ in the sliding mode error function, then the parameters $h_{j,k}, p_{j,k}, q_{j,k}$ in the rectify function $r_{j,k}(t)$ can be calculated by using Eqn. (30). The controller gains are selected as $b = 5$, $\gamma_1 = \gamma_2 = \gamma_3 = 10$, and the iteration number is set as 30. The simulation results are shown in Figs. 2–8.

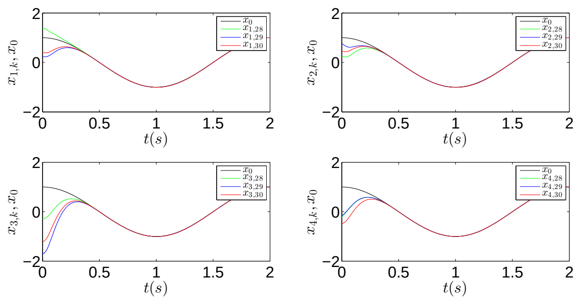

Fig. 3 – The leader's trajectory and all follower-agents' trajectories.

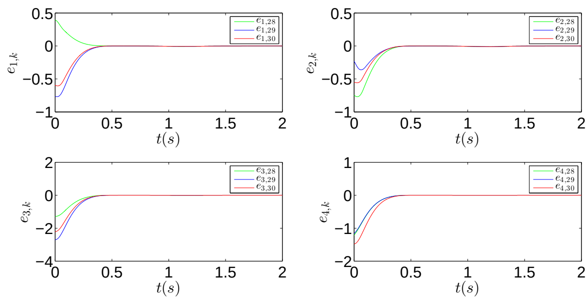

Fig. 4 – Tracking errors.

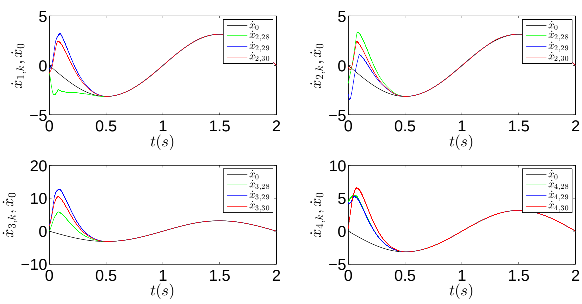

Fig. 5 – The leader's state and all follower-agents' states.

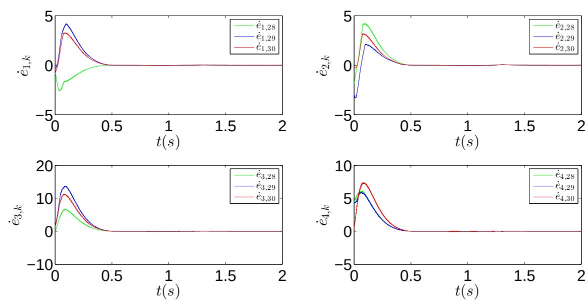

Fig. 6 – State errors.

Figure 2 shows the changes of the consensus errors associated with the four follower-agents at the last iteration. It can be seen that at the initial moment, the consensus errors are obviously not equal to 0, and they change constantly in the interval $[0, 0.5)$. Whereas at time $t_p$, the state shifts have been rectified, so that the consensus errors converge to almost 0. We can get a conclusion that at the 30th iteration, the follower-agents can not only reach consensus with the trajectory of the leader, but also can be agreed with the velocity of the leader.

In Figs. 3 and 5, the black solid lines represent the trajectory and state of the leader , while the red, green and blue solid lines represent the trajectories and states of the four follower-agents at the 28th, 29th and 30th iterations, respectively. The red, green and blue solid lines in Figs. 4 and 6, represent the tracking errors and state errors of the four follower-agents at the 28th, 29th and 30th iterations, respectively. Although there is significant discrepancy between the initial states of each iteration, the four follower-agents can completely track the leader at time $t_p$. These results show that the control law proposed in this paper can rectify arbitrarily state shifts, so that the multi-agent system can realize consensus tracking to the leader in the specified interval.

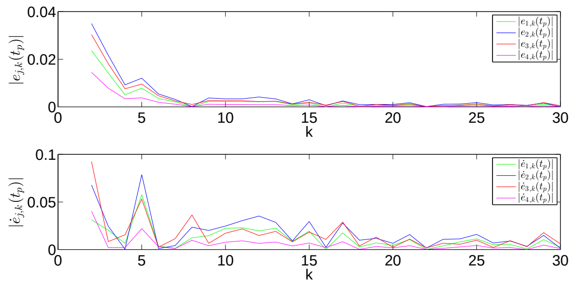

Fig. 7 – Tracking errors and state errors at time $t_p$ .

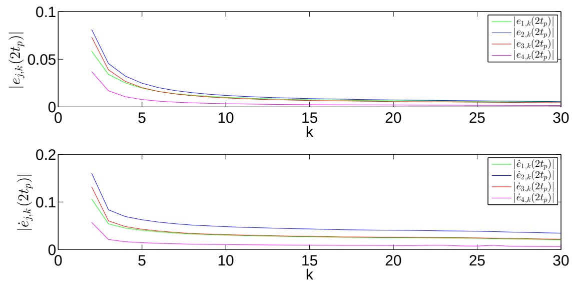

Fig. 8 – Tracking errors and state errors at time $2t_p$.

By observing Figs. 7 and 8, we obtain that the change of tracking errors and state errors between the follower agents and the leader is evident in 30 iterations, at time $t_p$ and $2t_p$, respectively. Because the initial state shifts need to be rectified in the interval $[0, t_p]$, the change at time $t_p$ is relatively volatile, but it is smooth at time $2t_p$. On the whole, tracking errors and state errors become smaller and smaller with the growth of the number of iterations $k$, that is, as $k$ approaches infinity, the tracking errors and state errors of the system eventually tend to 0.

## 6. CONCLUSION

This paper studied the consensus problem of a class of second-order leader-following nonparametric uncertain multi-agent systems with arbitrary initial state shifts. To solve the problem, a first-order attractor controller with the function of rectifying state shifts was proposed. Theoretical analysis showed that the controller could make the multi-agent system track the leader-agent completely in the specified interval. Finally, an example was given to demonstrate the effectiveness of the proposed algorithm.

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